Wireless Sensor Networks for Wastewater Treatment Control
Gains and limitations

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This talk will include the following:

1. Biological wastewater treatment using several DO sensors: reducing energy costs.
2. Reducing sampling time to increase sensor longevity.
3. Iterative Learning Control and Repetitive Control: promising control strategies.
4. Overview of ILC.
5. ILC in a Sequencing Batch Reactor (SBR).
Simulation scenarios

**S10**

- **Input:** 18446 m³/d
- **Output:** 385 m³/d
- **DO**
- **KLa**
- **DOref**

**S1**

- **Input:** 18446 m³/d
- **Output:** 385 m³/d
- **DO**
- **KLa**
- **DOref**

**S2**

- **Input:** 18446 m³/d
- **Output:** 385 m³/d
- **DO**
- **KLa**
- **DOref**

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DO set point optimization

Simulations are executed in a modified version of BSM1, where the aerated volume is divided in 10 totally mixed, equally sized compartments.

- **Purpose**: minimize daily KLa for a given flow-proportional effluent ammonia concentration.
- **Optimize** w.r.t. the $DO_{ref}$ profile. One value for each hour is applied, giving 24 optimization variables.
- **Each simulation runs for 14 days, and the last day is evaluated.**
Results

![Graph showing the relationship between average effluent SNH (mg/l) and average KLa (days⁻¹) for different categories S10, S1, and S2. The graph includes data points for each category, indicating trends and comparisons.](image-url)
Results, cont’d

![Graph showing DO vs. hours for different scenarios](image)

- **S10**
- **S1**
- **S2**

**DO** \(_{\text{ref}}\) \([\text{mg/l}]\)

**Hours:**

- 0
- 5
- 10
- 15
- 20

**DO** \(_{\text{ref}}\) values:

- S10: [4, 3, 2, 1]
- S1: [5, 4, 3, 2]
- S2: [6, 5, 4, 3]
Decreasing the sampling time

Consider now the original BSM1 with three aerated basins, controlled with discrete time PI controllers

\[ u_j(t) = \frac{K}{q} \left(1 + \frac{1}{T(1 - q)}\right) e_j(t), \]

where \( u_j(t) \) is \( KLa \) \([days^{-1}]\) and \( e_j(t) = DO_{ref}(t) - DO(t) \) is the control error at the \( j \):th aerated compartment.

Objective: minimize MSE over all three basins for 14 days, w.r.t. \( K \) and \( T \), for a given sampling time.
Results

![Graph showing MSE, DO vs. sampling time with Best case and Worst case lines]

MSE, DO \( [\text{mg}^2 \text{L}^{-2} \text{days}^{-1}] \)

- **Best case** line
- **Worst case** line

Sampling time \([\text{min}]\)

0 10 20 30 40 50 60

0 0.01 0.02 0.03 0.04 0.05 0.06 0.07

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ILC
Future work

- WSNs are cheap and easy to install (no wiring), which enables several sensors.
- Several sensor measurements can increase plant performance.
- Since WSNs are battery driven, the sampling time should be as low as possible.
- Solution: use ILC/RC to learn the daily input trajectories, then decrease sampling time.
The Arimoto Algorithm

Let \( y_k(t) = G(q)u_k(t) + d(t) \) be a discrete time LTI system, where

- \( t = 0, 1, \ldots, N - 1 \) is the time instants,
- \( N \) is the batch length,
- \( k = 0, 1, \ldots \) is the batch number,
- The disturbance \( d(t) \) is the same for each batch.

Consider the following control law: \( u_{k+1}(t) = u_k(t) + \gamma e_k(t + \delta) \), where

- \( e_k(t) = r(t) - y_k(t) \) is the control error,
- \( r(t) \) is the reference signal (same for each batch),
- \( \gamma > 0 \) and \( \delta \in \mathbb{Z} \) are design parameters.
The Arimoto Algorithm, example

A system $G(q) = \frac{1}{q^5 - 0.5q^4}$ is being controlled by a PI controller.

- $P = 0.1$ and $I = 0.6$,
- $d(t)$ is a step function (same for all $k$),
- ILC: $u_{k+1}^{ILC}(t) = u_k^{ILC}(t) + 0.4e_k(t + 5)$,
- Closed loop: $y_k(t) = G_c(q)u_k^{ILC}(t) + d(t)$.
The Arimoto Algorithm, example cont’d
The Arimoto Algorithm, example cont’d
A commonly used general ILC structure is
\[ u_{k+1}(t) = Q_k(q)(u_k(t) + L_k(q)e_k(t)). \]

- Design problem: Choose the filters \( Q_k(q) \) and \( L_k(q) \).
- \( L_k(q) \) is often based on the model \( \hat{G}(q) \).
- \( Q_k(q) \) must be 1 (or converge to 1 w.r.t. \( k \)) for convergence to zero error.
- Often \( Q_k(q) = Q(q) \) is a zero-phase butterworth filter to achieve smooth input.
Results, asymptotic noise influence

Assume a system with iteration-varying disturbance and measurement noise $z_k = Gu_k + d_k$, $y_k = z_k + n_k$, $e_k = r - z_k$, and the ILC strategy $u_{k+1} = u_k + L(e_k - n_k)$.

- $L = \gamma G^{-1} \Rightarrow \lim_{k \to \infty} E\|e_k\|^2 = 2/(2 - \gamma)E[d^T d] + \gamma/(2 - \gamma)E[n^T n]$ (M. Butcher, PhD thesis 2009).
- $L = \gamma G^T \Rightarrow \lim_{k \to \infty} E\|e_k\|^2 = 2E[d^T (2I - \gamma GG^T)d] + E[n^T \gamma GG^T (2I - \gamma GG^T)^{-1} n]$ (J. Nygren et.al. 2013).
If the disturbance $d_k$ is uncorrelated with $u_k$, we get
\[ E\|e_k\|^2 = e\|r - Gu\|^2 + E\|d\|^2. \]
Hence, the minimum possible variance is achieved if $r = Gu$.

**Theorem:** Consider the ILC law $u_{k+1} = u_k + \gamma_k L (e_k - n_k)$, where $\gamma_k$ is decreasing ($\sum_{j=0}^{\infty} \gamma_j^2 < \infty$) but not too fast ($\sum_{j=0}^{\infty} \gamma_j = \infty$). If $LG + G^TL^T$ is strictly positive definite, then
\[ \lim_{k \to \infty} E\|e_k\|^2 = \lim_{k \to \infty} E\|d\|^2 \] (J. Nygren et.al. 2013).
Optimization-based ILC

Choose \( u_{k+1}(t) \) (\( Q_k(q) \) and \( L_k(q) \)) such that a certain criteria \( J \) is minimized.

- **LQG-type criterion:** \( J = E[e_{k+1}^T e_{k+1} + u_{k+1}^T Su_{k+1}] \) (S. Mishra et.al., ACC 2010).

- **Input update penalty:** \( J = \|e_{k+1}\|_Y^2 + \|u_{k+1} - u_k\|_U^2 \) (N. Amann et.al., IEE Proc. C.T. 1996).

- **Minimize error power spectrum:** \( J = \Phi_e(\omega) \) (D.A. Bristow, ACC 2010).

- **Input/state error covariance matrix:** \( J = \text{Tr} E[MM^T], \) 
  \[ M = [u^*(t) - u_{k+1}(t), x^*(t + 1) - x_k(t + 1)] \) (S.S. Saab, TAC 2001).
Sequencing batch reactor (SBR) process

1. Fill
2. React
3. Settle
4. Withdraw
DO reference trajectory

- $S_O$ reference
- $S_{NH}$ (ammonium nitrogen)
- $S_{NO}$ (nitrate nitrogen)
ILC in SBR

- ASM1 is used for simulations.
- Gaussian measurement noise with st. dev. = 0.1 mg/l.
- First order model for ILC: $G(q) = \frac{b}{q-a}$.
- ILC algorithm: $u_{k+1} = u_k + \gamma L e_k$.
- $L = G^T$ (ABILC) or $L = I$ (Arimoto, MFPIILC).
- $\gamma$ was chosen to minimize MSE at the 4’th iteration ($k = 3$).
- Compared with a PI strategy with optimized parameters (w.r.t. MSE).
Arimoto algorithm (MFPIILC)
Adjoint-based algorithm (ABILC)
MSE for $a = 0.98$
MSE vs. pole placement

\[
\text{MSE} = \left( \text{mg O}_2 \text{l}^{-1} \right)^2
\]

- ABILC, 4th
- MFPILC, 4th
- ABILC, 10th
- MFPILC, 10th
- ABILC, 100th
- MFPILC, 100th

pole location
Decreasing step size

Now, $\gamma_k = \frac{\gamma_0}{k+1}$, where $\gamma_0$ minimizes MSE at the 4’th iteration.