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Urban Water Management - Modelling, Simulation and Control of the Activated Sludge Process

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Abstract

During the last few decades, wastewater treatment processes in urban water management have become more and more efficient and complex. Several factors such as urbanization, stricter legislations on effluent quality, economics, increased knowledge of the involved biological, chemical and physical processes as well as technical achievements have been important incentives for the development of more efficient procedures for wastewater treatment plants. Future requirements on more sustainable urban water systems, in combination with increasing wastewater loads, will most probably further increase the need for optimization and control of both existing and emerging wastewater treatment processes.

In this thesis estimation, modeling and control strategies are developed in order to improve the efficiency of the activated sludge process.

The first part of the thesis presents a JAVA based simulator of the activated sludge process. An overview of its features, with some emphasis on implemented control strategies, is given. In particular, a new control strategy for the internal recycling flow rate is described. A summary of experiences from using the simulator as a teaching and training tool is also given.

The second part of the thesis includes a derivation of reduced order models for the activated sludge process. The resulting models, a time-varying linear state-space model for the anoxic part and a time-varying bilinear state-space model for the aerobic part, are intended to be used for control applications.

In the third part, an adaptive control strategy for control of the nitrate level using an external carbon source is presented. The controller adapts and compensates for changes in the system dynamics since important system parameters are estimated adaptively and incorporated on-line in the control design. The estimated system parameters and model states also give guidance about the state of the process and the characteristics of the wastewater. Several simulation examples are used to illustrate the control law.

Finally, a new suboptimal control law for the bilinear quadratic regulator problem with infinite final time is presented. The control strategy is evaluated in a simulation study, where special concern is devoted to controlling the activated sludge process, using the bilinear model developed in the second part of this thesis.

List of publications

Chapter 1 is partly based on:

Ekman, M. and B. Carlsson (2001). Informationssystem inom VA-sektorn - En förstudie för projekt Lokalt reningsverk för Hammarby Sjöstad, etapp 1. *Technical report*, No. 32. Stockholm Vatten (In Swedish).

Chapter 2 is based on:

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Samuelsson, P., M. Ekman and B. Carlsson (2001). A JAVA based simulator of activated sludge processes. *Journal of Mathematics and Computers in Simulation*, **56**(4–5), 333–346.

Chapter 3, 4 and 5 is based on:

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M. Ekman (2003). Suboptimal control for the bilinear quadratic regulator problem. Technical Report. Div. of Systems and Control, Uppsala University. Uppsala, Sweden. (To appear)

M. Ekman (2003). Suboptimal control for the bilinear quadratic regulator problem: Application to the activated sludge process. *IEEE Transactions on Control Systems Technology*, (Submitted).

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Contents

Chapter 1

Introduction

During the last few decades, treatment of wastewater has become more and more important in the society. Several factors, such as urbanization, tighter effluent standards, economics, more knowledge on the involved biological, chemical and physical processes and technical achievements have resulted in more complex and efficient treatment processes. The rapid development taking place in areas like information technology, computer science and sensor technology has also influenced the operation and design of wastewater treatment plants (WWTP's) and other urban water systems. Future demands on more sustainable urban water systems will most probably increase the demand for information handling. Adequate information systems, including computer systems for automation and control, have a great role to play in order to obtain a more durable operation in terms of resources, energy, economy and hygienical and technical risks.

In Section 1.1 of this chapter some of the major motivations behind this thesis are presented. The objectives and contributions of this thesis are presented in Section 1.2. Section 1.3 includes a brief overview of current exploitation and future scenarios for information systems in urban water systems. An overview of control strategies, models and simulators for wastewater treatment systems is presented in Section 1.4. Finally, an outline of the chapters to follow is given in Section 1.5.

1.1 Motivation

The lack of advanced information systems in many parts of the urban water system of today contrasts to other areas such as paper and pulp and power industries. Advanced information systems could certainly play a central role, when developing new or modified systems in the future, where also householders and industries may take a more active part.

The urban water system of today is characterized by a large and complex infrastructure. One problem of this infrastructure is a lack of integration between the involved sub-systems. Integrated information systems is an important pre requisite for a more reliable operation. This applies also for drinking water systems, stormwater systems and households. Furthermore, information systems can be used to monitor discharge from individuals or groups of households and also give information to users, which may change their behavior. It is of great importance to achieve a safer and more sustainable operation in future centralized or decentralized water systems.

Some of the incentives and challenges that motivates the development and research of information systems for urban water systems are listed below, see also (Olsson and Newell 1999):

- How much savings of resources in terms of energy, chemicals and other operation costs can be achieved by a more effective use of information?

- Is it possible to obtain a more robust and less resource consuming operation by coordination between different subsystems?

- How can new information systems, including computer systems for automation and control, improve and simplify the operation?

- Is it possible to obtain urban water systems that are more adaptable to future changes by using effective information systems?

- How can information to the householders affect the attitudes and behavior of the users and thereby decrease the quantity of pollutions in the wastewater?

1.2 Objectives and Contributions of the Thesis

1.2.1 Objectives

The objectives of this thesis include:

- Provision of mathematical models and simulators which will be useful for education purposes, increase the knowledge about different process configurations, control structures and facilitate the operation of urban water systems.

- Methodology development and evaluation of information systems which may be suitable for an efficient operation, supervision and control of current and future urban water systems. Important disciplines include optimization theory, modeling, simulation, signal processing and control.

In a broader perspective, the objective of this thesis is to develop information systems for urban water systems which contribute to a sustainable and robust operation.

1.2.2 Contributions

The use of dynamic models, model calibration, estimation, simulation and control is central in this work. The major contributions include:

- A presentation of a JAVA based simulator for the activated sludge process. The simulator is aimed to be suitable for education and training purposes.

- The development of reduced order models for the anoxic and the aerobic compartments in an activated sludge process (ASP), aimed for control applications.

- An investigation of the identifiability properties, together with development of on-line methods for state and parameter estimation, of the model covering the anoxic part of the process, is performed.

- Model based and adaptive control strategies, utilizing simplified and reduced order models of the ASP, are suggested. In particular, the bilinear quadratic regulator problem with infinite final time is concerned.

1.3 Information Systems for Urban Water Systems

A general information system can be defined as the integrating procedure which is applied for collecting, storing, analyzing and processing measurable process parameters. The purpose is to utilize these parameters in a computer system so that the process can be controlled towards optimal production results. For a WWTP this will typically involve to achieve a good effluent water quality while the consumption of resources is kept as low as possible and to minimize the influence of external disturbances. An information system contains several components including networks, communication, instrumentation and monitoring as well as data collection, data storing, presentation, interfaces, detection, warning systems, signal processing, models, controllers, software sensors and hardware sensors.

1.3.1 Present Utilization of Information Systems in Urban Water Systems

Using the stream of information for control purposes requires that data is treated in an intelligent way in future information systems and that different helping tools are utilized maximally. However, this may contrast to todays utilization of implemented IT-tools at many WWTP's. In many aspects, the handling of wastewater systems are very conservative, as far as effective utilization of information technology and control systems is concerned. Security is rather looked for in form of using old solutions, see (Lundkvist 1999). An interesting and in some aspects very critical description on how IT-tools so far have been used in wastewater systems is given in (Alm 2000).

There is, however, a recent strong interest among operators and researchers to find IT based solutions. There are also examples of effective integration and utilization of information systems in water systems. (Pleau *et al.* 2000) give an example of how the volume of flooding can be reduced by using a control strategy where plant measurements and meteorological predictions are updated periodically. In order to avoid flooding and hydraulic overload, a control strategy has been implemented in Rya WWTP in Göteborg (Sweden). The control is based on hydrological and hydrodynamical models, see (Lumley 1996). Internet has in recent years become a more and more important tool for data acquisition. Some data acquisition and analysis softwares, for example the software Waste can be reached by Internet, see (Olsson *et al.* 1998). Progress have also been achieved in urban storm management where system analysis methods have been used, see (Loke *et al.* 1997) and (Arnbjerg-Nielsen *et al.* 1998).

A comprehensive compilation of the development within operation and control of WWTP's in Scandinavia during the last 20 years, is given in (Olsson *et al.* 1998). An overview of the present status and future trends of Instrumentation, Control and Automation (ICA) is presented in (Jeppsson 2001). One conclusion which can be drawn is that the level of ICA differs significantly between different countries, but most of the larger WWTP's in Europe are equipped with a SCADA (Supervisory Control And Data Acquisition) system, although these are generally more used for data acquisition than for control. Another conclusion is that there exists a significant need for objective evaluation of different control strategies and more need for an integrated and plant-wide control.

1.3.2 Future Integrated Information System for Urban Water Systems

Information systems have traditionally acted as the intermediary structure between operators and the process. In the future, the information from households and industries will probably be more important than today for the operation and optimization of the water treatment. The information system may also be forced to handle an increasing stream of information from e.g. new sensors.

The specific properties of urban water systems infer special requirements on the installed information systems. Operational staff can not be available 24 hours per day. Therefore, when a computer system for automation and control is constructed, distributed systems with redundancy is often demanded. Often it is also necessary to divide the control objects into several smaller units, so that only a part of the system is affected if the system crashes. Water systems are often spread out over a large geographic territory. This infer special requirements on the communication capability. Sub-control systems have to work even if the communication with the central processor is broken.

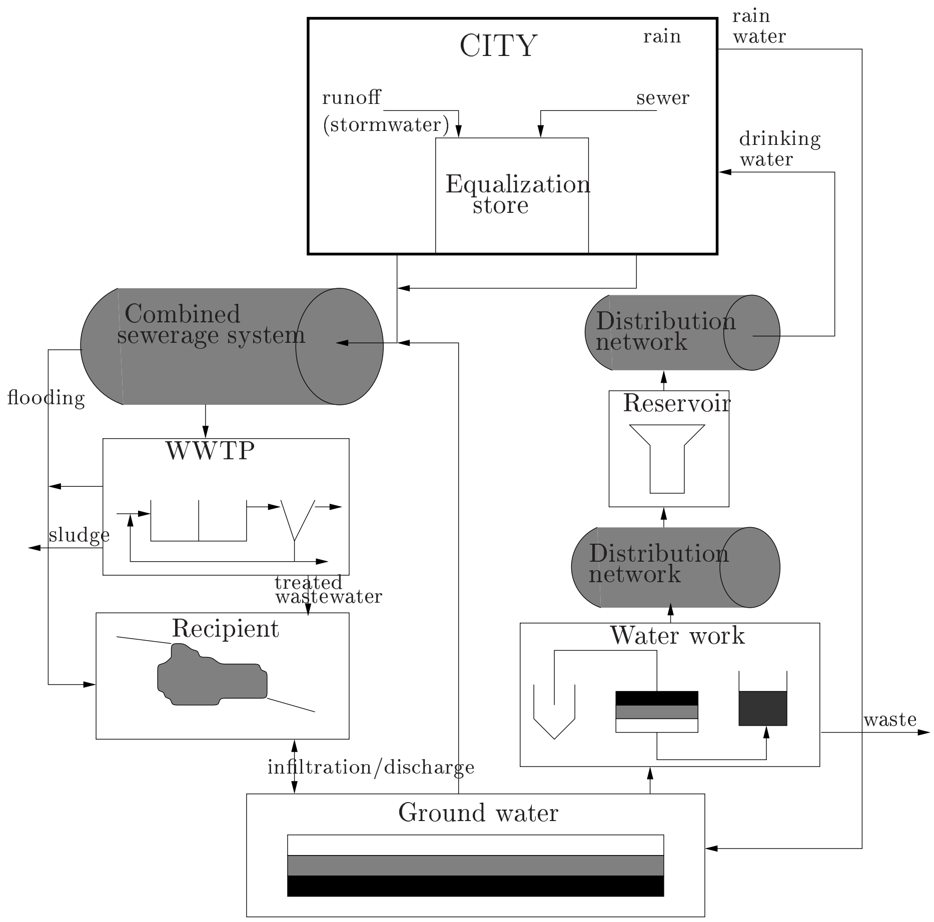

Figure 1.1: Interactions between urban water subsystems which form the integrated urban water system.

Today, urban water systems are often large, geographically distributed systems containing several subsystems and subprocesses with several more or less strong interactions, this is illustrated in Figure 1.1. A technical challenge is to co-ordinate and integrate many subsystems to achieve a more effective and co-ordinated operation. There exists a need to develop information techniques together with control strategies and sensors to accomplish this, see (Olsson and Newell 1999). An effective information system may also facilitate the possibility to adapt to future changes of the urban water system. A future scenario will probably involve that important information from different subsystems is even more integrated in the information system.

Some examples of applications where information systems may be useful are given below:

- Integrated control where information from, for instance, the sewerage system is used for early warning systems and feedforward strategies, see e.g. (Lumley 2001).
- Forecasting the water flow in sewerage networks using meteorological data, see for instance (Pleau *et al.* 2000) and (Lumley 1996).
- Data acquisition and signal processing. Utilizing new measurements both on- and off-line, which will give an increasing amount of information.
- Disturbance detection and monitoring. This includes for example, discovering different process changes or the status of a pump. Fault detection is also the backbone for early warning systems. (Rosen 2001) gives examples of early warning and monitoring systems applied to wastewater treatment where multivariate methods are used.
- Indirect control of households. As mentioned, relevant information about pollutions in the wastewater may affect the attitude and behavior of the users.

The design of future information systems and what system configurations will be used, is very much dependent on the technological development within computer and network communication. A critical factor is also to what extent the new technology will be used within the water system society. The specific properties of urban water systems which infer special requirements on the information systems give a hint on how the systems will be developed in the future. Intranet/Internet will probably be the dominating platform for the networks.

Examples of how present Internet technology may be used for monitoring, supervision and control is given in (Cianchi *et al.* 2000) and (Fatta *et al.* 2000). Interesting thoughts about future urban water system can be found in (Olsson and Newell 1999).

1.4 Modeling and Control of Wastewater Treatment Systems

Today, there exist several different mathematical models covering different parts of the urban water system. In this section an introduction to modeling parts of wastewater treatment systems is given. In particular, mathematical models describing the activated sludge process and the settler are discussed.

Mathematical modeling is fundamental in wastewater treatment design. A model may be a tool for answering questions about the process without performing practical experiments. When using simple models it is possible to analytically investigate the behavior of the model, which may give some insight about the process. For more complicated models, analytical methods are often not possible to use, but still the model may give useful information by numerical experiments, or in other words, simulations.

A model may be used in many different ways; some typical general applications include:

- Forecasting. It may include to predict future performance of the plant, for example after a rain storm.
- Design of the process. Instead of using simple "rule of thumb", simulation studies using dynamic models may be advantageous.
- Fault detection and monitoring. A model can be used for early detection of, for instance, sensor failures.
- Design and analysis of control systems. In order to design and analyze a control strategy it is often necessary to use some kind of model for the

process, since it is often too expensive, dangerous or time-consuming to perform practical experiments on a real process.

- Education. Simulators can be used for education and training.
- Research. A great part of the natural and technical science includes development and testing of models (hypotheses).

1.4.1 Mathematical Models for ASP's

A common concept for biological wastewater treatment is the ASP. The ASP is a biological process in which microorganisms oxidize and mineralize organic matter. A basic description of the ASP is given in Appendix A.1. The possibilities of utilizing mathematical models for several important applications have also been a driving force in the development of models describing the ASP. Today, there exist several models describing the ASP. The most widely used is the IAWQ activated sludge model No. 1 (ASM1), see (Henze *et al.* 1987). Three important processes described by the model are: removal of organic matter, nitrification and denitrification. The model contains 13 different components and the behavior of each component is described by a nonlinear differential equation. Besides pure biomass growth, ASM1 also contains other processes such as microbial decay, hydrolysis and ammonification. A description of ASM1 is given in Appendix B.1. Other more recent models for the ASP are ASM2, which also deals with biological phosphorus removal, and ASM3, which is a modified ASM1. Today there also exist models corresponding to ASM1 for anaerobic processes (ADM1), see (Batstone *et al.* 2002).

1.4.2 Modeling the Settling Process

Sedimentation (settling) is commonly used in wastewater treatment systems. The sedimentation of particles occurs due to the gravity force from the differences in density between the particles and the fluid. A successful sedimentation is very important for the overall efficiency of a plant and this process is often the weak link in many WWTP's. The sludge in the bottom of the settler contains high concentrations of the microorganisms necessary for the biological processes and is therefore recirculated back to the bioreactor, see Figure A.1 in Appendix A.1.

There exist several different settler models, see (Grijspeerdt *et al.* 1995) for a comparative study between different settler models. See also (Jeppsson 1996) for a thorough evaluation of different settler models with respect to consistency and robustness. A settler model, which has been proved to be reliable, is the model by (Takács *et al.* 1991). This model is briefly described in Appendix A.2.

1.4.3 Simulators

The main problem in all kind of simulations is to find a suitable model and to calibrate the model. This is also very valid for models describing ASP's. Typical tasks for the users are how to:

- Characterize influent water. It may also be important to have knowledge about the dynamic changes in the influent water characteristics.

- Calibrate the model for a specific WWTP. This is often a very time consuming process, where different laboratory experiments have to be combined with the calibration of model parameters.

The purpose of the simulation decides how careful one has to be in the steps described above. If the purpose requires a very accurate model, then the model has to be thoroughly calibrated and validated.

Several models and simulators have been developed the last decades. A comprehensive overview of different commercial simulators for ASP's can be found in (Olsson and Newell 1999). A list of some simulators for WWTP's is given in Table 1.1. One example where EFOR and GPS-X have been

Table 1.1: A list of some simulators for WWTP's.

Simulator	Company	Web address
GPS-X	Hydromantis	www.hydromantis.com
Simba	IFAK	www.ifak.fgh.de/regelung/
Efor	DHI	www.efor.dk/efor/
West	Hemmis	www.hemmis.com/product/west
JASS	Uppsala Univ.	www.syscon.uu.se/JASS

successfully used as a simulation tool can be found in (Lumley *et al.* 1994). A JAVA based simulator, which can be run via Internet is presented in Chapter 2.

1.4.4 Characteristics and Operational Objectives for Wastewater Treatment Plants

A strong motivation for an effective control of WWTP's is to achieve a more sustainable and robust operation with respect to use of chemicals and energy consumption. This, in turn, will lead to considerable economical benefits. Development towards tighter effluent demands and more complex WWTP's will probably intensify the focus on more automation of the treatment process in the future.

Some properties and attributes of biological wastewater treatment, which significantly affect the design and implementation of automatic control, are given below, see further (Lindberg 1997) and (Olsson and Newell 1999):

- The process has significant nonlinearities.
- Strong interactions between different process variables.
- Strong external couplings, from sewers to the process.
- The microorganisms change their behavior and their distributions.
- The settling, which is very sensitive for hydraulic disturbances.

The main objective for controlling the wastewater treatment process is to optimize the operation in order to minimize expenses and keep the effluent concentrations at a low level, despite disturbances affecting the process. The control of the process has to fulfill several operational goals in order to handle the main objectives of the wastewater treatment. Some examples of general operational objectives for WWTP's are given below (Olsson and Newell 1999):

- To grow the right biomass population.
- To maintain good mixing where appropriate.
- To keep adequate dissolved oxygen (DO) concentration.

- To keep adequate air flow.
- To achieve good settling properties.
- To avoid settler overload.
- To avoid denitrification in the settler.

1.4.5 An Overview of Control Strategies for Wastewater Treatment Systems

The control of the wastewater treatment processes in present WWTP's can be divided into three main field or supervisory goals:

- Control of organic removal.
- Control of phosphorus removal.
- Control of nitrogen removal.

In a biological reactor with nitrogen and phosphorus reduction competition between the three objectives given above may exist. Conflicts may appear if one, for example, tries to fulfill several simultaneous reactions and requirements.

Some common control strategies today in WWTP's are:

- Manual control. The control signal is changed by the operator.
- Feedforward control. The control signal is controlled by measurable disturbances. A common example is dosage of precipitation chemicals based on influent flow (flow proportional control).
- Time control. Typically the control signal has one value during the day and another during the night.

A question that arises is why the control strategies above not always are good enough? The main problem is that the methods above is not taking advantage of feedback from the control output. Consequently, there is no guarantee that the process output is close to the desired value. Control strategies which are not using feedback are called open control strategies. Using open-loop control implies, for example, a considerable risk to add too much precipitation chemicals, which may result in increasing operational costs and chemical discharges. However, one advantage with open-loop control is that there is often no need for sensors in the controlled process.

A number of possible control strategies and fields of research, mainly for ASP's, will be outlined below. However, similar and interesting control problems and control strategies also exist for other wastewater treatment processes and water systems. There are also many different flowsheet configurations which result in different control problems.

- Control of the DO (Dissolved Oxygen) concentration. It is important to have a satisfactory concentration of DO in order to maintain the growth of nitrifying bacteria. The DO concentration is controlled by the air flow rate, and the dynamics is relatively fast. DO control has been practiced for many years, and several variants exist. One of the first computer controlled DO controllers was implemented in Käppala (Sweden) WWTP (Olsson and Hansson 1976). The DO control was improved in the same WWTP at Käppala by implementing a self-tuning DO controller (Olsson *et al.* 1985). See also (Lindberg and Carlsson 1996*b*) for nonlinear control of the DO. An approach using auto tuning is found in (Carlsson *et al.* 1994).

- Supervisory control of the DO. The ammonium concentration can be controlled by manipulating the DO set-point. Examples can be found in e.g. (Lindberg and Carlsson 1996*b*), (Lindberg 1998), (Suescun *et al.* 2001), (Galarza *et al.* 2001), (Serralta *et al.* 2001) and (Ingildsen 2002).

- Control of external carbon source. An addition of external carbon source is commonly used in order to control the denitrification. Chapter 4 and (Ekman *et al.* 2003) present such a control strategy. Examples of carbon dosage control can further be found in (Hellström and Bosander 1990), (Isaacs *et al.* 1993), (Hallin *et al.* 1996), (Yuan *et al.* 1997) (Lindberg and Carlsson 1996*a*), (Barros and Carlsson 1997), (de Arruda and Barros 2001), (Carlsson and Milocco 2001) and (Samuelsson and Carlsson 2001).

- Control of the nitrification by manipulations of the aeration volume.

See for instance (Samuelsson and Carlsson 2002), (Krause *et al.* 2001) and (Brouwer *et al.* 1998).

- Control of hydraulic variables. Examples of different hydraulic variables that can be controlled in an ASP are the internal recirculation flow rate, the sludge recycling flow rate and the excess sludge flow rate.

  The quantity of sludge in the system can be controlled by manipulating for example the excess sludge flow rate.(Rhenström and Carlsson 2001) describes a control strategy where the excess sludge flow rate is used for controlling the sludge level in a secondary clarifier. Automatic control of the excess sludge flow rate is also described in (Ekster 2001) and (Vrecko *et al.* 2002*b*).

  The sludge blanket height (SBH) can be controlled by using the sludge recycling flow rate as a control signal. (Yuan *et al.* 2001) showed that the SBH should be controlled dynamically in order to optimize the nitrification and denitrification during different load situations.

  The recirculation flow of nitrate in a predenitrifying system is often a fast process. This implies that the nitrate concentration in the incoming water to the anoxic compartments may be changed rapidly by manipulating the internal recirculation flow rate. A control strategy where the internal recirculation flow rate is used as a control signal is presented in Chapter 2, (Ekman and Samuelsson 2001), (Rhenström and Carlsson 2001) and (Vrecko *et al.* 2002*a*). Other examples of internal recirculation flow rate control can be found in (Vrecko *et al.* 2002*b*) and (Ingildsen 2002).

- Control of the addition of precipitation chemicals. Precipitation chemicals are commonly used for phosphorous removal. See for example (Hellström *et al.* 1984), (Devisscher *et al.* 2001) and (Ingildsen 2002). Control of biological phosphorus removal can be found in (Munk-Nielsen *et al.* 2001).

- Integrated control. Wastewater treatment systems are often exposed to varying disturbances due to weather and unintentional discharges from industries and householders. Characteristic for urban water systems is the strong interactions that exist between different parts of the urban water flows. It is therefore desirable to have some form of integrated control, where information from the different parts of the urban water system is utilized to optimize the operation.

Examples of integrated control is to level out the influent flow to WWTP's by using the sewer network as a buffer volume. Experiences exist for instance from Sweden in Malmö (Aspegren *et al.* 1996) and Göteborg (Lumley 1996). Information from e.g. hydrological and meteorological models may be used in order to forecast the flows of the incoming water to WWTP's. The European project SMAC (SMArt Control of wastewater systems) is focusing on grey-box models in order to discover relationships between flows in sewers and processes in water systems for integrated control, see further (Thornberg *et al.* 2001).

- Control of anaerobic processes. Anaerobic fermentation is a relatively common treatment process in wastewater systems, for example in anaerobic digestion of sludge. Today there exist many examples of modeling, optimization and control of anaerobic processes in the literature. In (Polito-Bragga *et al.* 2001) a control strategy has been developed for a combined wastewater treatment process based on a UASB-reactor followed by an activated sludge system. Implementing an analysis of several advanced control strategies for anaerobic digestion processes has been done in (Harmand and Steyer 2001). An adaptive linearized fuzzy-controller has been implemented in a pilot plant and is described in (Bernard *et al.* 2000). (Steyer *et al.* 2001) discusses the advantages of using advanced instrumentation and implementation of advanced information systems applied on anaerobic processes.

- Multivariable and model based control. The last years several model based and multivariable approaches for controlling wastewater treatment systems have been suggested in the literature. Model based control strategies are proposed in, for example, (Bastin and Dochain 1990), (Lindberg 1997), (Lukasse 1999) and (Weijers 2000). A multivariable approach can be found in Chapter 5. See further e.g. (Lindberg 1998), (García-Sanz and Ostolaza 1998) and (Weijers *et al.* 1997) for other multivariable control strategies. An overview on the use of models in control systems of WWTP's is given in (Vanrolleghem 2003).

1.5 Outline of the Thesis

In this section, a brief summary of the four remaining chapters is presented.

A JAVA based simulator denoted JASS (a Java based Activated Sludge process Simulator), is presented in Chapter 2. The simulator is implemented in the programming language JAVA and can hence be run using an ordinary Internet browser. The simulator is easy to interpret and run. This is believed to make the simulator special suited for education and training purposes. Special concern has been devoted in deriving a control strategy based on internal recycling for control of the nitrate level. Some other implemented control strategies, including dissolved oxygen (DO) control, supervisory DO control and control of external carbon dosage, are also briefly presented. The implemented automatic control strategies may be a tool for promoting the benefits of using automatic control in wastewater treatment plants.

In Chapter 3, the development of reduced order models for the anoxic and the aerobic compartments in an ASP is presented. The resulting models, a time-varying linear state-space model for the anoxic part and a time-varying bilinear state-space model for the aerobic part, are aimed to be suitable for control applications. An analysis of the identifiability properties is performed for the model of the anoxic part of the process.

An evaluation of an adaptive control strategy for controlling the nitrate level using an external carbon source, is presented in Chapter 4. The strategy is based on the derived reduced order model. The controller adapt and compensate for changes in the system dynamics since important system parameters are estimated adaptively. The estimated system parameters and model states also give guidance about the state of the process and the characteristics of the wastewater.

Chapter 5 presents the development of a suboptimal control law of the bilinear quadratic regulator problem with infinite final time. Special emphasize is devoted to illustrate the performance of the derived control law applied to the activated sludge process.

Chapter 2

Control Strategies in a JAVA Based Activated Sludge Process Simulator

Modeling and simulation studies are becoming increasingly common in the wastewater treatment area. Typical application areas include process design, controller design, optimization, forecasting, and research. Several simulators and models of the activated sludge process (ASP) have been developed during the last decades. A good overview of different simulators is given in (Olsson and Newell 1999). See also Table 1.1 in Chapter 1 for homepages of some popular simulation softwares. It is fair to conclude that most simulators have not been used very much by plant personnel. There are several reasons for this, but one major cause is that many simulators so far have not been very user friendly. In this chapter, a simulator denoted JASS (a Java based Activated Sludge process Simulator), which is believed to be more user friendly than most existing simulators, is described. Some of its features are summarized below.

1. The simulator is written in the programming language JAVA and can hence be run using an ordinary Internet browser such as Netscape Communicator or Microsoft Explorer.

2. The simulator is easy to interpret and run. For example, to change a flow rate, the user needs just to click on the corresponding pump symbol, and enter the desired flow rate in a dialog box.

3. A novel graphical user interface (GUI) is used to present process information in a comprehensive way.

4. Special concern has been devoted to implement automatic control strategies including dissolved oxygen (DO) control, supervisory DO control, control of external carbon dosage and nitrate control based on internal recycling.

Items 1-3 are believed to make the simulator closed to ideal for education and training purposes. With the increased use of the Internet, the simulator may be used also for distance education. Item 4 may be a tool to promote the benefits of using automatic control in wastewater treatment plants.

The JASS simulator was first developed by (Samuelsson 1998) and an updated version is described in (Samuelsson *et al.* 2001). A new version of JASS, denoted JASS v. 3.0, has recently been implemented and can be found at the URL address: *www.syscon.uu.se/JASS/*. This newer version of JASS has been developed in a Master thesis work, see (Grusell 2002), with the main purpose of improving the graphical user interface and increase the flexibility of the program.

In this chapter we give an overview of the JASS simulator with some emphasis on implemented control strategies. In particular, a new controller for the internal recycling flow rate is described. A summary of teaching experiences using the simulator is also given.

The outline of this chapter is as follows: In Section 2.1 an overview of the simulator in terms of underlying model and implementation is discussed. Section 2.2 presents the implemented control strategies. Special concern is devoted on the derivation of the internal recycling flow rate controller. JASS as a teaching and training tool is presented in Section 2.3. Conclusions are given in Section 2.4

2.1 Overview of the Simulator

The underlying process model used in the simulator is the IAWQ activated sludge model No. 1 (ASM1), see (Henze *et al.* 1987) and Appendix B.1 for a general model description. ASM1 is probably the most used advanced model for the activated sludge process. Three different processes are described by the model: removal of organic matter, nitrification and denitrification. The model contains 13 different components and the behavior of each component is described by a nonlinear differential equation. The settler unit is modeled as a traditional one-dimensional layer model given by (Takács *et al.* 1991), see also Appendix A.2.

In the default version of the simulator, an activated sludge process with pre-denitrification is implemented. The implemented process consists of a basin divided into 10 completely mixed compartments, each modeled by ASM1, and a settler. The number of compartments is fixed. The settler is modeled with 10 vertical layers. The configuration is depicted in Figure 2.1.All differential equations are solved with a fourth order Runge-Kutta method with time step 0.01 h. Since the simulated ASP is divided into ten completely mixed compartments, the use of an object oriented language, such as JAVA is natural. Each compartment is implemented as an instance of a class, which makes the structure of the program simple. The settler is an instance of another class. Another advantage with JAVA is its support for multithreading and graphics. Two threads are implemented in the program, one for handling events and one for running the simulation. For a more detailed information on the program implementation see (Samuelsson 1998) for the older version of JASS and (Grusell 2002) for the newer version JASSv.3.0.

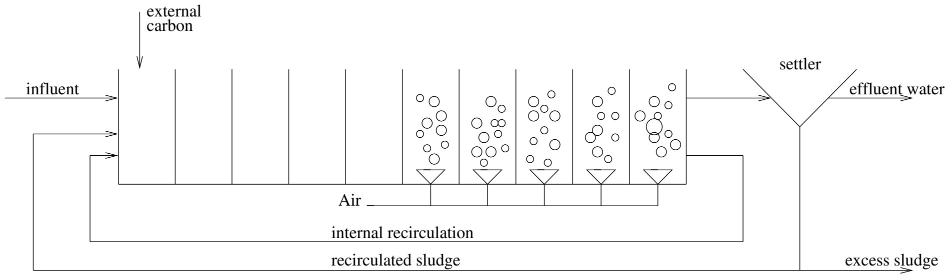

Figure 2.1: Schematic layout of the activated sludge process in the simulator.

The simulator is operated from a GUI where also the status of the plant is shown in real time, see Figure 2.2. In the lower part of the GUI, the layout of the simulated plant is shown. The aim of this picture is to illustrate

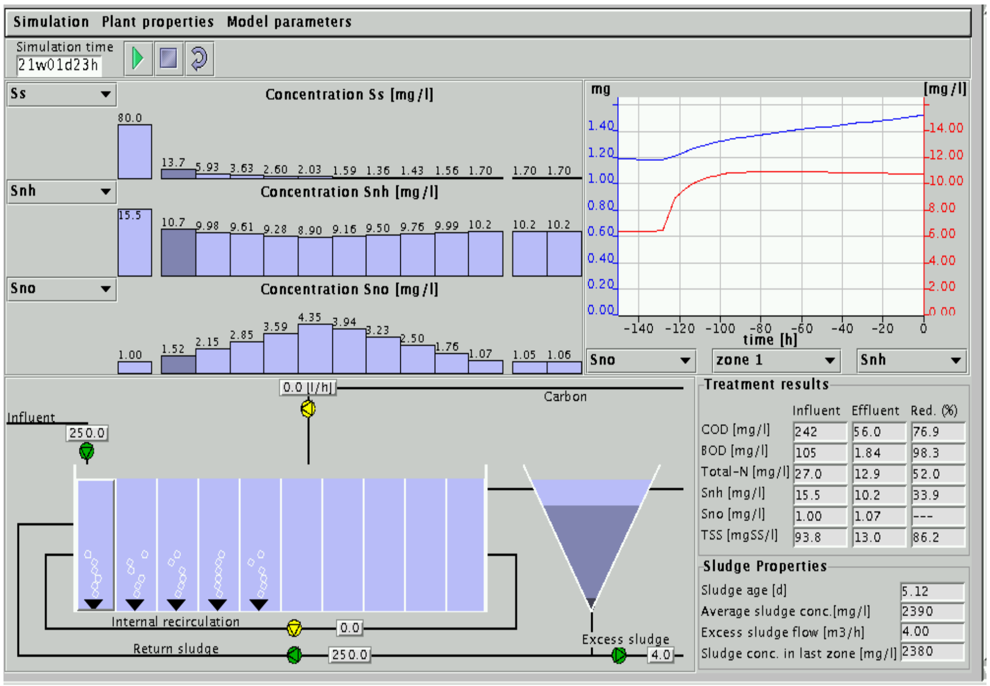

Figure 2.2: The graphical user interface in the simulator.

some fundamental settings of the model, for instance which compartments that are aerated and active pumps. The picture also contains clickable text fields. When the cursor is in a text field and the mouse is clicked, new dialog windows, either used for presenting data or changing process parameters appear. For instance, the simulation is controlled with the five buttons in the *Simulation* control panel located in the upper left corner of the GUI. The buttons are: *start*, *pause*, *stop*, *load* and *save*. Most of the functions in the simulator are very easy to learn and use. For more information on how to run the simulator see (Grusell 2002)

To improve the presentation of simulated process components, three bar diagrams are used. Each bar diagram displays the concentration of a selectable component in influent water, all compartments, the recirculated sludge and the effluent water. Numerical values of the concentrations are also demonstrated. In the GUI an ordinary graph is also used. Here, the concentrations of two components in a selectable compartment can be plotted versus time.

2.2 Automatic Control Strategies

In the current version of the simulator, four control strategies are implemented:

- DO control. The DO is controlled by the air flow rate with a standard PID controller which can be tuned from the GUI.

- Supervisory DO control. In the simulator, a supervisory DO controller has been implemented. Given a desired ammonium level and measurements of the ammonium concentration in the last aerated compartment, a supervisory controller calculates the set-point for the DO controller presented above. If the ammonium set-point concentration is chosen properly, the controller may give a lower average DO set-point, compared to a constant DO, which saves energy. Other advantages may be a better control of effluent ammonium, lower nitrate concentration in the effluent because of improved denitrification (due to a lower average DO) and a decreased need of external carbon (Lindberg 1997). A standard PID controller which can be tuned from the GUI is implemented.

- External carbon flow rate control. The flow rate of external carbon is used to control the nitrate level in the last anoxic compartment. A combined feedforward-feedback strategy based on a simple mass balance derived from a simplified model of the ASM1 is applied. The controller is described in (Samuelsson and Carlsson 2001).

- Internal recycling control of the nitrate concentration in the last aerobic compartment. This controller is described below.

2.2.1 Control of Internal Recycling Flow Rate

In a pre-denitrifying system the denitrification process occurs in the anoxic compartments at the beginning of the ASP and the nitrification in the aerobic compartments at the end of the ASP. The nitrogen rich water in the last aerobic compartment is consequently necessarily recirculated back to the first anoxic compartment. A reasonable control strategy for controlling the nitrate concentration in the last aerobic compartment is to adjust the internal recycling flow rate so that the nitrate level is kept close to a given set-point, see for example (Galarza *et al.* 2000). We will derive a feedforward controller based on a simplified mass balance. The feedforward controller is then combined with a feedback controller.

Derivation of the controller

In order to derive the simple feedforward mass balance part of the controller, he following assumptions are used:

A2.1. No other biological reactions than denitrification and nitrification are affecting the dynamics of the nitrogen components.

A2.2. Nitrification only occurs in aerated compartments, and denitrification only in the anoxic compartments.

A2.3. No biological reactions occur in the settler.

A2.4. The influent nitrate concentration is zero..

The feedforward controller is based on a steady state mass balance for ammonium and nitrate in the anoxic and aerobic compartments. The parameters used in the derivation are illustrated in Figure 2.3.

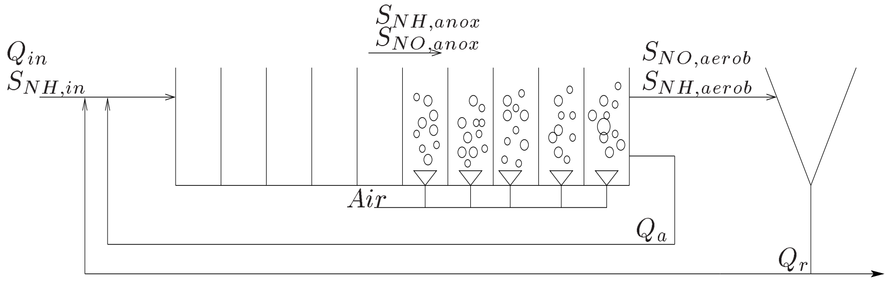

Figure 2.3: Parameters used in the derivation of the feedforward controller. Q_{in} is the influent flow rate, Q_r the recirculated sludge flow rate and Q_a the internal recycling flow rate.

Using assumptions (**A2.1**)-(**A2.4**), the mass balance for ammonium in the anoxic and the aerobic compartments is given by:

$$Q_{in}S_{NH,in} + (Q_a + Q_r)S_{NH,aerob} = (Q_{in} + Q_a + Q_r)S_{NH,anox} \quad (2.1)$$

$$(Q_{in} + Q_a + Q_r)S_{NH,anox} = (Q_{in} + Q_a + Q_r)S_{NH,aerob} + R_{NIT} \quad (2.2)$$

where R_{NIT} denotes the reaction rate of the nitrification process and $S_{NH,in}$ is the influent ammonium concentration. The subscript *anox* and *aerob* denotes the concentration of nitrate, S_{NO}, and ammonium, S_{NH}, in the last anoxic compartment and last aerobic compartment, respectively. The flow rates are defined in Figure 2.3.

Similarly, the mass balance of nitrate in the anoxic and aerobic compartments is given by:

$$(Q_a + Q_r)S_{NO,aerob} = (Q_{in} + Q_a + Q_r)S_{NO,anox} + R_{DEN} \tag{2.3}$$

$$(Q_{in} + Q_a + Q_r)S_{NO,anox} = (Q_{in} + Q_a + Q_r)S_{NO,aerob} - R_{NIT} \tag{2.4}$$

where R_{DEN} denotes the reaction rate of the denitrification process. Combining the two first equations (2.1) and (2.2) and inserting this in equation (2.4) leads to the expression:

$$\begin{aligned}(Q_{in} + Q_a + Q_r)S_{NO,anox} &= (Q_{in} + Q_a + Q_r)S_{NO,aerob} \\ &+ Q_{in}S_{NH,aerob} - Q_{in}S_{NH,in}.\end{aligned} \tag{2.5}$$

If the nitrate concentration in the last compartment, $S_{NO,aerob}$, is replaced with a set-point, $S_{NO,ref}$, and time indexes are included, the following feed-forward control law is obtained:

$$Q_a(t) = \frac{Q_{in}(t)(S_{NH,in}(t) - S_{NH,aerob}(t))}{S_{NO,ref}(t) - S_{NO,anox}(t)} - (Q_{in}(t) + Q_r(t)) \tag{2.6}$$

There are some possibilities to further simplify (2.6). The following assumption is often reasonable:

A2.5. The denitrification process in the last anoxic compartment is complete.

Assumption **A2.5** typically holds (approximately) when the influent water has a sufficiently amount of soluble readily biodegradable substrate or when an external carbon source is used. This means that the nitrate level in the last anoxic compartment is $S_{NO,anox} = 0$, and (2.6) can be simplified to:

$$Q_a(t) = \frac{Q_{in}(t)(S_{NH,in}(t) - S_{NH,aerob}(t))}{S_{NO,ref}(t)} - (Q_{in}(t) + Q_r(t)) \tag{2.7}$$

To further increase the control performance and to avoid stationary errors, a PI controller is also included in the control law. The complete control law then consist of the following feedforward+PI controller:

$$Q_a(t) = \frac{Q_{in}(t)(S_{NH,in}(t) - S_{NH,aerob}(t))}{S_{NO,ref}} - (Q_{in}(t) + Q_r(t)) +$$
$$K(S_{NO,aerob}(t) - S_{NO,ref}(t)) + K_I \int_{t_0}^{t} (S_{NO,aerob}(\tau) - S_{NO,ref}(\tau)) d\tau \quad (2.8)$$

where K and K_I are the PI-parameters.

Simulation Results

An external carbon flow rate controller, a supervisory DO controller and the internal recycling flow rate controller, (2.8), were running simultaneously during the simulations. The nitrate set-point in the last anoxic compartment for the external carbon flow rate controller was chosen to 0.1 mg/l, and the ammonium set-point in the last aerobic compartment to 4 mg/l. The upper bound for the internal recycling flow rate was chosen to be 1250 m^3/h. The set-point for the internal recycling flow rate controller was $S_{NO,ref} = 5$ mg/l during both simulations. For comparison, simulations were also performed using: 1) a PI controller without feedforward for controlling the internal recycling flow rate; 2) a constant internal recirculation flow rate. The same PI parameters were used in the pure PI case as in the combined PI and feedforward case. Figure 2.4 shows the control performance when step changes are applied to the influent flow rate, and in Figure 2.5 the simulation result from step changes in the influent ammonium concentration is illustrated. It has been found from various simulations that a faster disturbance rejection can be achieved when a feedforward internal recirculation flow rate controller is used, especially for disturbances in influent flow rate.

The performance of the internal recycling flow rate controller is dependent on the nitrate concentration in the last anoxic compartment and thus dependent on the behavior of the external carbon flow rate controller. The impact of the internal recycling flow rate controller is more significant if the nitrate concentration in the last anoxic compartment is kept at a low value. In order for the supervisory DO controller to keep the ammonium set-point in the last aerobic compartment, the PI parameters for the internal recycling flow rate controller must be tuned with great care. The feedforward strategy is believed to have some advantages for disturbances rejection. Illustrations of some simulation results are also given in (Rhenström and Carlsson 2001).

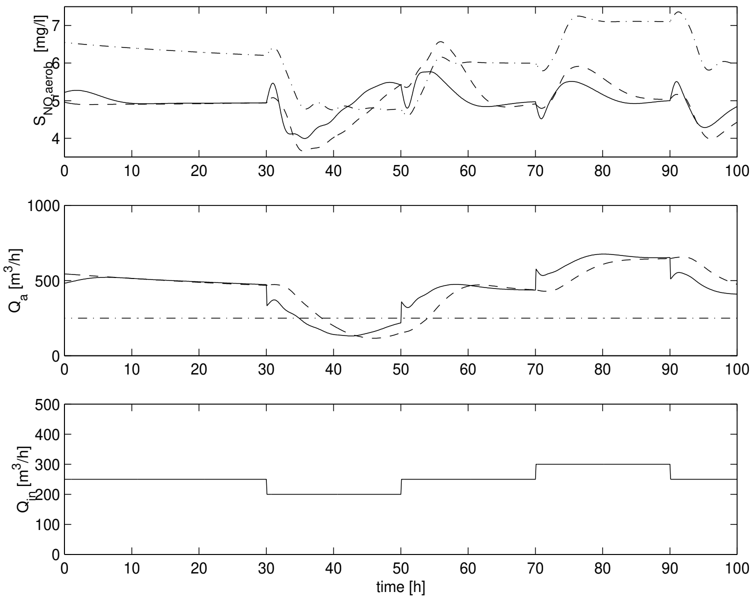

Figure 2.4: Comparison between the feedforward+PI controller (2.8), (solid lines), a PI controller (dashed lines) and a constant internal recirculation flow rate (dashed-dotted lines). Upper plot: The nitrate concentrations in the last aerobic compartment. Middle plot: The internal recirculation flow rates. Lower plot: Influent flow rates.

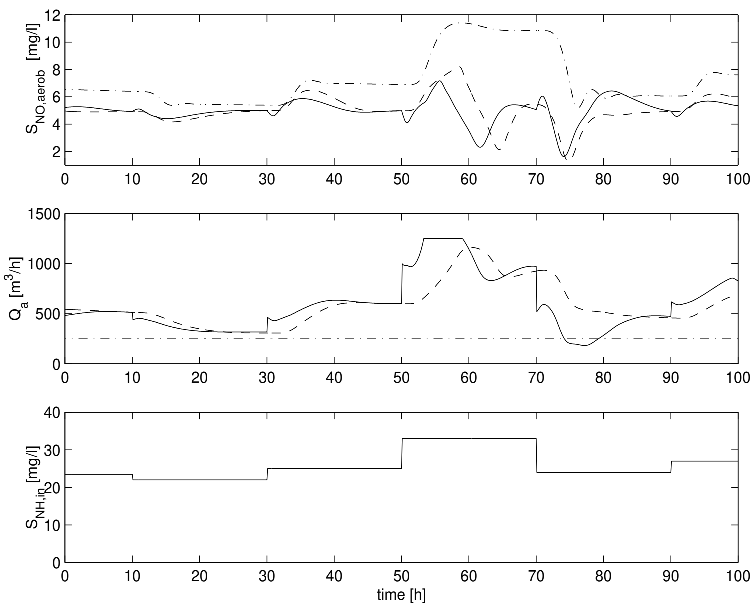

Figure 2.5: Comparison between the feedforward+PI controller, (2.8), (solid line), a PI controller (dashed line) and a constant internal recirculation flow rate (dashed-dotted line). Upper plot: The nitrate concentration in the last aerobic compartment. Middle plot: The internal recirculation flow rate. Lower plot: Influent ammonium concentration.

2.3 JASS as a Teaching and Training Tool

Since 1998 we have used the JASS simulator as an integrated teaching tool for undergraduate students and wastewater treatment plant (WWTP) personnel. Currently, several hundred students have been using the simulator in our course "Wastewater treatment" given in the Aquatic and Environmental MSc program at Uppsala University. Further, around 80 participants from Swedish WWTP's have been using the simulator in a 3 day course entitled "Automatic Control in WWTP's". This course has currently been given six times. The simulator has also been used by some other universities and special editions of the simulator have been developed for three WWTP's. The simulator is run via Internet with a standard browser. This feature is especially appealing for distance courses. It can be concluded that the simulator has been very appreciated by students and plant personnel. Below we give some experiences from the courses.

The simulator is mainly used for illustrating various aspects of nitrogen removal in the ASP. Typical issues that are studied is advantages and disadvantages with pre- and post-denitrification processes, for example, the need for an external carbon source in the latter case. Other aspects which are illustrated is the risk for wash-out if a too low sludge age is chosen, and the risk for sludge overflow in the settler if a too high sludge age is selected. Another use of the simulator is to illustrate and motivate various automatic control strategies.

A problem when the simulator is used for students is to find suitable pen and paper problems for the students to do as preparation exercises. It is important to have instructions which include some theoretical exercises and verifications, since this reduces the risk that the students just push buttons and passively write down the simulation results without any reflections. It is also important to emphasize that the simulator uses a very idealistic model of an ASP. For example, the well known problem with malfunctioning sensors are not encountered in the simulator. Further, the difficult problem to calibrate a model like the ASM1 for a specific plant must not be underestimated.

When using the simulator as a training and teaching tool for personnel from WWTP's, it is important to recognize that their background and skills differ a lot. Also, the experience of using computers may be limited for some participants. In the course, the morning sessions are devoted to theory, including automatic control and microbiology. In the afternoon sessions, the simulator is used to further illustrate various aspects of the activated sludge process. In order for the simulation session to be appreciated by all types of plant personnel it is important to have quite simple (but meaningful) laboratory instructions. It may also be good to have non compulsory exercises, where more advanced material is covered. We usually start the next session after a simulation session with a briefing of the lab exercises. We have found that the user interface used in the JASS simulator has been easy to work with and interpret by the plant personnel. A big advantage is that users almost immediately can start to simulate and interpret the results. On the other hand, the operators quite quickly want to start to simulate their "own" plant. A drawback with the JASS simulator is that it is not possible to make big changes in the basic layout without programming.

2.4 Conclusions

A JAVA based activated sludge process simulator has been presented. The simulator can be reached over Internet and operated from a web browser. The platform is user friendly and easy to interpret. A number of control strategies have been implemented. The simulator can be run with traditional DO level control as well as with supervisory DO control. Further, two feedforward/feedback control strategies, which control the external carbon dosage and the internal recycling flow rate, have been implemented. The simulator has been used for academic education as well as in courses for personnel from wastewater treatment plants. The simulator is then used as an integrated teaching tool to illustrate different process alternatives in nitrogen removal using the activated sludge process. The feature that the simulator can be run from Internet with a standard browser is of special interest considering future distance courses.

Chapter 3

Reduced Order Models for the Activated Sludge Process

As mentioned in Section 1.4.1, the IAWQ activated sludge model No. 1 (ASM1) is a widely used model for simulation of the activated sludge process (ASP). The model includes 13 different components, and the dynamic behavior of each component is described by a nonlinear differential equation, see (Henze *et al.* 1987) and Appendix B.1). See also (Bastin and Dochain 1990) and (Dochain and Vanrolleghem 2001) for some general bioreactor models. Due to its large complexity, it is far from trivial to use the ASM1 directly for controller design or theoretical analysis of the system. In order to design model based controllers, reduced order state-space models of the ASM1 are derived in this section. The model reduction is based on lumping of components together with some simplifying assumptions, see also (Jeppsson 1996). The procedure aims to maintain the basic mechanistic structure and the significant reactions in ASM1, while describing the system with a lower order model. Another reason for lumping components is that several of the components in ASM1 are not measurable. By lumping appropriate components, measurable variables may be obtained.

The chapter is organized as follows: In Section 3.1 the derivation of a linear time varying state-space model of an anoxic compartment is presented. An analysis of the identifiability properties of the model is also performed in this section. Section 3.2 presents the derivation of a time varying bilinear model for an pre-denitrifying ASP with two compartments, one anoxic and one aerobic.

3.1 Derivation of a State-Space Model for an Anoxic Compartment

Initially in the model reduction, one anoxic compartment in a denitrifying system will be considered. The design of an external carbon flow rate controller in Chapter 4 will be based on the model derived in this section.

The following assumptions are introduced:

A3.1. The biological reactions in the compartment is described by ASMI and the compartment is completely mixed.

A3.2. The volume in the compartment is constant, V [m^3].

A3.3. The concentration of dissolved oxygen is zero, $S_O = 0$ [mg/l].

A3.4. The impact of an external carbon source is modeled as an addition of readily biodegradable substrate $S_S(t)$ [mg COD/l].

A3.5. The slowly biodegradable substrate, $X_S(t)$ [mg COD/l], is not included in the soluble COD (Chemical Oxygen Demand).

The dilution rate is $D(t) = Q(t)/V$, where $Q(t)$ is the time varying inflow to the compartment. The control signal is the diluted concentration, see also (Samuelsson and Carlsson 2001):

$$u(t) = \frac{Q_c}{V} S_{S,C}(t) \tag{3.1}$$

where $Q_C(t)$ is the flow rate of the external carbon source and $S_{S,C}(t)$ [mg COD/l] is the COD concentration of the carbon source.

Assumptions **A3.1**, **A3.2** and **A3.4** result in that the dynamic behavior of the nitrate concentration, $S_{NO}(t)$, and the readily biodegradable substrate, $S_S(t)$, in a completely mixed anoxic reactor can be described by the mass balance:

$$\frac{dS_{NO}(t)}{dt} = -R(t) + D(t)(S_{NO,in}(t) - S_{NO}(t)) \tag{3.2}$$
$$\frac{dS_S(t)}{dt} = \alpha R(t) + D(t)(S_{S,in}(t) - S_S(t)) + H(t) + u(t) \tag{3.3}$$

where the subscript in denotes the concentrations in the influent of the anoxic compartment. $H(t)$ is a hydrolysis term. Assumption **A3.3** implies that

$$R(t) = \frac{1-Y_H}{2.86Y_H}\hat{\mu}_H(\frac{S_S(t)}{K_S + S_S(t)})(\frac{S_{NO}(t)}{K_{NO} + S_{NO}(t)})\eta_g X_{B,H}(t) \tag{3.4}$$
$$\alpha = -\frac{2.86}{1-Y_H}, \tag{3.5}$$

where Y_H is the yield of the heterotrophic biomass. The time varying concentrations and constant parameters are lumped into one unknown variable $R(t)$, which is the process rate term. For an explanation of the different parameters in (3.4)-(3.5) and their typical default values, see (Henze *et al.* 1987).

One difficulty in using the model (3.2)-(3.3) for control purposes is that $S_S(t)$, in general, is not directly measurable on-line. In practice, standard laboratory COD measurements on filtered samples of the wastewater is often considered to be equal to the amount of $S_S(t)$, see (Jeppsson 1996). However, all soluble biodegradable matter is not necessarily readily biodegradable and moreover, particulate biodegradable matter may not be slowly biodegradable. Some guidelines for the determination of COD components is defined in (Petersen 2000), where it is proposed that the soluble COD should contain readily biodegradable substrate $S_S(t)$, soluble inert organic matter $S_I(t)$ and part of the slowly biodegradable substrate $X_S(t)$.

In order to obtain an appropriate state-space representation of the soluble COD, it is natural to define a new state variable as the sum of $S_S(t)$ and $S_I(t)$, especially if Assumption **A3.5** is taken under consideration. $S_I(t)$ is inert and therefore considered to be unaffected by biological reactions, i.e. the dynamics of $S_I(t)$ is modeled as $\dot{S}_I(t) = D(t)(S_{I,in}(t) - S_I(t))$. By introducing the new state variable $S_{COD}(t) = S_S(t) + S_I(t)$, the system (3.2)-(3.3) can be rewritten as:

$$\frac{dS_{NO}(t)}{dt} = -R(t) + D(t)(S_{NO,in}(t) - S_{NO}(t)) \tag{3.6}$$

$$\frac{dS_{COD}(t)}{dt} = \alpha R(t) + D(t)(S_{COD,in}(t) - S_{COD}(t)) + H(t) + u(t) \tag{3.7}$$

The complete process rate term in (3.4) is given by the relationship $R(t) = r_{S_S}(t)S_S(t)$, where

$$r_{S_S}(t) = \frac{1-Y_H}{2.86Y_H}\hat{\mu}_H(\frac{1}{K_S + S_S(t)})(\frac{S_{NO}(t)}{K_{NO} + S_{NO}(t)})\eta_g X_{B,H}(t).$$

Since $S_S(t)$ is hard to measure on-line, it is desirable to derive a corresponding relationship $R(t) = r_{S_{COD}}(t)S_{COD}(t)$, in order to formulate an observable and controllable state-space representation of the system (3.6)-(3.7). The fact that $S_S(t) = S_{COD}(t) - S_I(t)$, gives $R(t) = r_{S_S}(t)(S_{COD}(t) - S_I(t))$. A reasonable assumption is that $S_{COD} > 0$, and $r_{S_{COD}}(t)$ is then given by the identity:

$$R(t) = r_{S_S}(t)\frac{S_S(t)}{S_S(t) + S_I(t)}S_{COD}(t) = r_{S_{COD}}(t)S_{COD}(t). \tag{3.8}$$

The system (3.6)-(3.7) can now be rewritten as:

$$\frac{dS_{NO}(t)}{dt} = -r_{S_{COD}}(t)S_{COD}(t) + D(t)(S_{NO,in}(t) - S_{NO}(t)) \tag{3.9}$$

$$\frac{dS_{COD}(t)}{dt} = \alpha r_{S_{COD}}(t)S_{COD}(t) + D(t)(S_{COD,in}(t) - S_{COD}(t)) + H(t) + u(t) \tag{3.10}$$

or:

$$\begin{bmatrix} \dot{S}_{NO}(t) \\ \dot{S}_{COD}(t) \end{bmatrix} = \begin{bmatrix} -D(t) & -r_{S_{COD}}(t) \\ 0 & \alpha r_{S_{COD}}(t) - D(t) \end{bmatrix} \begin{bmatrix} S_{NO}(t) \\ S_{COD}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} D(t)S_{NO,in}(t) \\ D(t)S_{COD,in}(t) \\ H(t) \end{bmatrix} \tag{3.11}$$

Considering $S_{NO}(t)$ as the output, the model (3.11) is controllable and observable if $r_{S_{COD}}(t) \neq 0$, see (Wilson 1993) for controllability and observability of time varying linear systems.

An extension of the model (3.11) to the case of two anoxic compartments is derived in Appendix C.1.

3.1.1 Derivation of an Augmented State-Space Model

A lot of research has been devoted to develop optical sensors that could replace the traditional laboratory COD measurements. However, today it is still not common to have on-line measurements of the soluble COD (S_{COD}) at wastewater treatment plants (WWTP's). One possibility to estimate soluble COD on-line may be to establish an empirical relationship between measurements of COD from filtered wastewater and measured TOC (Total Organic Carbon), see also (Olsson and Newell 1999).

In this section, the soluble COD will be estimated from on-line measurements which utilize sensors of more standard type. By lumping together unmeasurable ASM1 variables, new variables related to TSS (Total Suspended Solids) and total COD are obtained.

In order to obtain a measurable variable (see Remark **4.1** in Section 4.2), a new lumped variable is defined as:

$$X_{TSS}(t) = X_B(t) + X_I(t) + X_S(t), \tag{3.12}$$

where the total active biomass, $X_B(t)$, is the sum of heterotrophic and autotrophic biomass $X_B(t) = X_{B,H}(t) + X_{B,A}(t)$. The inert and particulate inert matter from biomass decay, as defined in ASM1, are combined into one matter $X_I(t)$. Another variable is defined as the sum of $X_{TSS}(t)$ and $S_{COD}(t)$:

$$X_{COD}(t) = X_{TSS}(t) + S_S(t) + S_I(t) = X_{TSS}(t) + S_{COD}(t). \tag{3.13}$$

$X_{COD}(t)$ can be regarded as the total COD. Summing up the reaction rates for the components in (3.12)-(3.13) (see Appendix B.1), the mass balance of the new variables (3.12)-(3.13) and the nitrate, $S_{NO}(t)$, can be written as:

$$\frac{dS_{NO}(t)}{dt} = -R(t) + D(t)(S_{NO,in}(t) - S_{NO}(t)) \tag{3.14}$$

$$\begin{aligned}\frac{dX_{TSS}(t)}{dt} &= -\alpha Y_H R(t) + D(t)(X_{TSS,in}(t) - X_{TSS}(t)) \\ &\quad - H(t)\end{aligned} \tag{3.15}$$

$$\begin{aligned}\frac{dX_{COD}(t)}{dt} &= -2.86R(t) + D(t)(X_{COD,in}(t) - X_{COD}(t)) \\ &\quad + u(t)\end{aligned} \tag{3.16}$$

Using (3.8), and exploiting the relationship $S_{COD}(t) = X_{COD}(t) - X_{TSS}(t)$ inserted in (3.8), the system (3.14)-(3.16) is obtained as:

$$\begin{aligned}
\frac{dS_{NO}(t)}{dt} &= -D(t)S_{NO}(t) + r_{S_{COD}}(t)X_{TSS}(t) - r_{S_{COD}}(t)X_{COD}(t) \qquad (3.17)\\
&\quad + D(t)S_{NO,in}(t)\\
\frac{dX_{TSS}(t)}{dt} &= (\alpha Y_H r_{S_{COD}}(t) - D(t))X_{TSS}(t) - \alpha Y_H r_{S_{COD}}(t)X_{COD}(t) \qquad (3.18)\\
&\quad + D(t)X_{TSS,in}(t) - H(t)\\
\frac{dX_{COD}(t)}{dt} &= 2.86 r_{S_{COD}}(t)X_{TSS}(t) - (2.86 r_{S_{COD}}(t) + D(t))X_{COD}(t) \qquad (3.19)\\
&\quad + D(t)X_{COD,in}(t) + u(t)
\end{aligned}$$

Using the relationship $X_{COD,in}(t) = X_{TSS,in} + S_{COD,in}$, a state-space representation of the system (3.17)-(3.19) can be written as:

$$\dot{z}(t) = \begin{bmatrix} -D(t) & r_{S_{COD}}(t) & -r_{S_{COD}}(t) \\ 0 & \alpha Y_H r_{S_{COD}}(t) - D(t) & -\alpha Y_H r_{S_{COD}}(t) \\ 0 & 2.86 r_{S_{COD}}(t) & -2.86 r_{S_{COD}}(t) - D(t) \end{bmatrix} z(t)$$
$$+ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t) + \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} D(t)S_{NO,in}(t) \\ D(t)X_{TSS,in}(t) \\ H(t) \\ D(t)S_{COD,in} \end{bmatrix} \qquad (3.20)$$

where $z(t) = \begin{bmatrix} S_{NO}(t) & X_{TSS}(t) & X_{COD}(t) \end{bmatrix}^T$. The hydrolysis $H(t)$ and the influent soluble COD to the anoxic compartment, $S_{COD,in}$, may be regarded as non-measurable process disturbances. Note that the system (3.20) is not observable if $S_{NO}(t)$ is the only output. A minimal realization of (3.20) is simply the system (3.11), which is obtained by replacing the states $X_{COD}(t)$ and $X_{TSS}(t)$ with the new state variable $S_{COD}(t) = X_{COD}(t) - X_{TSS}(t)$. This will be exploited when designing the controllers in Section 4.2.

The system (3.20) is extended to the case of two anoxic compartments in Appendix C.2.

3.1.2 Theoretical Identifiability

One important question is if the unknown parameter, $r_{S_{COD}}(t)$, in the suggested model (3.20) is theoretically identifiable. A simple method for investigating theoretical identifiability is the Taylor series expansion of observations method, see for example (Jeppsson 1996) and (Dochain and Vanrolleghem 2001) for application on models for the ASP. Based on the analysis in Appendix B.2, the following result holds.

Conjecture 3.1.1 *The reaction rate term $r_{S_{COD}}(t)$ in (3.20), is theoretically identifiable under the following assumptions:*

A3.6. *$r_{S_{COD}}(t)$ and $D(t)$ are constants, i.e. not varying with time.*

A3.7. *The nitrate concentrations $S_{NO,in}(t)$ [mg/l] and $S_{NO}(t)$ [mg/l], are measurable.*

A3.8. *The concentrations of total suspended solids, $X_{TSS,in}(t)$ [mg COD/l] and $X_{TSS}(t)$ [mg COD/l] are measurable.*

A3.9. *The incoming soluble COD ($S_{COD,in}$) [mg COD/l] is measurable.*

Comment: Assumption **A3.9** is only temporary and will be somewhat relaxed in Section 4.3.2. Assumption **A3.6** is not true in practice and is only temporarily used in order to investigate the theoretical identifiability.

The Taylor series expansion of observations method can also be applied to (3.9)-(3.10). It is interesting to compare the identifiability results from (3.9)-(3.10) with the expression (B.4) in Appendix B.2. By investigating the theoretically identifiability of a constant $r_{S_{COD}}$ in the model (3.9)-(3.10), we can conclude that a constant $r_{S_{COD}}$ cannot be identified if $S_{COD}(t)$ is unmeasurable, since $r_{S_{COD}}(t)$ and $S_{COD}(t)$ always occur as a combined variable. Thus, the conclusion that can be drawn from the identifiability results is that in the case of unmeasurable $S_{COD}(t)$, a constant $r_{S_{COD}}$ can at least be theoretically identifiable using the model (3.20), if measurements of nitrate, TSS and $S_{COD,in}(t)$ are available and if $D(t)$ is a constant.

For the case when Y_H is unknown, the following conjecture based on the analysis in Appendix B.3, can be made.

Conjecture 3.1.2 *Consider the same assumptions as in Conjecture 3.1.1 and the following additional assumption:*

A3.10. *$H(t)$ is constant.*

The heterotrophic yield, Y_H, is then theoretically identifiable.

3.2 Derivation of a Bilinear Model for the Activated Sludge Process

In this section one completely mixed anoxic compartment, one completely mixed aerobic compartment and a settler in a pre-denitrifying system is considered, see Figure 3.1.

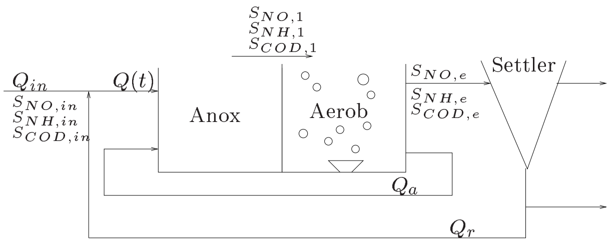

Figure 3.1: Schematic picture of the activated sludge process for one anoxic and one aerobic compartment.

Modeling general nonlinear systems using bilinear system structures often gives a more accurate approximation to nonlinear systems than a linear approximation. Bilinear approximation might also be acceptable in a wider region of the state-space around some operating point than a linear one, see (Mohler 1991). Here, the ASM1 will be reduced to a sixth order bilinear state-space model. As in Section 3.1, the model reduction is based on lumping of components together with some simplifying assumptions, in order to maintain the basic mechanistic structure and the significant reactions in ASM1. The model will be used in Chapter 5, where a suboptimal bilinear control law is applied to an activated sludge process.

The six states in the resulting model describes the dynamic behavior of three different components in the compartments. The considered components are the concentrations of nitrate, $S_{NO}(t)$, soluble COD, $S_{COD}(t)$, and the ammonium, $S_{NH}(t)$.

The components in the first anoxic and second aerobic compartment are denoted by the subscripts 1 and e respectively. The components in the incoming water are denoted by the subscript in, see Figure 3.1.

3.2.1 The Aerobic Compartment

The model of the aerobic compartment is derived in this section. The assumptions **A3.1**, **A3.2** and **A3.5** are used, together with the additional assumption:

A3.11. No biological processes occur in the settler, and the settler dynamics are neglected.

Remark 3.1: Taking Assumption **A3.11** under consideration, implies that the soluble matters in the settler is unaffected and thus, the settler unit can be neglected when deriving the models for the soluble components.

By using Assumption **A3.11** we have that the concentration of soluble components in the effluent water and in the return sludge flow, S_r, is approximately equal to the concentration of soluble components in the aerobic compartment, i.e. $S_r = S_e$. The following general mass balance then holds for any soluble component S_e in the aerobic compartment:

$$\frac{dS_e(t)}{dt} = R_s(t) + D(t)(S_1(t) - S_e(t)) + \frac{Q_a}{V}(S_1(t) - S_e(t)), \tag{3.21}$$

where $R_s(t)$ is the general reaction term and Q_a is the internal recycling flow rate. $D(t) = Q(t)/V$ is the dilution rate, and $Q(t) = Q_{in}(t) + Q_r(t)$ is the sum of the influent flow to the compartment and the sludge return flow, see Figure 3.1.

In connection with (3.21) and by using the reaction terms in ASM1, the dynamic behavior of $S_{NO,e}(t)$, $S_{COD,e}(t)$ and $S_{NH,e}(t)$, can for one completely mixed aerobic reactor be described by the mass balance:

$$\begin{aligned}\frac{dS_{NO,e}(t)}{dt} &= -\bar{R}_1(t) + \bar{R}_2(t) + D(t)(S_{NO,1}(t) - S_{NO,e}(t)) \\ &+ \frac{Q_a}{V}(S_{NO,1}(t) - S_{NO,e}(t))\end{aligned} \tag{3.22}$$

$$\begin{aligned}\frac{dS_{COD,e}(t)}{dt} &= \alpha_1\bar{R}_1(t) - \bar{R}_3(t) + D(t)(S_{COD,1}(t) - S_{COD,e}(t)) \\ &+ \frac{Q_a}{V}(S_{COD,1}(t) - S_{COD,e}(t)) + \bar{H}_e(t)\end{aligned} \tag{3.23}$$

$$\begin{aligned}\frac{dS_{NH,e}(t)}{dt} &= -\alpha_3\bar{R}_1(t) + \alpha_2\bar{R}_2(t) - \alpha_4\bar{R}_3(t) + Am_e(t) \\ &+ D(t)(S_{NH,1}(t) - S_{NH,e}(t)) + \frac{Q_a}{V}(S_{NH,1}(t) - S_{NH,e}(t)),\end{aligned} \tag{3.24}$$

$\bar{H}_e(t)$ and $Am_e(t)$ are the hydrolysis and ammonification term, respectively. The reaction rate terms are given by:

$$\begin{aligned}\bar{R}_1(t) =& \frac{1-Y_H}{2.86Y_H}\hat{\mu}_H\left(\frac{S_{S,e}(t)}{K_S + S_{S,e}(t)}\right)\left(\frac{K_{0,H}}{K_{O,H} + S_O}\right)\times \\ & \left(\frac{S_{NO,e}(t)}{K_{NO} + S_{NO,e}(t)}\right)\eta_g X_{B,H,e}(t)\end{aligned} \tag{3.25}$$

$$\bar{R}_2(t) = \frac{1}{Y_A}\hat{\mu}_A\left(\frac{S_{NH,e}(t)}{K_{NH} + S_{NH,e}(t)}\right)\left(\frac{S_O(t)}{K_{O,A} + S_O(t)}\right)X_{B,A,e}(t) \tag{3.26}$$

$$\bar{R}_3(t) = \frac{1}{Y_H}\hat{\mu}_H\left(\frac{S_{S,e}(t)}{K_S + S_{S,e}(t)}\right)\left(\frac{S_O(t)}{K_{O,H} + S_O(t)}\right)X_{B,H,e}(t) \tag{3.27}$$

$$\begin{aligned}\alpha_1 =& -\frac{2.86}{1-Y_H}, \quad \alpha_2 = -(i_{XB}Y_A + 1) \\ \alpha_3 =& -\alpha_1\alpha_4, \quad \alpha_4 = Y_H i_{XB}.\end{aligned} \tag{3.28}$$

Y_A is the yield for the autotrophic biomass. For an explanation of the different parameters in (3.25)-(3.28) and their typical default values, see as before (Henze *et al.* 1987).

From (3.26) we can define the relationship $\bar{R}_2(t) = \bar{r}_{S_{NH}}(t)S_{NH,e}(t)$. Replacing $S_{S,e}$ in the numerator of (3.25) and (3.27) by $S_{COD,e}$ and using the same procedure as in Section 3.1, also gives the relationship $\bar{R}_i(t) = \bar{r}_{S_{COD,i}}(t)S_{COD,e}(t)$, $i = 1,\ 3$. Utilizing this, the system (3.22)-(3.24) can be rewritten as:

$$
\begin{aligned}
\frac{dS_{NO,e}(t)}{dt} &= -\bar{r}_{S_{COD,1}}(t)S_{COD,e}(t) + \bar{r}_{S_{NH}}(t)S_{NH,e}(t) && (3.29)\\
&+ D(t)(S_{NO,1}(t) - S_{NO,e}(t)) + \frac{Q_a}{V}(S_{NO,1}(t) - S_{NO,e}(t))\\
\frac{dS_{COD,e}(t)}{dt} &= \alpha_1\bar{r}_{S_{COD,1}}(t)S_{COD,e}(t) - \bar{r}_{S_{COD,3}}(t)S_{COD,e}(t) + \bar{H}_e(t) && (3.30)\\
&+ D(t)(S_{COD,1}(t) - S_{COD,e}(t)) + \frac{Q_a}{V}(S_{COD,1}(t) - S_{COD,e}(t))\\
\frac{dS_{NH,e}(t)}{dt} &= -\alpha_3\bar{r}_{S_{COD,1}}(t)S_{COD,e}(t) + \alpha_2\bar{r}_{S_{NH}}(t)S_{NH,e}(t) && (3.31)\\
&- \alpha_4\bar{r}_{S_{COD,3}}(t)S_{COD,e}(t) + D(t)(S_{NH,1}(t) - S_{NH,e}(t))\\
&+ \frac{Q_a}{V}(S_{NH,1}(t) - S_{NH,e}(t)) + Am_e(t).
\end{aligned}
$$

3.2.2 The Anoxic Compartment

The differential equations determining the dynamic behavior of the components $S_{NO,1}(t)$ and $S_{COD,1}(t)$ is derived in Section 3.1, and an extended version of the model (3.11) will be utilized. Thus, assumptions (**A3.1**)-(**A3.5**) are also used in this section. The dynamic behavior of the components in the anoxic compartment is somewhat simpler compared to the aerobic reactor. Due to the Assumption **A3.3**, there is only one process rate term that influence the process. Augmenting the model (3.11) with the state $S_{NH,1}$, and including the dynamics from the internal recycling flow, results in the model:

$$
\begin{aligned}
\frac{dS_{NO,1}(t)}{dt} &= -r_{S_{COD}}(t)S_{COD,1}(t) + D_r(t)S_{NO,e}(t) && (3.32)\\
&- D(t)S_{NO,1}(t) + \frac{Q_a}{V}(S_{NO,e}(t) - S_{NO,1}(t)) + D_{in}(t)S_{NO,in}(t)
\end{aligned}
$$

$$
\begin{aligned}
\frac{dS_{COD,1}(t)}{dt} &= \alpha_1 r_{S_{COD}}(t)S_{COD,1}(t) + D_r(t)S_{COD,e}(t) & (3.33)\\
&- D(t)S_{COD,1}(t) + \frac{Q_a}{V}(S_{COD,e}(t) - S_{COD,1}(t))\\
&+ D_{in}(t)S_{COD,in}(t) + \bar{H}_1(t) + u_3(t)\\
\frac{dS_{NH,1}(t)}{dt} &= -\alpha_3 r_{S_{COD}}(t)S_{COD,1}(t) + D_r(t)S_{NH,e}(t) & (3.34)\\
&- D(t)S_{NH,1}(t) + \frac{Q_a}{V}(S_{NH,e}(t) - S_{NH,1}(t))\\
&+ D_{in}S_{NH,in}(t) + Am_1(t),
\end{aligned}
$$

where $D_r(t) = Q_r(t)/V$ is the sludge return dilution rate and $D_{in}(t) = Q_{in}(t)/V$ the influent dilution rate. $u_3(t)$ is the external carbon source in (3.1).

3.2.3 Assembling the Models

In this section the models for the anoxic and the aerobic compartments are assembled. The control variables are defined as $u_1(t) = Q_a$, $u_2 = \frac{S_O}{K+S_O}$ and $u_3(t)$. In order to assemble the two models the following additional assumptions are introduced:

A3.2'. The volumes in the anoxic and the aerobic compartments are equal and constant, V [m^3].

A3.12. $K = K_{O,H} = K_{O,A}$,

The dynamics of the components of the two compartments can then be expressed in the bilinear state-space form:

$$
\dot{x}(t) = A(t)x(t) + N_1(t)x(t)u_1(t) + N_2x(t)u_2(t) + B_u u_3(t) + B_{\bar{d}}\bar{d}(t) + B_{\bar{w}}\bar{w}(t) \tag{3.35}
$$

where $x(t) = [S_{NO,1}(t)\ S_{COD,1}(t)\ S_{NH,1}(t)\ S_{NO,e}(t)\ S_{COD,e}(t)\ S_{NH,e}(t)]^T$, $\bar{d}(t) = [D_{in}(t)S_{NO,in}(t)\ D_{in}(t)S_{COD,in}(t)\ D_{in}(t)S_{NH,in}(t)]^T$ and $\bar{w}(t) = [\bar{H}_1(t)\ Am_1(t)\ \bar{H}_e(t)\ Am_e(t)]^T$. The systems matrices are:

$$
A(t) = \begin{pmatrix} A_{anox}(t) & D_r(t)I \\ D_r(t)I & A_{aerob}(t) \end{pmatrix}
$$

where I is a 3×3 unity matrix, and

$$A_{anox}(t) = \begin{pmatrix} -D(t) & -r_{S_{COD}}(t) & 0 \\ 0 & \alpha_1 r_{S_{COD}}(t) - D(t) & 0 \\ 0 & -\alpha_3 r_{S_{COD}}(t) & -D(t) \end{pmatrix}$$

$$A_{aerob}(t) = \begin{pmatrix} -D(t) & -\bar{r}_{S_{COD,1}}(t) & 0 \\ 0 & \alpha_1 r_{S_{COD,1}}(t) - D(t) & 0 \\ 0 & -\alpha_3 r_{S_{COD,1}}(t) & -D(t) \end{pmatrix}.$$

We have further:

$$N_1(t) = \begin{pmatrix} -\frac{1}{V}I & \frac{1}{V}I \\ \frac{1}{V}I & -\frac{1}{V}I \end{pmatrix}, \quad N_2(t) = \begin{pmatrix} 0 & 0 \\ 0 & n_2(t) \end{pmatrix}$$

where

$$n_2(t) = \begin{pmatrix} 0 & \bar{r}_{S_{COD,1}}(t) & \bar{r}_{S_{NH}}(t) \\ 0 & -\alpha_1 \bar{r}_{S_{COD,1}}(t) - \bar{r}_{S_{COD,3}}(t) & 0 \\ 0 & \alpha_3 \bar{r}_{S_{COD,1}}(t) - \alpha_4 \bar{r}_{S_{COD,3}}(t) & \alpha_2 \bar{r}_{S_{NH}}(t) \end{pmatrix}.$$

The remaining system matrices are $B_u = [S_{S,C}/V \; 0 \ldots 0]^T$, and

$$B_{\bar{d}} = \begin{pmatrix} I \\ 0 \end{pmatrix}, \quad B_{\bar{w}} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Chapter 4

Adaptive Control of the Nitrate Level in an Activated Sludge Process using an External Carbon Source

The degree of nitrogen removal in an ASP is dependent on the efficiency of two biological processes; nitrification and denitrification. By nitrification ammonium is oxidized to nitrate and in the denitrification process nitrate is transformed to nitrogen gas. Nitrate can be reduced to nitrogen gas by faculatative heterotrophic bacteria in an anoxic environment. An organic carbon source is needed as electron donor and to supply carbon for synthesis of cell components. If the Carbon/Nitrate ratio is too low in the influent water to the anoxic zone, an addition of external carbon source is required (Henze *et al.* 1995). An important question is which amount of carbon that should be added. A too high carbon dosage is expensive, may increase the biochemical oxygen demand (BOD) level in the effluent, increase the sludge production (Aspegren *et al.* 1992) and decrease the capacity of the microorganisms to use naturally existing carbon sources (Hallin and Pell 1996). On the other hand, a too low carbon dosage will leave the denitrification process carbon constrained, and the full capacity for nitrogen removal is not utilized.

Adaptive and model-based approaches for control and estimation of wastewater treatment plants have been frequently reported in the literature. Such an approach is, for example, found in (Bastin and Dochain 1990). The IWA

Activated Sludge Model No.1 (ASM1) described in (Henze *et al.* 1987) is frequently used for modeling of ASP's. (Jeppsson 1996) and (Weijers 2000) discuss the need for model reduction and different reduction approaches for the ASM1 when model based control strategies are suggested. In (Lukasse 1999) an adaptive MPC evaluated on an alternating ASP, with dissolved oxygen concentration as control input and ammonium and nitrate concentrations as controlled outputs, is presented. The reduced order model used in the control design is based upon ASM1, see (Lukasse *et al.* 1997). In (Lindberg 1997) several different controllers for a predenitrifying process are designed.

Some of the reported strategies consider control of the nitrate concentration using the external carbon dosage as an input signal. In (Lindberg and Carlsson 1996*a*) a direct adaptive strategy for controlling the nitrate concentration, based on a generalized minimum-variance (GMV) strategy, is evaluated in a predenitrifying pilot plant. A controller design based on the ASM1 is presented in (Yuan *et al.* 1997). The choice of set-point for the carbon controller is also considered. A feedforward strategy based on a simple mass balance is presented in (Samuelsson and Carlsson 2001). A related strategy can be found in (Carlsson and Milocco 2001), where a model based feedback-feedforward controller is designed using exact linearization on a simplified ASM1. A cascade control strategy for controlling the nitrate concentration in both the effluent and in the last anoxic compartment is proposed in (Cho *et al.* 2002).

In this chapter a model based adaptive control strategy is suggested. The model used for controller design is derived in Section 3.1. The time varying model parameters in (3.20) are estimated from measurements of nitrate concentration and total suspended solids (TSS). The derived state-space model (3.20) is then used in the design of a Linear Quadratic Gaussian (LQG) controller utilizing both feedback and feedforward from measurable disturbances. The suggested control strategy can be regarded as an indirect adaptive LQG controller, which is described in (Åström and Wittenmark 1989). The controller to adapts and compensates for changes in the system dynamics, since important system parameters are estimated adaptively. Another benefit with the suggested control strategy is that the estimated system parameters and states serve as a guidance about the state of the process and the characteristics of the wastewater.

The outline of the chapter is as follows. In Section 4.1 the estimation of reaction rate terms and the heterotrophic yield is discussed. The model structures that are used are derived in Section 3.1. The LQG controller design is outlined in Section 4.2. Simulation results using the LQG controller together with comparisons with traditional PID control and PI control with feedforward are presented in Section 4.3. Finally, conclusions are given in Section 4.4.

4.1 Estimation of the Reaction Rate Term

The transformed reaction rate term, $r_{S_{COD}}(t)$ appears in the models (3.11) and (3.20), derived in Section 3.1. Since $r_{S_{COD}}(t)$ is unknown, it has to be estimated in an appropriate way. One approach is to approximate the reaction rate term as constant, which gives the possibility to use off-line estimation techniques. With an additional assumption that $D(t)$ is constant, the system matrices in (3.11) and (3.20) will be time invariant, which simplifies the control synthesis. One drawback with this approach is that if the assumption regarding the constant reaction rate term is inexact, important information about the system dynamics may be lost.

In this section, the reaction rate term will be estimated on-line. Allowing $r_{S_{COD}}(t)$ to be time varying implies the possibility for the models to adapt to important system dynamics. The time varying reaction rate term also gives guidance about the state of the process. $r_{S_{COD}}(t)$ will be estimated from (3.20) with an Extended Kalman Filter (EKF).

4.1.1 Estimation of the Reaction Rate Term with known Heterotrophic Yield, Y_H

In this section one anoxic compartment is considered and it is assumed that the yield of the heterotrophic biomass, Y_H, is known in the model (3.20).

In order to use an EKF discrete-time algorithm, the model (3.20) can be sampled and augmented according to the following steps:

1. The model (3.20) is sampled which gives the discrete-time system:

$$z(t+1) = A_s(t)z(t) + B_H(t)H(t) + B_u(t)u(t) + B_m(t)m(t). \quad (4.1)$$

A zero-order hold sampling is used, see e.g. (Åström and Wittenmark 1990).

2. In order to obtain the Jacobian matrix in the EKF-algorithm (see Appendix C.3) the model (3.20) is linearized with respect to $z(t)$ and $r_{S_{COD}}(t)$ and then sampled.

3. Next, the unknown parameters $H(t)$ and $r_{S_{COD}}(t)$ are modeled as AR-processes:

$$\begin{aligned} r_{S_{COD}}(t+1) &= \phi r_{S_{COD}}(t) + e(t) \\ H(t+1) &= \phi H(t) + v(t), \end{aligned} \tag{4.2}$$

   where $e(t)$ and $v(t)$ are white noise. A choice $\phi = 1$ would correspond to a random walk process. In what follows, we will choose $\phi = 0.99$, which can be seen as a filtered random walk.

4. The system (4.1) is then augmented with $H(t)$ and $r_{S_{COD}}(t)$ as additional states:

$$\begin{aligned} \begin{bmatrix} z(t+1) \\ r_{COD}(t+1) \\ H(t+1) \end{bmatrix} &= \begin{bmatrix} A_s(t) & 0 & B_H(t) \\ 0 & 0.99I & 0 \\ 0 & 0 & 0.99I \end{bmatrix} \begin{bmatrix} z(t) \\ r_{COD}(t) \\ H(t) \end{bmatrix} \\ &+ \begin{bmatrix} B_u(t) \\ 0 \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} B_m(t) \\ 0 \\ 0 \end{bmatrix} m(t) + \begin{bmatrix} 0 & 0 \\ I & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} e(t) \\ v(t) \end{bmatrix}. \end{aligned} \tag{4.3}$$

5. By augmenting the Jacobian matrix in the same manner as in step 4, the hydrolysis, $H(t)$, the reaction rate term, $r_{S_{COD}}(t)$, and the unmeasurable state, $X_{COD}(t)$, can now be updated using one step in the discrete-time EKF algorithm. After updating the model (3.20) with the new estimates, the procedure is repeated.

See Appendix C.3 for a summarized discrete-time EKF algorithm. Another approach is to use a continuous-discrete EKF, which has the benefit to keep the model in the continuous form, while the measurements are discrete in time. For more practical aspects of extended Kalman filtering, see (Gelb 1975). The estimation of the unknown states, the reaction rate terms and the hydrolysis terms for two anoxic compartments is illustrated by simulations in Section 4.3.2.

4.1.2 Estimation of the Heterotrophic Yield, Y_H

In Section 4.1.1, the heterotrophic yield, Y_H, was assumed to be known. Y_H can be determined using respirometry by addition of an amount of wastewater COD and measurements of the substrate oxidation , see e.g. (Sollfrank and Gujer 1991) and (Petersen 2000). Y_H will be estimated on-line from measurements of $S_{NO}(t)$ and $X_{TSS}(t)$ using a discrete-time EKF. A related strategy can be found in (Samuelsson 2001) where Y_H and the hydrolysis in one anoxic compartment is estimated from measurements of $S_{NO}(t)$ and $S_S(t)$ using a Kalman filter.

Let the parameter β be defined as $\beta = \alpha Y_H$. The heterotrophic yield is then obtained from $Y_H = \frac{\beta}{\beta - 2.86}$ using (3.3). The unknown parameter β is modeled as an AR-process, see (4.2), and the state vector in (4.3) is augmented with the new state β. The unknown states can be estimated in the same way as in Section 4.1.1 using a discrete-time EKF. Here, the tracking problem involves simultaneous estimation of the four unknown parameters β, $r_{S_{COD}}(t)$, $H(t)$ and $X_{COD}(t)$. The estimation of the heterotrophic yield, reaction rate terms and the hydrolysis terms, for two anoxic compartments is illustrated by simulations in Section 4.3.4.

4.2 Controller Design

In this section the case of two anoxic compartments is considered. The components in the first and second compartment are denoted by the subscripts 1 and 2 respectively. The LQG controller design is based on the two models (C.1) and (C.7) in Appendix C.1 and Appendix C.2. The augmented state-space model (C.7) is used for the estimation part. If $D(t)$ and $r_{S_{COD,i}}(t) \neq 0$, $i = 1,\ 2$, are assumed to be constants, the model (C.1) is a minimal realization, which is not the case for the model (C.7). Therefore, the controller design is based on model (C.1).

The assumptions (**A3.7**)-(**A3.9**) in Section 3.1.2 are used. However, **A3.7** and **A3.8** are modified to hold for two anoxic compartments, and **A3.9** is somewhat relieved in order to hold for measurement methods of standard type:

A4.1. The nitrate concentrations, $S_{NO,in}(t)$ [mg/l] and $S_{NO,2}(t)$ [mg/l], are measurable.

A4.2. The concentrations of total suspended solids, $X_{TSS,in}(t)$ [mg COD/l], $X_{TSS,1}(t)$ [mg COD/l] and $X_{TSS,2}(t)$ [mg COD/l] are measurable.

A4.3. The incoming soluble COD ($S_{COD,in}$) [mg COD/l] is measured in twenty-four hour intervals from laboratory tests.

Remark 4.1. In practice, insoluble particles can be measured on-line using, for example, turbidimeters. Turbidity is related to the content of suspended solids (SS) and there are also laboratory tests for solids that can calibrate turbidity measurements, see (Olsson and Newell 1999). However, insoluble particles also contains other materials than organic particles. It is also necessary to convert the measurement of TSS to have the same unit as $X_{TSS}(t)$. A survey of new and existing technologies for measuring different components and variables in wastewater, including SS, is presented in (Vanrolleghem and Lee 2003).

The LQG controller will be designed in discrete-time. Hence, the first step is to sample the system (C.1) using zero-order hold sampling. In order to obtain a feedforward action in the LQG controller, the measurable disturbance $d(t)$ from (C.2) is included in the state vector in (C.1). The dynamics of $d(t)$ is modeled as an AR-process, see (4.2) in Section 4.1.1. Several different strategies for how to design an integrating controller exist. The approach used here is based on differentiating the system and deriving an incremental controller, see e.g. (García *et al.* 1989).

After rewriting (C.1) in a discrete and differential form, with $d(t)$ included in the state vector, a feedforward-feedback integrating LQG controller can be designed. The differentiation of the model results in the disappearance of constant process disturbances in the model. To see this, assume that:

A4.4. The hydrolysis terms $H_i(t)$ are slowly time varying.

This will result in that $H_i(t)$ can be omitted from (C.1) when deriving the LQG controller. The sampled and differentiated system can now be written as, see further ((Lindberg 1997)):

$$\left[\begin{array}{c} \Delta x_n(t+1) \\ y(t+1) \end{array} \right] = A_d(t) \left[\begin{array}{c} \Delta x_n(t) \\ y(t) \end{array} \right] + B_d(t)\Delta u(t) \tag{4.4}$$

where $A_d(t)$ and $B_d(t)$ are the discrete-time system matrices obtained from sampling and differentiating (C.1), and $y(t) = S_{NO,2}(t)$ is the output signal. Further, $\Delta x_n(t+1) = x_n(t+1) - x_n(t)$ and so on. The differentiated state vector is $\Delta x_n(t) = \left[\Delta x(t) \quad \Delta d(t)\right]^T$, where

$$\Delta x(t) = \left[\begin{array}{cccc} \Delta S_{NO,1}(t) & \Delta S_{NO,2}(t) & \Delta S_{COD,1}(t) & \Delta S_{COD,2}(t) \end{array} \right]^T,$$

and the differentiated control signal can then be computed as

$$\begin{aligned} \Delta u(t) = -L \left[\begin{array}{c} \Delta \hat{x}_n(t) \\ y(t) \end{array} \right] + L_w w(t) = \\ - \left[\begin{array}{cc} L_{x_n} & L_y \end{array} \right] \left[\begin{array}{c} \Delta \hat{x}_n(t) \\ y(t) \end{array} \right] + L_w w(t) \end{aligned} \tag{4.5}$$

where $w(t)$ is the reference signal (set-point of the nitrate). A suitable choice of L_w is $L_w = L_y$, because the set-point $w(t)$ and output $y(t)$ are then amplified equally. The state feedback (and disturbance feedforward) vector L is found by minimizing the cost function:

$$\lim_{n\to\infty} \sum_{t=1}^{n} \Big(\left[\begin{array}{c} \Delta x_n(t) \\ y(t) \end{array} \right]^T Q_1 \left[\begin{array}{c} \Delta x_n(t) \\ y(t) \end{array} \right] + \Delta u(t)^T Q_2 \Delta u(t) \Big) \tag{4.6}$$

where Q_1 and Q_2 are positive definite penalty matrices, see also, e.g. (Åström and Wittenmark 1990).

An EKF observer is used for estimating the unmeasurable states in $\Delta x(t)$ and the reaction rate terms, $r_{S_{COD,i}}(t)$, in (C.1). After zero-order hold sampling, the model (C.7) is augmented with the reaction rate terms and the hydrolysis terms H_i. The soluble COD in the two compartments can now be estimated using the relationship, $\hat{S}_{COD,i}(t) = \hat{X}_{COD,i}(t) - X_{TSS,i}(t)$, where $\hat{X}_{COD,i}(t)$ is estimated by the EKF. The system matrices, A_d and B_d, in (4.4) are updated at each sampling instant using the EKF estimates of $\hat{r}_{S_{COD,i}}(t)$.

The controller design in this section can be regarded as an indirect adaptive LQG controller, which is illustrated in Figure 4.1. A recursive least squares (RLS) estimation procedure has also been evaluated, but recent results indicate that the EKF performs better.

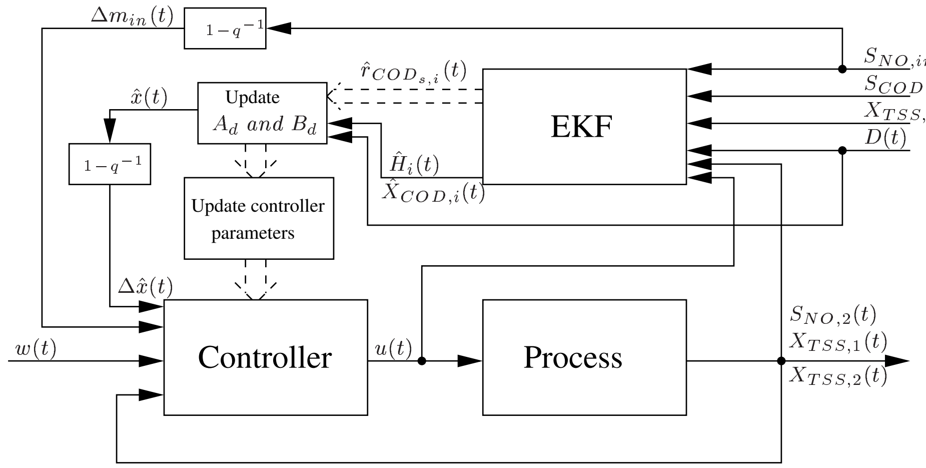

Figure 4.1: Block diagram of the adaptive LQG controller with EKF.

4.3 Simulation Studies

4.3.1 Simulation Set Up

The feedback-feedforward adaptive LQG control law given by (4.5) was studied on the ASM1 in a post-denitrifying ASP. The simulated ASP consisted of eight aerobic compartments, two anoxic compartments and one settler. Matlab was used to implement the model. The parameters in the model were the default values given in (Henze *et al.* 1987). The total volume of the plant was $6000m^3$. Each anoxic compartment had the volume $V_{anox} = 1000m^3$, and each aerobic compartment had the volume $V_{aerobic} = 500m^3$. The excess sludge flow rate was $Q_w = 16m^3/h$. The plant was simulated using influent data from the file dryinfluent.mat and storminfluent.mat from the COST benchmark, see htpp://www.ensic.unancy.fr/COSTWWTP/ and (Alex *et al.* 1999).

In the tuning of the controller parameters, influent data were taken from the file dryinfluent.mat. A traditional PI controller was used for controlling the level of dissolved oxygen (DO) in the aerobic compartments, and the DO set-point was chosen to be $2mg/l$ in each compartment. The return sludge flow rate was controlled so that a constant ratio between the return sludge flow rate and the influent flow rate was obtained.

4.3.2 Simulations Using an LQG Controller assuming Y_H known

In this section the assumptions (**A4.1**)-(**A4.3**) are used. $S_{NO,1}(t)$, $X_{COD,1}(t)$ and $X_{COD,2}(t)$ were estimated using an EKF and $S_{COD,i}(t)$ was estimated as, $\hat{S}_{COD,i}(t) = \hat{X}_{COD,i}(t) - X_{TSS,i}(t)$. The data used for estimation and control were sampled with a sampling period of 1 minute without noise and delay on the sensors.

The external carbon source was assumed to be ethanol with $S_{S,C}$ = 1200000 mg (COD)/l. An upper bound on the external carbon flow rate was introduced and chosen as 5 m^3 ethanol/day. The design variables in the LQG controller were chosen in a way so that the controller behaved as fast as possible without exceeding the upper bound on $u(t)$ when influent data from the file dryinfluent.mat was used. The nitrate set-point was $4mg/l$. For the LQG controller the penalty matrices in (4.6) were chosen as $Q_1 = diag[0, 0, 0, 0, 1, 1, 1]$ and $Q_2 = 25000$. The initial value of the covariance matrix in the EKF was chosen as $P(0) = I$, where I denotes the unity matrix, and the design variables as $R_1 = diag[10, 1, 10, 1, 1, 1, 1, 1, 1, 0.01]$ and $R_2 = 0.01I$, see Appendix C.3.

For comparison, a traditional PID controller and a PI controller with feedforward (Samuelsson and Carlsson 2001) were also implemented. The feedforward part uses measurements of the influent $S_S(t)$ of the first anoxic compartment. The control parameters in the traditional PID controller and the PI controller with feedforward were tuned to make the controllers as fast as possible without exceeding the upper bound on $u(t)$, when simulating using influent data from the file dryinfluent.mat. The upper bound on $u(t)$ and the nitrate set-point were the same as for the LQG controller. It is worth noticing that none of the controllers are optimally tuned.

Results and Discussion

The simulation results for the three different controllers are illustrated in Figure 4.2. The seven last days of the simulations using influent data from the storminfluent.mat file are shown in the plots. By comparing the LQG controller and a traditional PID controller without feedforward, it is seen that the LQG controller gives a faster disturbance rejection than the PID controller. The PI controller with feedforward, in the same figure, has a similar performance as the LQG controller. During the larger disturbances at day 2 and 4 the controllers reach the upper bound on $u(t)$. The total consumption of external carbon during the period, (u_{cons}), and the variance of the control error, (e_{var}), for the three controllers are shown in Table 1.

Table 4.1: The variance of the control error, (e_{var}), and the total consumption of external carbon during the period, (u_{cons}), for the three controllers.

Controller	e_{var}	u_{cons}
LQG	0.468	16.17
PI-feedforward	0.440	16.41
PID	1.175	16.37

Notice that the PI controller with feedforward uses measurements of both $S_{NO,in}(t)$ and $S_{S,in}(t)$. Previous work where the LQG design and the estimation procedure is based on on-line measurements of $S_{COD}(t)$ has shown that the LQG controller gives faster disturbance rejection than the PI controller with feedforward, see (Ekman *et al.* 2002). However, one drawback with the LQG design is the implementation part, which is more complex compared to the PI controller with feedforward.

Figure 4.3 illustrates the EKF estimation of $r_{S_{COD,1}}(t)$ and $r_{S_{COD,2}}(t)$. The EKF also provides a good estimate of the total COD and satisfactory estimates of the hydrolysis terms in both anoxic compartments. The convergence of the estimates of $r_{S_{COD,i}}(t)$ and $H_i(t)$ is improved with the reasonable assumption **A4.3**. Figure 4.4 and 4.5 illustrate the estimates of $X_{COD,i}(t)$ and $H_i(t)$, respectively. All fourteen days from the storminfluent.mat file are shown in the plots.

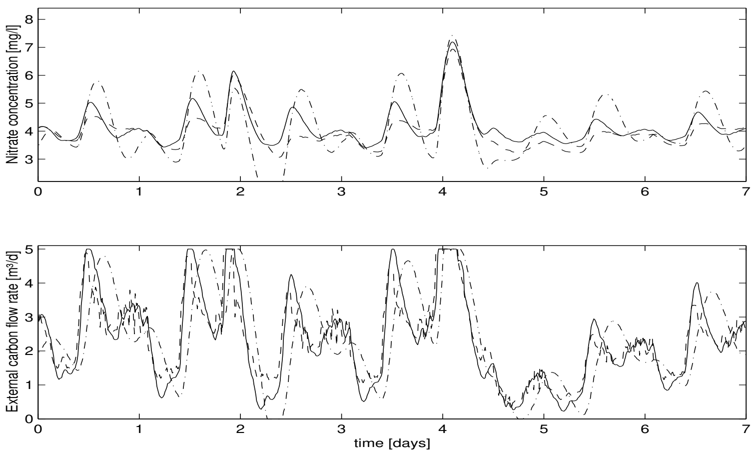

Figure 4.2: Upper plot: The solid line shows the nitrate concentration in the last anoxic compartment when the LQG controller is used, dashed line the nitrate concentration when PI with feedforward is used, and dashed-dotted line the nitrate concentration when a traditional PID controller is used. Lower plot: The external carbon flow rates are shown for the three controllers, respectively.

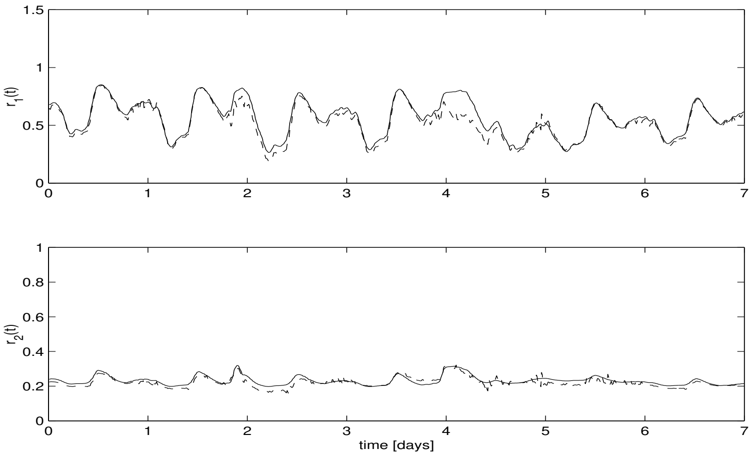

Figure 4.3: Upper plot: The dashed line shows the EKF estimated value of the reaction rate term $r_{S_{COD},1}(t)$ and the solid line the true value. Lower plot: The dashed line shows the EKF estimated value of the reaction rate term $r_{S_{COD},2}(t)$ and the solid line the true value.

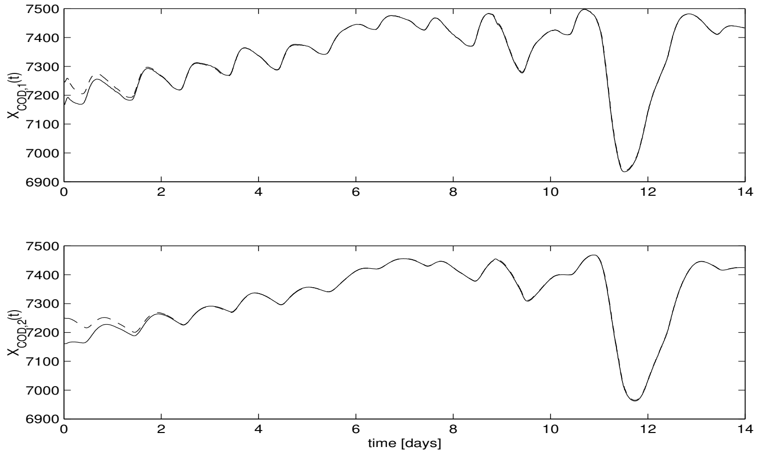

Figure 4.4: Upper plot: The dashed line shows the EKF estimated value of $X_{COD,1}(t)$ and the solid line the true value of $X_{COD,1}(t)$. Lower plot: The dashed line shows the EKF estimated value of $X_{COD,2}(t)$ and the solid line the true value of $X_{COD,2}(t)$.

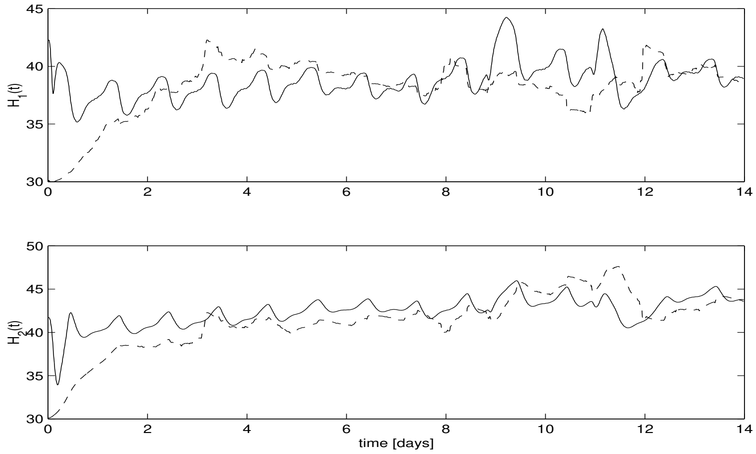

Figure 4.5: Upper plot: The dashed line shows the EKF estimated value of the hydrolysis term $H_1(t)$ and the solid line the true value. Lower plot: The dashed line shows the EKF estimated value of the hydrolysis term $H_2(t)$ and the solid line the true value.

4.3.3 Estimating the Reaction Rate Terms, $r_{S_{COD,1}}(t)$ and $r_{S_{COD,2}}(t)$, during Toxic Load Event

In order to investigate the performance of the EKF estimates of $r_{S_{COD,1}}(t)$ and $r_{S_{COD,2}}(t)$ during abnormal conditions, a toxic load event was simulated using the dryinfluent.mat file. It was assumed that the toxic load results in an increased decay of microorganisms and a decrease of the specific growth rate. The process conditions are changed more drastically in the aerobic compartments, since the nitrifying bacteria are generally more sensitive to toxic compounds than other bacteria.

The same LQG controller as in Section 4.3.2 was used in the simulation. After day one, the decay rate coefficient, b_a, for autotrophic biomass is suddenly increased with 60 % and the decay rate coefficient, b_h, for heterotrophic biomass is increased with 30 %. At the same event, the maximum specific growth rate for autotrophic biomass is decreased with 50 % and the maximum specific growth rate for heterotrophic biomass decreased with 30 %. This may simulate the behavior of the plant when a toxic compound suddenly appears in the wastewater. The coefficients are thereafter changed stepwise back to normal values, and after 8.3 days the biological properties of the microorganisms are recovered to normal conditions.

Results and Discussion

Figure 4.6 illustrates the EKF estimation of $r_{S_{COD,1}}(t)$ and $r_{S_{COD,2}}(t)$. It is seen that especially $r_{S_{COD,2}}(t)$ is difficult to estimate during the toxic load event. The explanation is probably a too poor excitation of the input signal, since the external carbon flow rate, $u(t)$, is zero during long periods. However, the result shows that at least the estimate of $r_{S_{COD,1}}(t)$ may be used as a toxic load indicator.

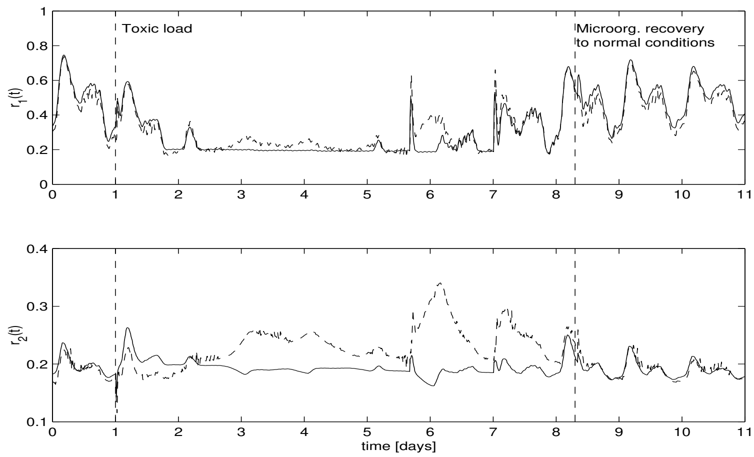

Figure 4.6: Upper plot: The dashed line shows the EKF estimated value of the reaction rate term $r_{S_{COD,1}}(t)$ and the solid line the true value. Lower plot: The dashed line shows the EKF estimated value of the reaction rate term $r_{S_{COD,2}}(t)$ and the solid line the true value.

4.3.4 Estimation of Y_H

As in Section 4.3.2, the assumptions (**A4.1**)-(**A4.3**) are used. In order to make the estimation procedure simpler, it was also assumed that the nitrate in the first anoxic compartment, $S_{NO,1}(t)$, was measured. An LQG controller was used simultaneously with the estimation of the unknown parameters, in order to control the nitrate concentration in the last anoxic compartment. In the estimation, influent data were taken from the file dryinfluent.mat. The data used for estimation were sampled with an interval of 1 minute without delay on the sensors. However, measurement noise with a standard deviation of 0.1 was added to the nitrate concentrations $S_{NO,1}(t)$, $S_{NO,2}(t)$ and $S_{NO,in}(t)$ and with a standard deviation of 0.05 to the SS concentrations $X_{TSS,1}(t)$, $X_{TSS,2}(t)$ and $X_{TSS,in}(t)$.

The initial value of the covariance matrix in the EKF was chosen as $P(0) = diag[1, 1, 1, 1, 1, 1, 5, 5, 10, 50, 50]$, and the design variables as $R_1 = diag[1, 1, 1, 1, 1, 1, 0.05, 0.05, 0.001, 3, 3]$ and $R_2 = 0.01I$. The initial value for the state vector was $[8, 6, 7200, 7200, 7250, 7250, 1, 0.53, 10, 10]^T$.

Results and Discussion

Figure 4.7 shows the external carbon flow rate generated by the LQG controller during the simulation. This is the input used to excite the system. Figure 4.8 illustrates the EKF estimation of Y_H. Although the convergence is fairly slow, it is seen from the plot that the estimate of Y_H comes fairly close to the true value. The estimation performance, especially for the hydrolysis terms and Y_H, seems to be rather dependent on the choice of design variables R_1, R_2, and $P(0)$, and the design variables can only be chosen in a small range to avoid biased estimates. Figure 4.9 shows that the EKF provides good estimates of the reaction rate terms and Figure 4.10 illustrates a satisfactory estimates of the hydrolysis terms.

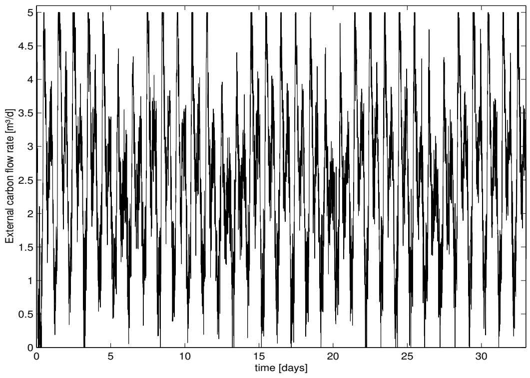

Figure 4.7: The input signal (external carbon flow rate generated by the LQG controller) used to excite the system.

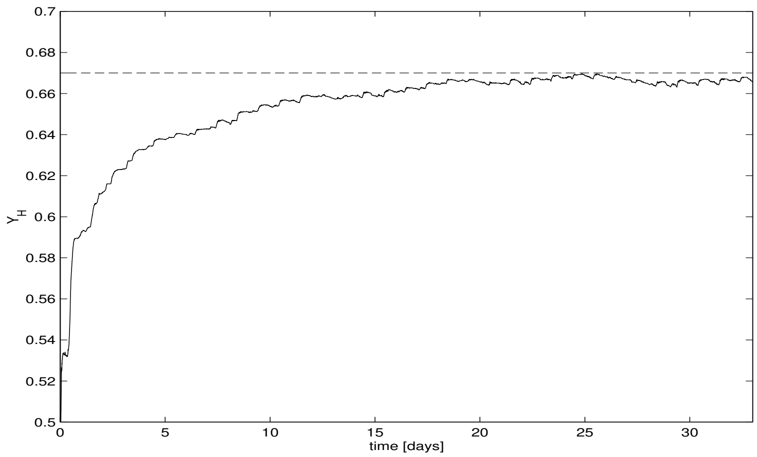

Figure 4.8: Simulations performed with a measurement noise with standard deviation 0.1 on the nitrate sensors, and with a measurement noise with standard deviation 0.05 on the SS sensors. Solid line shows the value of Y_H estimated with the EKF. Dashed line shows the true value.

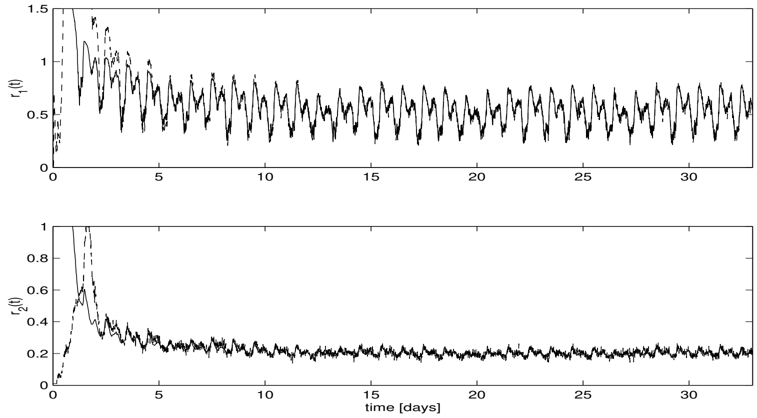

Figure 4.9: Simulations performed with a measurement noise with standard deviation 0.1 on the nitrate sensors, and with a measurement noise with standard deviation 0.05 on the SS sensors. Upper plot: The dashed line shows the EKF estimated value of the reaction rate term $r_{S_{COD,1}}(t)$ and the solid line the true value. Lower plot: The dashed line shows the EKF estimated value of the reaction rate term $r_{S_{COD,2}}(t)$ and the solid line the true value.

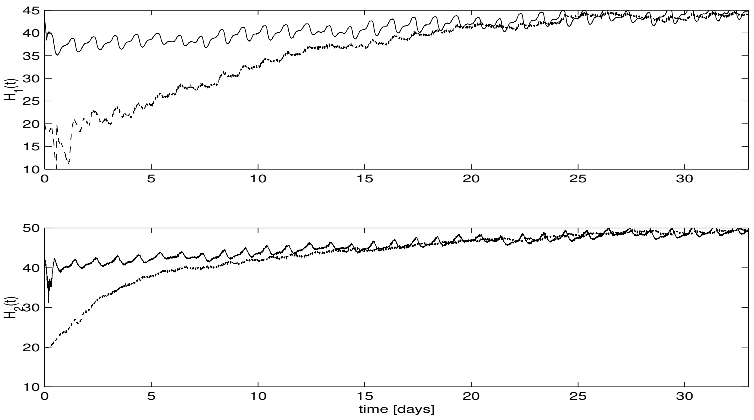

Figure 4.10: Simulations performed with a measurement noise with standard deviation 0.1 on the nitrate sensors, and with a measurement noise with standard deviation 0.05 on the SS sensors. Upper plot: The dashed line shows the EKF estimated value of the hydrolysis term $H_1(t)$ and the solid line the true value. Lower plot: The dashed line shows the EKF estimated value of the hydrolysis term $H_2(t)$ and the solid line the true value.

4.4 Conclusions

In this chapter, a model based control strategy for control of the nitrate level in the last anoxic compartment, using an external carbon source is presented. By lumping some states in the ASM1, a feasible state space model for estimation was derived. The model was then used for on-line design of an integrating LQG controller. This strategy can be regarded as an indirect adaptive LQG control. The controller was evaluated in a simulation study and performed well despite major process disturbances. In the EKF estimation procedure, it is possible to use sensors of more standard type. Interesting topics for further research is to extend the controller for internal flow rate control and supervisory dissolved oxygen control. Also an evaluation using pilot or full scale experiments could be done. The strategy for estimating the reaction rate terms may also be further improved.

Chapter 5

Suboptimal Control of the Bilinear Quadratic Regulator Problem

Physical systems are in general nonlinear by nature, and linear models are often insufficient to describe the behavior of such physical processes. The study of nonlinear systems has become more active and efforts have been made to overcome the complexity and analytical difficulties of nonlinear systems. However, the methods for analyzing and synthesizing controllers for nonlinear systems are still not as well developed as their linear counterparts.

Many processes, such as heat conduction, chemical reactions, fluid flows, distillation columns and biological processes, are adequately described by bilinear models, see for example (Mohler 1991). Overviews of bilinear systems in theory and applications are given in (Bruni *et al.* 1974) and (Mohler and Kolodziej 1980).

The optimal quadratic-cost control problems for bilinear systems have been considered by a number of authors. In most cases it is not possible to give explicit expressions for the optimal feedback control law. Some of the obtained optimal control laws rely on a quadratic cost function modified by incorporations of nonnegative state-dependent penalizing functions, see e.g., (Ryan 1984), (Cebuhar and Costanza 1984), (Tzafestas *et al.* 1984) and (Benallou *et al.* 1988). (Hofer and Tibken 1988) presents an iterative method for the solution of the finite-time optimization problem. In (Aganovic and

Gajic 1993) the recursive scheme in (Hofer and Tibken 1988) is applied together with the reduced-order method for weakly coupled control problems studied in (Su and Gajic 1991), in order to obtain a nearly optimal closed-loop control. The idea of decomposing the system into subsystems is further exploited in (Kim and Lim 2001). In (Cebuhar and Costanza 1984) both the finite-time and the infinite-time case are treated. The infinite-time case has also been investigated in for example (Tzafestas *et al.* 1984) and (Benallou *et al.* 1988). The problem of finding stabilizing feedback controllers for bilinear systems has also been discussed by a number of authors. Some of the results in the literature are partly applicable on the control law developed here. For instance, stabilizing feedback control laws which are quadratic in the states have been treated in (Jacobson 1977), (Slemrod 1978), (Gutman 1981), (Tzafestas *et al.* 1984) and (Benallou *et al.* 1988).

In this chapter, bilinear control systems with quadratic cost functions for the infinite-time case are considered. A suboptimal control law is obtained by approximating the solution of the state-dependent algebraic Riccati equation (ARE) using a Taylor series expansion around a fixed point in the state-space. A similar technique is used in (Wernli and Cook 1975), where the power series is expanded around a temporary variable for more general non-linear systems. The direct solution to the Hamilton-Jacobi equation through expansion in a power series for bilinear systems was analyzed in (Cebuhar and Costanza 1984). In this method a general power series expansion is used. For systems of higher dimensions the general power series expansion usually becomes rather complex. The proposed method in this chapter results in a simpler power series expansion and offline calculations of at most one ARE and a sequence of Lyapunov equations. Furthermore, some stability properties of the proposed suboptimal control law are discussed. A strategy in order to achieve a more robust behavior is also suggested.

The chapter is organized as follows. In Section 5.1, the problem statement and definitions are introduced. The derivation of the suboptimal control law is presented in Section 5.2. Some stability properties of the suggested control law, and a strategy in order to achieve a more robust behavior of the suboptimal control are presented in Section 5.3. In Section 5.4 , several illustrative examples are presented, in which the derived suboptimal control law is compared with other control laws found in the literature. The suggested bilinear control law is applied on an ASP and is compared with an LQ controller in

Section 5.5. Conclusions are given in Section 5.6.

5.1 Definitions and Review of Previous Results

Consider the following bilinear system:

$$\dot{x} = Ax + xNu + Bu, \quad x(0) = x_0, \tag{5.1}$$

The optimal control problem under consideration is to find a control signal u such that the performance index (or cost function):

$$J(x(0), u(\cdot)) = \frac{1}{2}\int_0^T (m(x) + u^T Q_2 u)dt, \tag{5.2}$$

is minimized. Here, the quadratic performance index, $m(x) = x^T Q_1 x$ and $T \to \infty$, will be considered. The following basic notations and assumptions are used:

$$x \in \mathbb{R}^n, \tag{5.3a}$$
$$u \in U \subseteq \mathbb{R}^m, \quad t \in [0, \infty) \tag{5.3b}$$
$$A \in \mathbb{R}^{n\times n}, \quad B \in \mathbb{R}^{n\times m} \tag{5.3c}$$
$$xN = \sum_{j=1}^{n} x_j N_j, \quad N_j \in \mathbb{R}^{n\times m}, \quad j = 1, ..., n, \tag{5.3d}$$
$$Q_1 \in \mathbb{R}^{n\times n}, \quad Q_1 = Q_1^T \succ 0, \tag{5.3e}$$
$$Q_2 \in \mathbb{R}^{m\times m}, \quad Q_2 = Q_2^T \succ 0, \tag{5.3f}$$

where $Q_2 \succ 0$ means that Q_2 is positive definite.

The following assumption is made:

A5.1. The matrix

$$M(x) = [(xN + B) \quad A(xN + B) \ \dots \ A^{n-1}(xN + B)] \tag{5.4}$$

has rank n for all x under consideration.

Assumption **A5.1** implies that the pair (A, B) is controllable and that x stays in the controllability domain defined by:

$$X_c \triangleq \{x \in \mathbb{R}^n | (A, B + xN) \text{ controllable}\}. \tag{5.5}$$

The Hamilton-Jacobi equation for the problem is (Bryson and Ho 1975):

$$-\frac{\partial \bar{V}}{\partial t} = \min_u \left[\frac{1}{2} m(x) + \frac{1}{2} u^T Q_2 u + (\frac{\partial \bar{V}}{\partial x}) Ax + (\frac{\partial \bar{V}}{\partial x})(xN + B)u \right], \tag{5.6}$$

where the right hand-side is minimized by the optimal control,

$$u = -Q_2^{-1}(xN + B)^T \frac{\partial}{\partial x} \bar{V}^T(x, t, T). \tag{5.7}$$

It is assumed that:

A5.2. There exists an optimal control and a continuously differentiable positive definite function $\bar{V}(x, t, T) : \mathbb{R}^n \to \mathbb{R},\ t \in [0, T]$ satisfying the standard Hamilton-Jacobi equation (5.6).

(Moyland and Anderson 1973) prove that, with the assumptions **A5.1** and **A5.2**, where X_c in (5.5) is the whole space $\mathbb{R}^n$, the solution to the Hamilton-Jacobi equation (5.6) is independent of time. In connection with (5.2), the optimal performance index is then given by:

$$V(x) = \lim_{T \to \infty} \bar{V}(x, t, T) = \inf_{u \in U} J(x, u(\cdot)), \tag{5.8}$$

and the optimal control is given by (5.7) with $\bar{V}(x, t, T)$ replaced by $V(x)$

$$u = -Q_2^{-1}(xN + B)^T (\frac{\partial V}{\partial x})^T. \tag{5.9}$$

Furthermore, in (Lukes 1969) it is shown that there exists some set Ω, with the origin in its interior, where (5.9) and (5.8) exist, provided that the pair (A, B) is controllable and that (5.3e)-(5.3f) holds.

It is assumed that:

A5.3. The limit on the right side of (5.8) exists and is independent of t, in some set Ω, with the origin in its interior.

Since the effect of the time derivative of $\bar{V}(x,t,T)$ in (5.6) is assumed to vanish as the time approaches infinity, and replacing (5.8) and (5.9) in (5.6), the solution to the Hamilton-Jacobi equation can be simplified to the Hamilton-Jacobi-Bellman (HJB) equation:

$$\frac{1}{2}m(x) + (\frac{\partial V}{\partial x})Ax - \frac{1}{2}(\frac{\partial V}{\partial x})(xN+B)Q_2^{-1}(xN+B)^T(\frac{\partial V}{\partial x})^T = 0. \quad (5.10)$$

It can be shown that the form:

$$(\frac{\partial V}{\partial x}) = x^T S(x), \quad S(x) : \mathbb{R}^n \to \mathbb{R}^{n\times n} \quad (5.11)$$

is permitted if and only if the matrix $\frac{\partial}{\partial x}[S^T(x)x]$ is symmetric, for a proof see (Jacobson *et al.* 1980).

It is assumed that:

A5.4. There exists a symmetric matrix-valued function $S(x)$, where $\frac{\partial}{\partial x}[S^T(x)x]$ is symmetric.

This yields, by replacing (5.11) in (5.9), that the optimal control law can be written as:

$$u = -Q_2^{-1}(xN+B)^T S(x)x. \quad (5.12)$$

By replacing (5.11) in (5.10), $S(x)$ will be the solution to the state-dependent ARE:

$$Q_1 + S(x)A + A^T S(x) - S(x)(xN+B)Q_2^{-1}(xN+B)^T S(x) = 0. \quad (5.13)$$

A unique positive definite solution to (5.13) exists if the Assumption **A5.1** holds, see (Kalman 1960). A discussion of conditions under which (5.12) and (5.13) have a unique continuous solution $u(x)$ inside the regions $\|u\| < q$, where q is arbitrary, for all $x \in \mathbb{R}^n$ inside the region $\|x\| < r$ if r is sufficiently small, can be found in (Wernli and Cook 1975).

5.2 Approximative Solution to the HJB Equation

In most cases it is not possible to obtain a closed form solution to the state-dependent ARE (5.13). In this section an approximative solution to (5.13) is derived by constructing the control law as a power series. The generalized power series (Volterra series) expansion of (5.11) is investigated in (Cebuhar and Costanza 1984). The idea in this section is to apply a simpler Taylor series expansion of the matrix $S(x)$ in (5.13).

Using (5.3d), the state-dependent ARE (5.13) can be written as:

$$
\begin{aligned}
&Q_1 + S(x)A + A^T S(x) - S(x)\Big[BQ_2^{-1}B^T + (BQ_2^{-1}N_1^T + N_1Q_2^{-1}B^T)x_1 \\
&+ (BQ_2^{-1}N_2^T + N_2Q_2^{-1}B^T)x_2 + \cdots + (BQ_2^{-1}N_n^T + N_nQ_2^{-1}B^T)x_n + \\
&(N_1Q_2^{-1}N_2^T + N_2Q_2^{-1}N_1^T)x_1x_2 + \cdots + (N_1Q_2^{-1}N_n^T + N_nQ_2^{-1}N_1^T)x_1x_n \\
&+ \cdots + (N_{n-1}Q_2^{-1}N_n^T + N_nQ_2^{-1}N_{n-1}^T)x_{n-1}x_n + N_1Q_2^{-1}N_1^Tx_1^2 \\
&+ \cdots + N_nQ_2^{-1}N_n^Tx_n^2\Big]S(x) = 0.
\end{aligned}
\tag{5.14}
$$

The Taylor series expansion is applied to the matrix $S(x)$. It is assumed that:

A5.5. The elements in $S(x)$ have continuous partial derivatives up to order $k+1$ at all points in a neighborhood of the origin.

Then, the Taylor polynomial (or the MacLaurin polynomial) of degree k provides a kth-degree polynomial approximation to $S(x)$ in a neighborhood of the origin. As an illustration, it is assumed that the state x is of dimension two, and the Taylor series expansion is written as a function of two variables. However, the extension to higher dimensions is analogous. Thus, for the two-dimensional case, the Taylor matrix polynomial of degree k is:

$$
\hat{S}^k(x_1, x_2) = \sum_{i=0}^{k}\sum_{j=0}^{i} \hat{S}_{i-j,j}x_1^{i-j}x_2^j\frac{1}{j!(i-j)!}. \tag{5.15}
$$

Replacing $S(x)$ in (5.14) by (5.15) and separating out and collecting terms by powers of x, results in the following system of equations for $n = 2$:

$$\hat{S}_{0,0}A + A^T\hat{S}_{0,0} - \hat{S}_{0,0}BQ_2^{-1}B^T\hat{S}_{0,0} + Q_1 = 0 \tag{5.16a}$$

$$\hat{S}_{1,0}(A - BQ_2^{-1}B^T\hat{S}_{0,0}) + (A^T - \hat{S}_{0,0}BQ_2^{-1}B^T)\hat{S}_{1,0} = \\ \hat{S}_{0,0}(BQ_2^{-1}N_1^T + N_1Q_2^{-1}B^T)\hat{S}_{0,0} \tag{5.16b}$$

$$\hat{S}_{0,1}(A - BQ_2^{-1}B^T\hat{S}_{0,0}) + (A^T - \hat{S}_{0,0}BQ_2^{-1}B^T)\hat{S}_{0,1} = \\ \hat{S}_{0,0}(BQ_2^{-1}N_2^T + N_2Q_2^{-1}B^T)\hat{S}_{0,0} \tag{5.16c}$$

$$\hat{S}_{1,1}(A - BQ_2^{-1}B^T\hat{S}_{0,0}) + (A^T - \hat{S}_{0,0}BQ_2^{-1}B^T)\hat{S}_{1,1} = \\ \hat{S}_{0,0}(N_1Q_2^{-1}N_2^T + N_2Q_2^{-1}N_1^T)\hat{S}_{0,0} + \hat{S}_{1,0}BQ_2^{-1}B^T\hat{S}_{0,1} + \\ \hat{S}_{0,1}BQ_2^{-1}B^T\hat{S}_{1,0} + \hat{S}_{0,0}(BQ_2^{-1}N_2^T + N_2Q_2^{-1}B^T)\hat{S}_{0,1} + \\ \hat{S}_{0,1}(BQ_2^{-1}N_2^T + N_2Q_2^{-1}B^T)\hat{S}_{0,0} + \hat{S}_{0,0}(BQ_2^{-1}N_1^T + N_1Q_2^{-1}B^T)\hat{S}_{1,0} \\ + \hat{S}_{1,0}(BQ_2^{-1}N_1^T + N_1Q_2^{-1}B^T)\hat{S}_{0,0} \tag{5.16d}$$

$$\vdots$$

$$\hat{S}_{k,j}(A - BQ_2^{-1}B^T\hat{S}_{0,0}) + (A^T - \hat{S}_{0,0}BQ_2^{-1}B^T)\hat{S}_{k,j} = T_{k-1,j-1}, \tag{5.16e}$$

where $T_{k-1,j-1}$ is a $\mathbb{R}^{n\times n}$-valued function of $\hat{S}_{0,0}$, $\hat{S}_{1,0}$, $\hat{S}_{0,1}$, $\hat{S}_{1,1}$,...,$\hat{S}_{k-1,j-1}$. Equation (5.16a) is the standard ARE for the linear part. (5.16b)-(5.16e) are Lyapunov matrix equations. The solutions to (5.16b)-(5.16e) are unique and symmetric, since the right hand side of the equations (5.16b)-(5.16e) are symmetric and the matrix $(A - BQ_2^{-1}B^T\hat{S}_{0,0})$ is Hurwitz.

Often, some of the matrices N_j, $j = 1, ..., n$ are zero. For simplicity assume that:

$$\begin{aligned} N_j &\neq 0 \quad j = 1, .., n_1 \\ N_j &= 0 \quad j = n_1 + 1, .., n. \end{aligned}$$

Denote $\bar{x} = [x_1 \dots x_{n_1}]^T$ and $\tilde{x} = [x_{n_1+1} \dots x_n]^T$. The following result then holds.

Conjecture 5.2.1 *The expression inside the square bracket in (5.14) will not contain the states $\tilde{x}$.*

Conjecture 3.1 is verified by collecting the terms containing the states $\tilde{x}$. It is easily seen that for every term containing one of the states in $\tilde{x}$, the corresponding system of equations will have the same form:

$$\tilde{S}(A - BQ_2^{-1}B^T S_{0,0}) + (A - BQ_2^{-1}B^T S_{0,0})^T \tilde{S} = 0. \tag{5.17}$$

Since the matrix $(A - BQ_2^{-1}B^T S_{0,0})$ is Hurwitz, the unique solution to the Lyapunov matrix equation (5.17) is $\tilde{S} = 0$. The matrix $S(x)$ is then a function of the remaining states $\bar{x}$, and the Taylor series expansion needs only be applied to the matrix $S(\bar{x}) \; : \mathbb{R}^{n_1} \to \mathbb{R}^{n \times n}$.

By insertion of (5.15) in to (5.12), the suboptimal control law, as a function of two state variables, obtains the form:

$$u = -Q_2^{-1}(xN + B)^T (\sum_{i=0}^{k} \sum_{j=0}^{i} \hat{S}_{i-j,j} x_1^{i-j} x_2^j \frac{1}{j!(i-j)!})x. \tag{5.18}$$

In what follows, the general suboptimal control law for systems in one or several state variables will be defined as:

$$u = -Q_2^{-1}(xN + B)^T \hat{S}^k(\bar{x})x. \tag{5.19}$$

One advantage of the Taylor series expansion, in comparison with the generalized power series method derived in (Cebuhar and Costanza 1984), is that the procedure of collecting terms by powers of x is simpler for the Taylor series expansion method. The implementation part might also be easier since it only requires the computation of one ARE and $k-1$ Lyapunov matrix equations. The generalized power series method requires the computation of one ARE and matrix equations of the form $(\frac{1}{2}(A+A^T) - BQ_2^{-1}B^T P_1)P_k = T_k$, $k = 2, 3, ...$, where P_1 is the solution of the ARE and T_k is a $\mathbb{R}^{n \times (k+1)}$ -valued function of P_1, P_2, ...,P_{k-1}.

Using assumption **A5.5**, the Taylor series (5.15) has a positive radius of convergence, if the function $S(\bar{x})$ is analytic in a neighborhood of $\bar{x} = 0$. However, the control law (5.18), or (5.19) for $n > 2$, may still be useful outside the region of convergence.

Remark 5.1. If $B = 0$, the system (5.1) is called homogeneous in the state. The Taylor series expansion method and the control law (5.19) may still be applicable to that class of bilinear systems. Equation (5.16a) is then reduced to the Lyapunov matrix equation $S_{0,0}A + A^T S_{0,0} + Q_1 = 0$. It is known that a unique solution exists if and only if $\lambda \in \sigma(A) \Rightarrow -\bar{\lambda} \notin \sigma(A)$, where λ are the eigenvalues of A and $\sigma(A)$ the spectrum of A. This is satisfied if A is Hurwitz. If Q_1 is positive definite, then $S_{0,0}$ is also positive definite, if A is Hurwitz.

5.3 Stability of the Suboptimal Control Law

The problem of finding stabilizing feedback controllers for bilinear systems has been treated by a number of authors. Some of the results in the literature are applicable to the control law (5.19), especially if only the first term in the Taylor series is used.

The following basic theorem is partly adopted from (Jacobson 1977).

Theorem 5.3.1 *Assume that the system (5.1) is controlled with (5.9) and Assumption* ***A5.3*** *holds, where Ω is the whole space $\mathbb{R}^n$, and $V(x)$ in (5.8) is positive definite and radially unbounded. The optimal closed-loop system is then globally asymptotically stable.*

Proof Let $V(x)$ in (5.8) be a candidate Lyapunov function of the closed-loop system. The rate of change along any optimal trajectory is then given by:

$$
\begin{aligned}
\dot{V}(x) &= (\frac{\partial V}{\partial x})\left[Ax - (xN+B)Q_2^{-1}(xN+B)^T(\frac{\partial V}{\partial x})^T\right] = \\
&= -\frac{1}{2}m(x) - \frac{1}{2}(\frac{\partial V}{\partial x})(xN+B)Q_2^{-1}(xN+B)^T(\frac{\partial V}{\partial x})^T = \\
&= -\frac{1}{2}x^TQ_1x - \frac{1}{2}(\frac{\partial V}{\partial x})(xN+B)Q_2^{-1}(xN+B)^T(\frac{\partial V}{\partial x})^T < 0, \;\; \forall x \neq 0.
\end{aligned}
$$

Thus, $V(x)$ is a Lyapunov function for the optimal system. ■

The assumption $m(x) > 0 : \forall x \neq 0$ implies that $(\frac{\partial V}{\partial x})Ax < 0 : \forall x \neq 0$. The requirement that $m(x)$ should be positive definite can be relaxed to $m(x) \geq 0$ and $(\frac{\partial V}{\partial x})Ax \leq 0$. The control law (5.9) still globally asymptotically stabilizes the system (5.1) under the condition of the following theorem.

Theorem 5.3.2 *Suppose there exists a positive definite and radially unbounded function $V(x) : \mathbb{R}^n \to \mathbb{R}$, which is once continuously differentiable such that $(\frac{\partial V}{\partial x})Ax \leq 0$. Then, if there is no non-zero $x \in \mathbb{R}^n$ for which $(\frac{\partial V}{\partial x})Ax$ and $(\frac{\partial V}{\partial x})(xN + B)$ are both zero,*

$$u = -Q_2^{-1}(xN + B)^T(\frac{\partial V}{\partial x})^T$$

globally asymptotically stabilizes the system (5.1).

Proof See e.g. Theorem 5 in (Glad 1987), or Theorem 2.5.4 in (Jacobson 1977). ■

For the suboptimal control law (5.19) we have the following lemma (which is weaker than the result from Theorem 5.3.1):

Lemma 5.3.1 *Assume that (A,B) is a stabilizable pair, then the suboptimal control law (5.19) makes the closed-loop system of (5.1) asymptotically stable at the equilibrium point $x = 0$.*

Proof using the control law (5.19) results in the closed-loop system:

$$\begin{aligned} \dot{x} = f(x) = Ax - (xN + B)Q_2^{-1}(xN + B)^T\hat{S}^k(\bar{x})x = \\ Ax - BQ_2^{-1}B^T\hat{S}_0x - \psi(\bar{x})x, \end{aligned} \tag{5.20}$$

where $\hat{S}_0$ is the matrix solution from the ARE (5.16a) for an arbitrary n. We have $\psi(\bar{x}) = O(\bar{x})$, and $f : D \to \mathbb{R}^n$ is continuously differentiable for a D in a neighborhood of the origin (Khalil 1996). Moreover, $\frac{\partial \psi}{\partial x}(\bar{x})|_{x=0} = 0$, and since (A,B) is a stabilizable pair the Jacobian matrix

$$\frac{\partial f}{\partial x}(x)|_{x=0} = A - BQ_2^{-1}B^T\hat{S}_0 \tag{5.21}$$

has all eigenvalues strictly in the left half-plane. This proves that the origin for the system (5.20) is asymptotically stable. ■

Remark 5.2. For the homogeneous class of bilinear systems it has to be assumed that A is Hurwitz to guarantee asymptotic stability at the origin. It has been shown, for example in (Jacobson 1977), (Gutman 1980) and

(Benallou *et al.* 1988), that if A is Hurwitz, then a feedback control of the form $u = -Q_2^{-1}(xN + B)^T Sx$, where S satisfies

$$SA + A^T S = -Q \tag{5.22}$$

and Q is positive definite, will make the closed-loop system of the bilinear system globally asymptotically stable. If A is non-Hurwitz, then the globally asymptotic stabilization depends on whether a positive definite solution to (5.22), where Q is positive semidefinite, can be found or not. Here, the result from Theorem 5.3.2 can be useful.

Stability results for the case when A is non-Hurwitz can be found in e.g. (Slemrod 1978) and (Gutman 1981). A comprehensive study covering the cases when A has eigenvalues on the imaginary axis, or when A is strictly unstable, can be found in (Gutman 1980).

In (Tzafestas *et al.* 1984) it is shown that a quadratic feedback control law, where the positive definite matrix satisfies the ARE of the type (5.16a), will stabilize (5.1). The following theorem can be established.

Theorem 5.3.3 *Suppose that the system (5.1) is controlled with*

$$u = -Q_2^{-1}(xN + B)^T Sx, \tag{5.23}$$

where the symmetric $n \times n$ positive definite matrix S satisfies the ARE:

$$SA + A^T S - SBQ_2^{-1}B^T S + Q = 0. \tag{5.24}$$

Consider further a cost function of the form:

$$J = \frac{1}{2}\int_0^\infty (L(x) + u^T Q_2 u)dt. \tag{5.25}$$

The feedback control law (5.23) will then minimize the cost function (5.25) and simultaneously make the closed-loop system asymptotically stable, if $L(x)$ in (5.25) is chosen as:

$$L(x) = x^T \left[Q + S((xN + B)Q_2^{-1}(xN + B)^T - BQ_2^{-1}B^T)S\right] x, \tag{5.26}$$

and Q satisfies

$$Q_1 + SBQ_2^{-1}B^T S \preceq Q, \tag{5.27}$$

where Q_1 and Q_2 are subjected to the conditions (5.3e)-(5.3f).

Proof The proof is partly adopted from Theorem 2 of (Tzafestas *et al.* 1984), where the theorem is shown for the finite time case.

The optimal control law (5.23) inserted in (5.10) and replacing $m(x)$ by $L(x)$ in (5.10), yields the steady-state HJB

$$\begin{aligned}0 &= \frac{1}{2}x^T\left[Q + S\left((xN+B)Q_2^{-1}(xN+B)^T - BQ_2^{-1}B^T\right)S\right]x+\\ &\frac{1}{2}x^T S(xN+B)Q_2^{-1}(xN+B)^T Sx\\ &+ x^T SAx - x^T S(xN+B)Q_2^{-1}(xN+B)^T Sx\\ &= \frac{1}{2}x^T\left[Q + SA + A^T S - SBQ_2^{-1}B^T S\right]x. \qquad (5.28)\end{aligned}$$

From (5.28) one obtains a ARE identical to (5.24), which reveals that the cost function (5.25) is minimized.

Taking $V(x) = x^T Sx$ as a candidate Lyapunov function of the closed-loop system, and using (5.28) one obtains

$$\begin{aligned}\dot{V}(x) &= x^T SAx - x^T S(xN+B)Q_2^{-1}(xN+B)^T Sx =\\ &= -\frac{1}{2}x^T(Q - SBQ_2^{-1}B^T S)x\\ &- x^T S((xN+B)Q_2^{-1}(xN+B)^T Sx < 0 \quad \forall x \neq 0. \qquad (5.29)\end{aligned}$$

Thus, the closed-loop system is globally asymptotically stable. ■

Comments on Theorem 5.3.3: Note that the control law (5.23) will be equal to (5.19) if only the first matrix term in $\hat{S}^k(\bar{x})$ is used. One question arising is: When is it possible to find a matrix Q satisfying (5.27)? If A is Hurwitz, such a Q can always be found. This can for example be seen by considering the Lyapunaov matrix equation (5.22). Assume that $\bar{S}$ is the unique positive solution to (5.22) for an arbitrarily positive definite $\bar{Q}$ (such a unique solution always exists if A is Hurwitz). A Q satisfying (5.27) can then be found by inserting $Q = \bar{Q} + \bar{S}BQ_2^{-1}B^T\bar{S}$ in the ARE (5.24). The solution will be $S = \bar{S}$, and obviously $Q - SBQ_2^{-1}B^T S = \bar{Q} \succ 0$. The same transformation can be used for the case when A is non-Hurwitz and there exists a positive definite solution to (5.22), with Q positive semidefinite, see **Remark 5.2**. Moreover, if it is not possible to find a positive definite Q

satisfying (5.22), then there exists no Q satisfying (5.27). ■

Lemma 5.3.1 shows that (5.23) will make the closed-loop system asymptotically stable at $x = 0$. Since the Jacobian matrix (5.21) is a stability matrix, one can always estimate the stability domain (or the region of attraction) of the origin (Khalil 1996). We shall assume that the stability domain can be chosen as:

$$\Omega_\delta = \{x : V(x) \leq \delta\} \text{ for some } \delta > 0. \tag{5.30}$$

The following corollary is then obtained, which extends Theorem 5.3.3.

Corollary 5.3.1 *Assume that (A,B) is a stabilizable pair. Then the control law (5.23) in Theorem 5.3.3 makes the closed-loop system of (5.1) asymptotically stable at the equilibrium point $x = 0$. Moreover, Ω_δ in (5.30) is an estimate of its stability domain, if $V(x)$ is a Lyapunov function for the system in a neighborhood of the origin.*

Proof The proof follows easily from the Lyapunov-Poincaré theorem, see e.g. (Khalil 1996). ■

In practice, it is necessary to truncate the Taylor series of $S(x)$. It is then possible that some of the desirable stability properties will be lost, see Theorem 5.3.1 and Theorem 5.3.2. In order to further investigate the stability properties of the truncated feedback control law, the techniques of the inverse optimal control problem can be used. One can define a function $\tilde{m}(x)$ such that the truncated $\hat{S}^k(\bar{x})x = (\frac{\partial \Phi}{\partial x})^T$ in (5.19) and the corresponding truncated optimal cost $\Phi(x)$ is the solution to the HJB equation, where $m(x)$ in (5.2) is replaced by $\tilde{m}(x)$. $\tilde{m}(x)$ can be determined by the following equation (Glad 1987).

$$\tilde{m}(x) = -\min_{u \in U}[\frac{1}{2}u^T Q_2 u + (\frac{\partial \Phi}{\partial x})(Ax + (xN + B)u)]. \tag{5.31}$$

If $\tilde{m}(x)$ is positive definite and there exists a unique minimum, where the truncated $\Phi(x) > 0$ is radially unbounded and (5.19) is at least continuous, then according to Theorem 5.3.1, the optimally closed-loop system is globally asymptotically stable. $\tilde{m}(x)$ is given from (5.31) as:

$$\tilde{m}(x) = -x^T \hat{S}^k(\bar{x})Ax + \frac{1}{2}x^T \hat{S}^k(x)(xN + B)Q_2^{-1}(\bar{x}N + B)^T \hat{S}^k(\bar{x})x. \tag{5.32}$$

It is easily seen from (5.32) that if

$$x^T \hat{S}^k(\bar{x})Ax < 0 \tag{5.33}$$

is satisfied, the suboptimally controlled system will be globally asymptotically stable. Note that if l is odd, $\tilde{m}(x)$ can never be positive definite. However, the tests for positive definiteness of (5.32), or negative definiteness of (5.33), may be difficult, especially for polynomials of more than two variables.

5.3.1 Robustness of the Suboptimal Control Law

Theorem 5.3.1 and 5.3.2 show that optimal bilinear feedback has certain intrinsic stability properties. This is also true for more general nonlinear optimal feedback. The robustness properties include infinite gain margin, where the gain can be static, dynamic or nonlinear. One way of investigating stability robustness is to study a perturbed system. Generally, the robustness is analyzed with respect to gain margin and gain reduction, and the perturbations considered are often static or dynamic input perturbations. A survey of robustness of nonlinear state feedback can be found in (Glad 1987).

In this section the truncated suboptimal control law (5.19) will be considered as a nominal input $u_{nom}(x)$ perturbed additively by the perturbation $u_p(x)$,

$$u = -Q_2^{-1}(xN + B)^T \hat{S}^k(\bar{x})x = -u_{nom}(x) + u_p(x). \tag{5.34}$$

In connection with Assumption **5.3**, it is assumed that there exists some set Ω with $x = 0$ in its interior, where the optimal control law is given by $u_{nom}(x)$ with the associated optimal cost $\Phi_{nom}(x)$, and where an arbitrary $m(x)$ in (5.2) satisfies:

$$m(0) = 0; \quad m(x) \geq 0 \quad \forall x \neq 0. \tag{5.35}$$

If $u_{nom}(\cdot)$ and $\Phi_{nom}(\cdot)$ are continuously differentiable on Ω and satisfy the corresponding stationary HJB equation, then with the Assumption **A5.1** the equilibrium point $x = 0$ is asymptotically stable. The necessary optimality condition of the HJB equation imposes that the optimal control law is given by

$$u_{nom}(x) = -Q_2^{-1}(xN + B)^T (\frac{\partial \Phi_{nom}}{\partial x})^T. \tag{5.36}$$

Defining its stability domain as Ω_δ, the following theorem can be established (Geromel and Yamakami 1985).

Theorem 5.3.4 *Suppose that the system (5.1) is controlled by the additively perturbed control law (5.34), where* $u_p(x) : \mathbb{R}^n \to \mathbb{R}^m; u_p(0) = 0$*, then the closed-loop system remains asymptotically stable with stability domain* Ω_δ *if*

$$m(x) > u_p(x)^T Q_2 u_p(x) \quad \forall x \neq 0 \in \Omega_\delta. \tag{5.37}$$

Proof Taking $\Phi_{nom}(x)$ as a candidate Lyapunov function, we can determine:

$$\begin{aligned}
\dot{\Phi}_{nom}(x) &= (\frac{\partial \Phi_{nom}}{\partial x})[Ax - (xN + B)(u_{nom}(x) + u_p(x))] = \\
&= -\frac{1}{2}m(x) - \frac{1}{2}u_{nom}(x)^T Q_2 u_{nom}(x) + (\frac{\partial \Phi_{nom}}{\partial x})(xN + B)u_p(x),
\end{aligned} \tag{5.38}$$

where the associated HJB equation is used in the second equality. From (5.36) we have

$$(\frac{\partial \Phi_{nom}}{\partial x})(xN + B) = u_{nom}(x)^T Q_2 \tag{5.39}$$

and (5.38) turns out to be

$$\begin{aligned}
&\dot{\Phi}_{nom}(x) = -\frac{1}{2}m(x) - \frac{1}{2}u_{nom}(x)^T Q_2 u_{nom}(x) + u_{nom}(x)^T Q_2 u_p(x) \\
&+ \frac{1}{2}u_p(x)^T Q_2 u_p(x) - \frac{1}{2}u_p(x)^T Q_2 u_p(x) = \\
&= -\frac{1}{2}m(x) + u_p(x)^T Q_2 u_p(x) - \frac{1}{2}(u_{nom}(x) - u_p(x))^T Q_2 (u_{nom}(x) - u_p(x)) \\
&\leq -\frac{1}{2}\left[m(x) - u_p(x)^T Q_2 u_p(x)\right].
\end{aligned} \tag{5.40}$$

If (5.37) is satisfied we get

$$\dot{\Phi}_{nom}(x) < 0 \quad \forall x \neq 0 \in \Omega_\delta \tag{5.41}$$

and the theorem is proved. The proof is also given for more general nonlinear systems affine in the input in Theorem 2 of (Geromel and Yamakami 1985). ■

Comments on Theorem 5.3.4: Note that if the nominal input is defined as in (5.23) and if Q satisfies (5.27), then Ω_δ is the whole space $\mathbb{R}^n$. It is then obvious from Theorem 5.3.3 that the closed-loop system will remain globally asymptotically stable if $u_p(x)$ satisfies the condition (5.37). Moreover, $m(x)$ will be of the degree 4, since from (5.25) $m(x) = L(x)$. The perturbation input will be of the degree $k+2$ where $k \geq 1$, since

$$u_p(x) = -Q_2^{-1}(xN + B)^T(\hat{S}^k(\bar{x}) - \hat{S})x,$$

where $\hat{S}$ satisfies (5.24) and, hence, is the first constant matrix term in the truncated Taylor matrix expansion $\hat{S}^k(\bar{x})$. This implies that

$$\frac{m(x)}{u_p(x)^T Q_2 u_p(x)} \to 0 \text{ as } \|x\|_2 \to \infty.$$

■

Condition (5.37) determines the maximum allowable magnitude of $u_p(x)^T Q_2 u_p(x)$. If $u_p(x)^T Q_2 u_p(x)$ grows faster than $m(x)$, the constraint in (5.37) will be exceeded for larger values of x. However, if the constraint in (5.37) is satisfied in a neighborhood of the origin, it might be possible to bound the magnitude of a known $u_p(x)$ in such a way that (5.37) will still be satisfied for values of x far from the origin. One possibility is to let the additive input perturbation in (5.34) for the truncated suboptimal control law (5.19) be replaced by the bounded signal:

$$u_e(x) = u_p(x)e^{-\frac{u_p(x)^T u_p(x)}{2\epsilon^2\|x\|_2^2}}, \tag{5.42}$$

where $\epsilon > 0 \ \ \epsilon \in \mathbb{R}$. The new input signal

$$u = u_{nom}(x) + u_e(x), \tag{5.43}$$

will approach the suboptimal truncated input (5.34) when x approaches the origin, and approach $u = u_{nom}(x)$ when x is increasing. Let $\lambda_{min}(\cdot)$ and $\lambda_{max}(\cdot)$ denote, respectively, the minimum and maximum eigenvalue of $(\cdot)$. Further, let

$$m(x) = x^T Q_1 x + \tilde{m}(x), \tag{5.44}$$

where $Q_1 \succ 0$ and $\tilde{m}(0) = 0$; $\tilde{m}(x) \geq 0$, then the following theorem can be formulated.

Theorem 5.3.5 *Suppose that the system (5.1) is controlled by the additively perturbed control law (5.43) and let m(x) be as in (5.44), then the closed-loop system remains asymptotically stable with stability domain Ω_δ if*

$$\epsilon < \Big(\frac{\lambda_{min}(Q_1)}{\lambda_{max}(Q_2)}e\Big)^{\frac{1}{2}} \tag{5.45}$$

Proof From condition (5.37) we have

$$m(x) > u_e(x)^T Q_2 u_e(x) = u_p(x)^T Q_2 u_p(x) e^{-\frac{u_p(x)^T u_p(x)}{\epsilon^2 \|x\|_2^2}} \leq$$
$$\leq u_p(x)^T u_p(x) e^{-\frac{u_p(x)^T u_p(x)}{\epsilon^2 \|x\|_2^2}} \lambda_{max}(Q_2).$$

Next, let $y = \frac{u_p(x)^T u_p(x)}{\epsilon^2 \|x\|_2^2}$, using the condition $y - \ln(y) \geq 1$ for $y > 0$, we obtain

$$\frac{u_p(x)^T u_p(x)}{\epsilon^2 \|x\|_2^2} - \ln\left(\frac{u_p(x)^T u_p(x)}{\epsilon^2 \|x\|_2^2}\right) \geq 1 \quad \Leftrightarrow$$
$$\Leftrightarrow \quad \ln\left(u_p(x)^T u_p(x)\right) - \frac{u_p(x)^T u_p(x)}{\epsilon^2 \|x\|_2^2} \leq -1 + \ln\left(\epsilon^2 \|x\|_2^2\right) \quad \Leftrightarrow$$
$$\Leftrightarrow \quad u_p(x)^T u_p(x) e^{-\frac{u_p(x)^T u_p(x)}{\epsilon^2 \|x\|_2^2}} \leq e^{-1}\epsilon^2 \|x\|_2^2,$$

and we will obtain the loose upper limit bound

$$u_p(x)^T u_p(x) e^{-\frac{u_p(x)^T u_p(x)}{\epsilon^2 \|x\|^2}} \lambda_{max}(Q_2) \leq e^{-1}\epsilon^2 \|x\|^2 \lambda_{max}(Q_2).$$

Further, we have $m(x) \geq x^T Q_1 x$. Considering the bounds

$$x^T Q_1 x \geq \lambda_{min}(Q_1)\|x\|^2,$$

imposes the condition

$$\lambda_{min}(Q_1) > e^{-1}\epsilon^2 \lambda_{max}(Q_2),$$

which gives (5.45), and the theorem is proved. ■

Comments on Theorem 5.3.5: If the nominal input is defined as in (5.23) and if Q satisfies (5.27), then it is obvious from Theorem 5.3.3 that the closed-loop system will remain globally asymptotically stable if $u_p(x)$ is replaced by the bounded signal $u_e(x)$ in (5.42), and if ϵ is chosen to satisfy condition (5.45). ■

The control law (5.42) is evaluated in a simulation study in Section 5.4.2.

5.4 Examples

In this section four illustrative examples are presented, where the derived Taylor series approximation control law is compared with other design methods found in the literature.

5.4.1 Example 5.1

The following scalar system and cost function were used in (Cebuhar and Costanza 1984):

$$\dot{x} = -x + (x+2)u, \quad x_0 = 1,$$
$$J = \frac{1}{2}\int_0^\infty (x^2 + 2u^2)dt.$$

For this system a closed form solution for the ARE can be found as:

$$S(x) = \frac{2(\sqrt{1+\frac{1}{2}(x+2)^2} - 1)}{(x+2)^2}, \tag{5.46}$$

and the control law is[1]

$$u = -\frac{(\sqrt{1+\frac{1}{2}(x+2)^2} - 1)}{(x+2)}2x.$$

The Taylor expansion of $S(x)$ in (5.46) at $x = 0$ for the first five terms is compared with the terms obtained from solving (5.16a)-(5.16e) in Table 5.1. In Figure 5.1, simulation results for the exact closed-loop solution are compared with the closed-loop solution obtained when the Taylor series control law derived in Section 5.2 is used. The Taylor series method comes very close to the true solution, using only two terms. For two terms the resulting control law is of order 3 in the state. Worth noticing is that the generalized power series scheme derived in (Cebuhar and Costanza 1984) generates the same control law as the Taylor series method for this scalar system.

[1] Note that the control law, $u = -[2x/(2+x)^2]\{\sqrt{[1+\frac{1}{2}(2+x)^2]} - 1\}$, given in (Cebuhar and Costanza 1984) (p. 622) is not correct.

Table 5.1: The first five "true" terms of the Taylor expansion compared with the approximations.

Terms	True	Approximation	Error
S_0	0.36603	0.36603	0
S_1	-0.07735	-0.07735	0
S_2	0.00990	0.01335	0.0034
S_3	0.00142	0.00027	-0.0011
S_4	-0.00155	-0.00127	0.0003

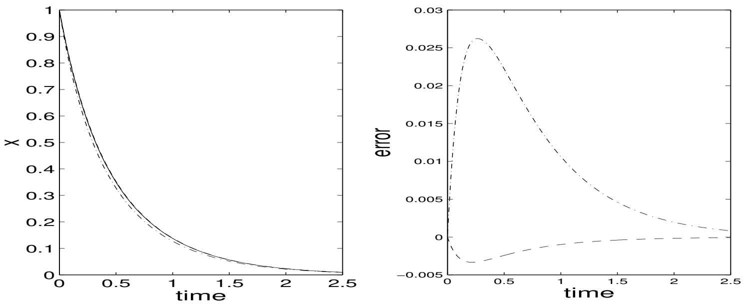

Figure 5.1: Closed-loop solutions for Example 5.1. Left figure: the solid line illustrates the true closed-loop solution, the dashed-dotted line the one-term Taylor series approximation and, finally the dashed line the two-term Taylor series approximation. Right figure: the dashed-dotted line shows the deviation from the "true" closed-loop solution for the one-term Taylor series and, the dashed line the error for the two-term Taylor series approximation.

5.4.2 Example 5.2

The following example was first considered in (Derese and Noldus 1980) to test their design method. The same system was used for design in (Benallou *et al.* 1988) for comparison with the one obtained in (Derese and Noldus 1980). Both design methods incorporate a state-dependent penalizing function in their cost function. The system considered is:

$$\dot{x} = \begin{pmatrix} 0 & 1 \\ -5 & -2 \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \end{pmatrix} x_1 u + \begin{pmatrix} 2 \\ 0 \end{pmatrix} x_2 u + \begin{pmatrix} 2 \\ 0 \end{pmatrix} u.$$

The cost function used for the Taylor series method is:

$$J = \frac{1}{2}\int_0^\infty (x^T Q_1 x + u^T Q_2 u)dt. \tag{5.47}$$

where

$$Q_1 = \begin{pmatrix} 6.02 & -0.06 \\ -0.06 & 5.11 \end{pmatrix}$$

and $Q_2 = 1$ are chosen to be same as in (Derese and Noldus 1980) and (Benallou *et al.* 1988). The control law obtained by (Derese and Noldus 1980) is:

$$u = -3.19x_1 + 0.41x_2$$

and for (Benallou *et al.* 1988)[2]:

$$u = -(xN + B)^T Sx, \tag{5.48}$$

where

$$S = \begin{pmatrix} 9.16 & 0.6 \\ 0.6 & 1.58 \end{pmatrix} \tag{5.49}$$

The control law derived in (Benallou *et al.* 1988) is based on the methods discussed in **Remark 5.2**. Thus, the matrix S satisfies the Lyapunov function (5.22) and the closed-loop system is globally asymptotically stable.

The behavior of the closed-loop systems for the three controllers were evaluated for different initial conditions. The results are shown in Figure 5.2 and Figure 5.3. The results indicate that the response for the state x_1 for the control law obtained by (Benallou *et al.* 1988) is rather fast, whereas the response for the state x_2 is rather slow. However, one important advantage of the control law obtained by (Benallou *et al.* 1988) is that it results in global asymptotic stability of the closed-loop system. This is not the case for the control law derived by (Derese and Noldus 1980) which can be seen in Figure 5.3, were the initial value $x_1(0) = 1$, $x_2(0) = -0.5$ is outside the boundary of the stability region. The control law from the Taylor series approximation is not globally asymptotically stable. This is due to the fact that $Q_1 - SBQ_2^{-1}B^TS$ is not positive definite. In order to obtain a more robust

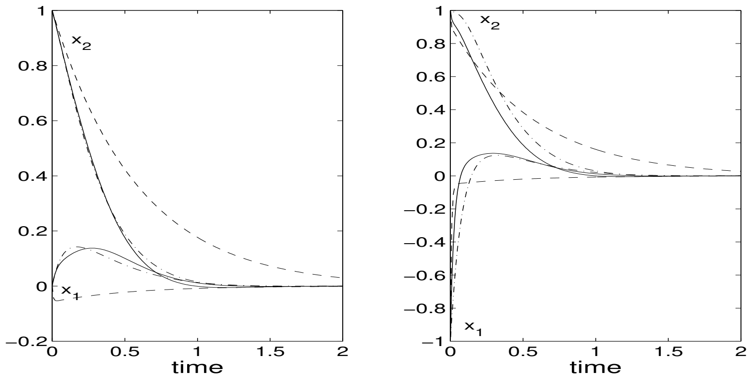

Figure 5.2: Closed-loop solutions for Example 5.2. The solid line displays the closed-loop solution using the Taylor series approximation method with $k = 2$. The dashed line shows the closed-loop solution using the control law by (Benallou *et al.* 1988). Finally, the dashed-dotted line illustrates the closed-loop solution using the control law by (Derese and Noldus 1980). Left figure: $x_1(0) = 0$ and $x_2(0) = 1$. Right figure: $x_1(0) = -1$ and $x_2(0) = 1$.

control for the Taylor series approximation method, the penalty matrix Q_1 was transformed to obtain the same matrix S in (5.49) as the solution to the ARE (5.16a), see comments on Theorem 5.3.3. Then, using only the first term in the Taylor series approximation, results in global asymptotic stability of the closed-loop system. However, if more than one term in the Taylor series is used, some further stability analysis has to be performed in order to confirm global asymptotic stability. The new transformed state penalty matrix, used in (5.47), is

$$Q_1 = \begin{pmatrix} 341.386 & 21.989 \\ 21.989 & 6.5596 \end{pmatrix}. \tag{5.50}$$

The robustness methods discussed in Section 5.3.1 were also tested in order to achieve a more robust control for the Taylor series approximation method. Choosing the nominal input $u_{nom}(t)$ as in (5.48), imposes the perturbation

[2]Note that the control law, $u^* = -0.6x_1^2 - 1.2x_2^2 - 16.7x_1x_2 - 9.2x_1 - 0.6x_2$, given explicit in (Benallou *et al.* 1988) (p. 1495) is not correct. Instead we have used the control law given by (5.48) and (5.49).

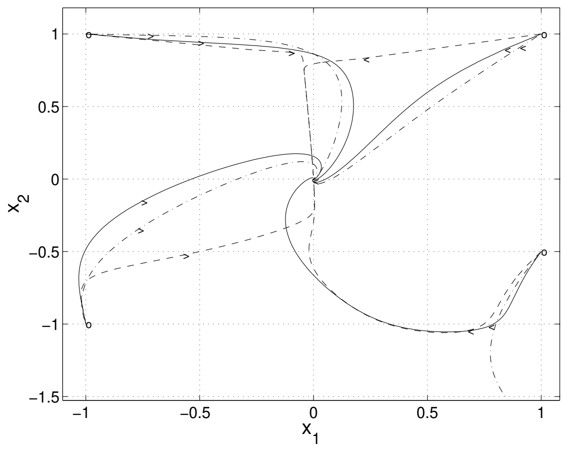

Figure 5.3: State-plane plot for Example 5.2. The solid line shows the state trajectories using the Taylor series approximation method with $k = 2$. The dashed line illustrates the trajectories using the control law by (Benallou *et al.* 1988) and the dashed-dotted line shows the path using the control law by (Derese and Noldus 1980).

input to be:

$$u_p(t) = -Q_2^{-1}(xN + B)^T(S^l(\bar{x}) - S)x,$$

where $S^l(\bar{x})$ is the truncated matrix polynomial, and the matrix S is from (5.49). Using (5.42) as a new perturbation input, and choosing $\epsilon < 3.725$, will guarantee a globally asymptotically stable closed-loop system.

The two new Taylor series approximation control laws with $k = 2$, together with the control law by (Benallou *et al.* 1988), were evaluated by comparing the solutions of the closed-loop system for different initial conditions. The results are shown in Figure 5.4 and Figure 5.5. The behavior of the Taylor series approximation, where (5.50) is chosen as the state penalty matrix, is quite similar to the behavior of the control law obtained by (Benallou *et al.* 1988). However, the Taylor series approximation control law where $u_p(t)$ is bounded, shows a faster response compared to the other two control laws.

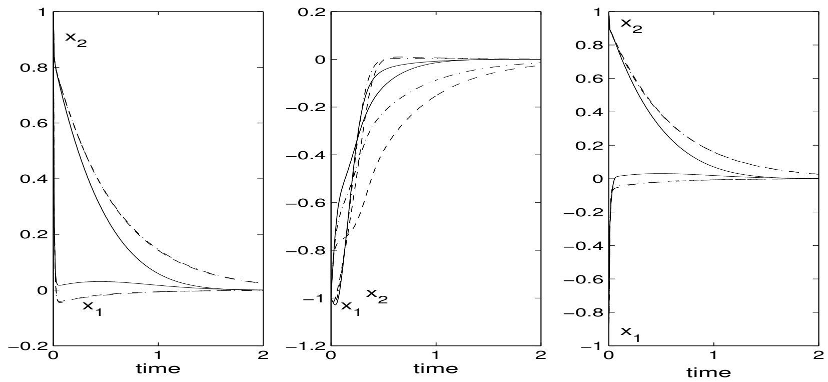

Figure 5.4: Closed-loop solutions for Example 5.2. The solid line displays the closed-loop solution using the Taylor series approximation method with bounded input and $k = 2$. The dashed line shows closed-loop solution using the Taylor series approximation with the new state penalty matrix (5.50) and $k = 2$. Finally, the dashed-dotted line illustrates the closed-loop solution using the control law by (Benallou *et al.* 1988). Left figure: $x_1(0) = 0$ and $x_2(0) = 1$. Middle figure: $x_1(0) = -1$ and $x_2(0) = 1$. Right figure: $x_1(0) = -1$ and $x_2(0) = 1$.

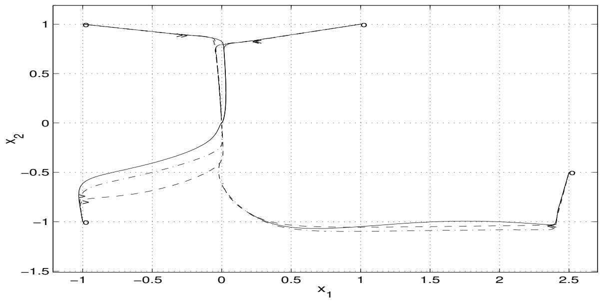

Figure 5.5: State-plane plot for Example 5.2. The solid line shows the state trajectories using the Taylor series approximation method with bounded input and $k = 2$. The dashed line illustrates the trajectories using the Taylor series approximation method with the new state penalty matrix (5.50) and $k = 2$. The dashed-dotted line shows the path using the control law by (Benallou *et al.* 1988).

5.4.3 Example 5.3

The next example is a second order bilinear system, which is homogeneous in the state:

$$\dot{x} = \begin{pmatrix} 0 & 1 \\ -1 & -2 \end{pmatrix} x + \begin{pmatrix} 0 & 0 \\ 0 & -2 \end{pmatrix} xu.$$

The Taylor series approximation control law is compared with the continuous feedback control law (5.51) derived in (Ryan 1984) and the true optimal control law (5.52), presented below. (Ryan 1984) used a state-dependent penalizing function

$$f(x) = \begin{cases} |u| - \frac{1}{2}r, & |u| > r, \\ \frac{1}{2}r^{-1}u^2, & |u| \le r, \end{cases}$$

in the cost functional

$$J = \int_0^\infty (x^T x + \frac{1}{2}u^T ru + f(x))dt,$$

and obtained the control law:

$$u = -sat[-x_2 r^{-1}(-2x_1 + 2x_2)]. \tag{5.51}$$

The state weighting matrix is $Q_1 = I$ and r is chosen as $r = 1$. The same weighting matrices are used in the quadratic cost functional for the Taylor series approximation method.

For this system it is also possible to find a closed-form solution to the state dependent ARE, which results in the "true" optimal control law

$$u = 2x_2(s_{12}x_1 + s_2x_2), \tag{5.52}$$

where

$$\begin{aligned} s_2 &= (\sqrt{\frac{1}{2}(1 + 2x_2^2 + \sqrt{1+4x_2^2}} - 1)/2x_2^2 \qquad (5.53) \\ s_{12} &= (\sqrt{1+4x_2^2} - 1)/4x_2^2. \\ s_1 &= s_2 + 2s_{12} + 4s_{12}s_2x_2^2 \end{aligned}$$

s_{12} are the off-diagonal elements and s_2 one of the diagonal element solutions to the state dependent ARE matrix $S(x)$ in (5.13). The diagonal element s_1 is not needed in the control law (5.52)

The solution to the Taylor polynomial $S(\bar{x})$ in (5.15) is an even function in powers of x_2. A satisfactory approximation is obtained in a neighborhood of the origin, which can be seen in Figure 5.6. The figure shows a plot of the true diagonal elements s_1 and s_2, compared with the Taylor series approximation for one, two and three terms. The behavior of the control law (5.51), (5.52),

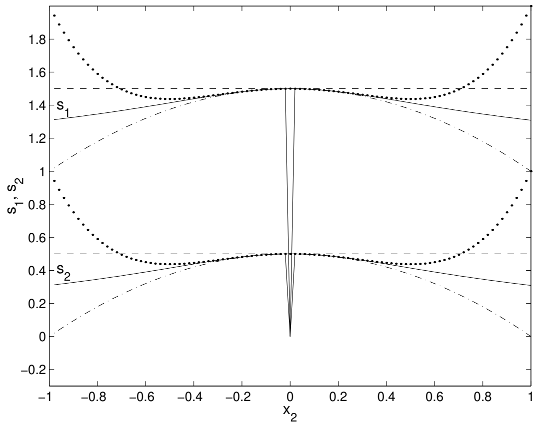

Figure 5.6: Diagonal elements for the ARE matrix in Example 5.3. The solid line shows the closed form solution s_1 and s_2 and the dashed line shows the Taylor series approximation using one term. The dashed-dotted line shows the Taylor series approximation using two terms. Finally, the dotted line illustrates the Taylor series approximation using three terms.

and the Taylor series approximation control law using three terms is quite similar when the initial states are within ± 1 and using the same weighting matrices. However, outside this region the Taylor series approximation and the control law (5.51) differ slightly from the true optimal path. This is illustrated in the state-plane plot in Figure 5.7.

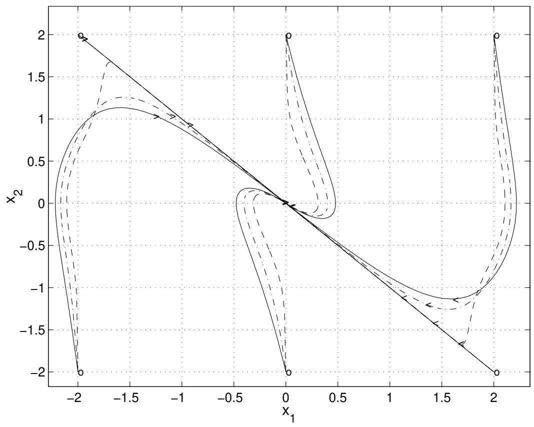

Figure 5.7: State-plane plot for Example 5.3. The solid line shows the true optimal trajectories, and the dashed line the trajectories using the Taylor series approximation method with three terms ($k = 4$). The dashed-dotted line shows the trajectories using the control law by Ryan.

5.5 Bilinear Suboptimal Control of an ASP

In the last example the derived control law is compared with an LQ controller. The controllers are simulated and tested using the IWA Activated Sludge Model No.1 (ASM1), which describes the biological nitrogen removal in municipal wastewater treatment plants, see (Henze *et al.* 1987). The simulated activated sludge process (ASP) consists of one aerobic compartment, one anoxic compartment and one settler in a pre-denitrifying ASP, see Figure 3.1. The parameters in the model are the default values given in (Henze *et al.* 1987). The volumes in the two compartments are constant and equal $V = 1000m^3$.

The controller design is based on a the simplified ASM1 model (3.35) derived in Chapter 3. The model (3.35) is a time varying state-space bilinear model of order six, where the states are the nitrogen S_{NO}, ammonium S_{NH} and the soluble COD S_{COD} in the two compartments. The states are assumed to be measurable. The model has three inputs. The first input is the internal recycling flow rate, u_1. The second input is $u_2 = \frac{S_O}{S_O+K}$, where S_O is the oxygen concentration in the aerobic compartment, and K is assumed to be a known constant. The third input, u_3, is the external carbon flow rate.

The design of the controller is made at a stationary point, x_s and u_s, and it may then be assumed that the model is time invariant within a sufficient small region around the stationary point. The following bilinear model is obtained, see Appendix D.1 for details:

$$0 = \dot{x_s} = Ax_s + N_1 x_s u_{s1} + N_2 x_s u_{s2} + B_u u_{s3} + B_{\bar{d}} \bar{d}_s + B_{\bar{w}} \bar{w}_s, \tag{5.54}$$

where $\bar{d}_s$ is considered to represent measurable disturbances and $\bar{w}_s$ unmeasurable disturbances at the stationary point. The chosen values of the states and inputs at steady-state are:

$$x_s = [0.025\ 102.01\ 8.49\ 4.57\ 42.4\ 2.91]^T, \quad u_s = [700\ 0.95\ 0.095]^T.$$

The system matrices are:

$$A = \begin{pmatrix} -0.8 & -0.0535 & 0 & 0.4 & 0 & 0 \\ 0 & -1.268 & 0 & 0 & 0.4 & 0 \\ 0 & -0.0267 & -0.8 & 0 & 0 & 0.4 \\ 0.8 & 0 & 0 & -0.8 & -0.668 & 0 \\ 0 & 0.8 & 0 & 0 & -6.592 & 0 \\ 0 & 0 & 0.8 & 0 & -0.334 & -0.8 \end{pmatrix}, \quad B_u = \begin{pmatrix} 0 \\ 1200 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix},$$

$$N_1(t) = \begin{pmatrix} -0.001I & 0.001I \\ 0.001I & -0.001I \end{pmatrix}, \quad N2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.668 & 3.136 \\ 0 & 0 & 0 & 0 & -2.2395 & 0 \\ 0 & 0 & 0 & 0 & -0.129 & -3.201 \end{pmatrix},$$

where I is the 3×3 unity matrix. The system matrices $B_{\bar{d}}$ and $B_{\bar{w}}$ are given in Section 3.2.3.

Neglecting the unmeasurable disturbances and introducing the new states, inputs and measurable disturbances, $\Delta x = x - x_s$, $\Delta u = u - u_s$, and $\Delta d = \bar{d} - \bar{d}_s$, a bilinear model around the operating point x_s can be obtained as, see (Gutman 1980)

$$\Delta\dot{x} = (A+\sum_{i=1}^{2} N_i u_{si})\Delta x + N_1 \Delta x \Delta u_1 + N_2 \Delta x \Delta u_2 + \sum_{i=1}^{2}(N_i x_s + B)\Delta u_i + B_d \Delta d. \tag{5.55}$$

An LQ controller is also designed on a model linearized around the same operating point x_s and u_s. However, the bilinear model is believed to cover more of the nonlinearities in ASM1 than the corresponding linear approximation. Note that the same cost function (5.2) can be used for both the LQ controller design and the Taylor series approximation method. The nitrate and the ammonium in the last aerobic compartment is chosen as outputs. To achieve integral action an integrating state, $x_i(t)$, with state feedback, L_i, is added in the control law. The control signals are then calculated as

$$u(t) = -L(\Delta x)\Delta x(t) - L_i x_i(t) \tag{5.56}$$

for the Taylor series approximation method, and

$$u(t) = -\bar{L}\Delta x(t) - L_i x_i(t) \tag{5.57}$$

for the LQ controller. The L_i-matrix is chosen relatively small as

$$L_i = \begin{pmatrix} 0.1835 & 0.6412 \\ -0.0046 & 0.0007 \\ 0.0005 & 0.0017 \end{pmatrix}.$$

Remark 5.3. By including measurable disturbances in the control law, an effective feedforward action may be obtained. For example, adding the term $-L_d d_{SNH}(t)$ in (5.56) and (5.57), where $d_{SNH}(t) = D_{in}(t)S_{NH,in}(t)$, will improve the disturbance rejection, if L_d is chosen in a proper way.

The weighting matrices used in (5.2) are $Q_1 = diag([0.01\ 0.01\ 0.01\ 40\ 0.01\ 50])$ and $Q_2 = diag([0.0005\ 60\ 70])$. The behavior of the closed system response, using a three terms Taylor series approximation, is compared with the LQ controller, which is illustrated in Figure 5.8. The nitrate and the ammonium concentrations in the second (aerobic) compartment are plotted in the figure. The concentrations in the last compartment are approximately the same as the effluent concentrations, and the plotted states are therefore denoted S_{NOe} and S_{NHe}, respectively. The result shows that the Taylor series approximation controller gives a better disturbance rejection compared to the LQ controller. Figure 5.9 compares the control signals used during the simulations. Figure 5.10 shows the influent disturbances used in the simulation. After 50 hours, $S_{NH,in}$ has changed from 25 mg/l to 40 mg/l, after 100 hours $S_{NH,in}$ has decreased to 30 mg/l, and finally, after 150 hours, it has changed back to 25 mg/l.

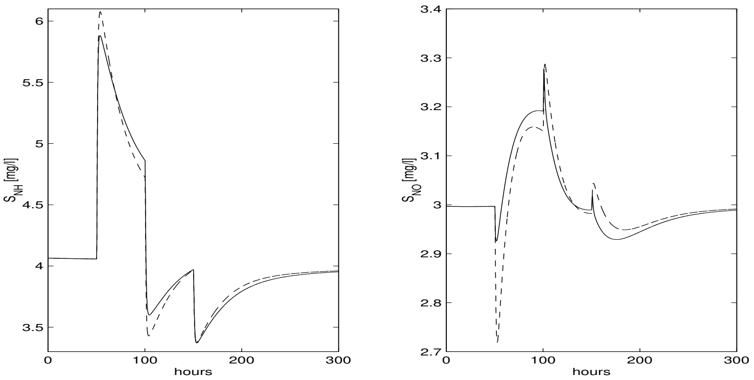

Figure 5.8: The solid line displays the result when the Taylor series approximation control law with three terms is used. The dashed line shows the result when the LQ control law is used. Left figure: Ammonium concentration S_{NHe} in the last aerobic compartment. Right figure: Nitrate concentration S_{NOe} in the last aerobic compartment.

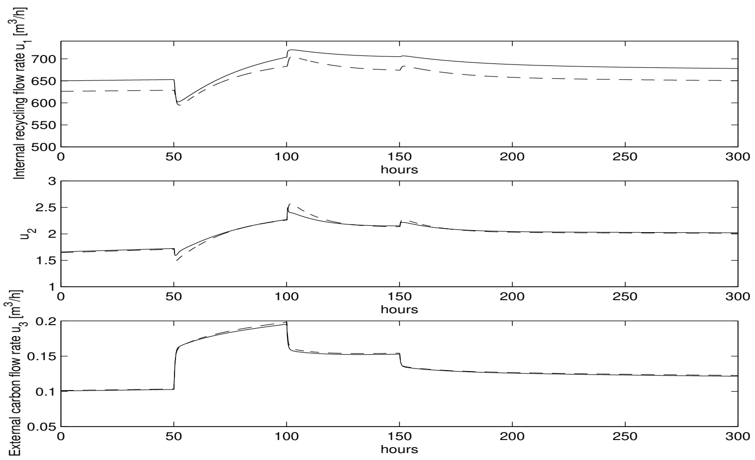

Figure 5.9: Control signals used for Example 5.4. The solid line displays the control signals when a Taylor series approximation with three terms is used. The dashed line shows the control signals from the LQ controller. Upper plot: Internal recycling flow rate u_1. Middle plot: $u_2 = \frac{S_O}{K+S_O}$. Lower plot: External carbon flow rate u_3.

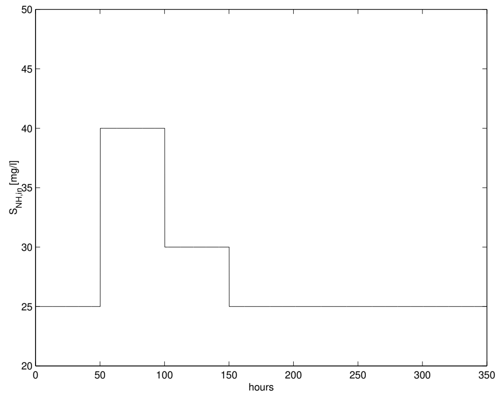

Figure 5.10: Influent variation of $S_{NH,in}$ used for Example 5.4.

5.6 Conclusions

A suboptimal control law to the bilinear quadratic regulator problem with infinite final time is developed. The solution to the state-dependent ARE is derived using a Taylor-series approach, which results in simple offline calculations of one ARE and a sequence of Lyapunov equations. Only few terms in the Taylor-series expansion were needed to obtain a good approximation to the solution. A strategy in order to achieve a more robust behavior of the truncated Taylor-series expansion was suggested. The performance of the resulting control law was compared to other feedback laws proposed in the literature.

Special emphasize was devoted to illustrate the performance of the developed control law applied to an ASP. Topics for further research could include a closer analysis of the stability properties of the resulting closed-loop system. An indirect adaptive controller approach for controlling the ASP, using a bilinear model structure, could also be a topic for future research.

Appendix A

Appendix to Chapter 1

A.1 The Activated Sludge Process

A common concept for biological wastewater treatment is the ASP. The ASP is a biological process in which microorganisms oxidize and mineralize organic matter. Before sewage enters the ASP, larger objects are removed by grates and screens, and suspended solids are removed by sedimentation. In the aerobic compartments, the active biomass is kept in suspension together with dissolved compounds and organic and inorganic particles. The organic fraction in the influent wastewater is mineralized and transformed into biomass. The biological step is followed by a settler, where the effluent water is separated from the sludge.

The basic ASP, which contains only aerated compartments and a settler, described above can be extended to include also biological nitrogen removal. Nitrogen in wastewater is present in several forms such as ammonia (NH_3), ammonium (NH_4^+), nitrate (NO_3^-), nitrite (NO_2^-) and as organic compounds. However, most of the nitrogen in the influent water is in the form of ammonium. In order to reduce the amount of nitrogen in the influent water, the ammonium may be converted into nitrate in the aerated compartments. This bacterial process is called nitrification. The nitrification process consists of two sub-processes, where primarily two different groups of autotrophic nitrifying bacteria oxidize ammonium. It is known that nitrification also may be accomplished by a variety of heterotrophic bacteria. Neglecting the temperature and pH dependency, the nitrification process can be described by the following simplified chemical reaction formula:

$$NH_4^+ + 1.5O_2 \Rightarrow NO_2^- + H_2O + 2H^+$$
$$NO_2^- + 0.5O_2 \Rightarrow NO_3^-.$$

The second step in the nitrogen reduction where nitrate is converted into gaseous nitrogen (N_2), is called denitrification. This process takes place in the anoxic compartments. In order for denitrification to be efficient it is crucial to limit the DO concentration, and thereby force the heterotrophic bacteria to use the nitrate oxygen for respiration, instead of DO. Denitrification can occur at high oxygen levels but the reaction is reduced considerably. The denitrification process can be described by the simplified formula:

$$2NO_3^- + 2H^+ \Rightarrow N_2 + H_2O + 2.5O_2.$$

For a more detailed description of the nitrification and denitrification processes, see for instance (Hallin 1998).

The two most common ways to configure the ASP for nitrogen removal are a post-denitrifying or a pre-denitrifying plant structure. In a post-denitrifying system the nitrification and the oxidation of organic material occurs in the first aerated compartments, whereas denitrification takes place in the following anoxic compartments, see Figure A.1. Since a great deal of the organic material from the influent wastewater is degraded in the aerobic compartments, a carbon source must be added to the anoxic compartment. In a pre-denitrifying plant, the aerated compartments are placed after the anoxic ones. Here, possible organic matter in the influent wastewater is better utilized by the denitrification bacteria. In order to denitrify the nitrate rich water from the end of the aerobic compartments, the water from the last aerobic compartment is generally recirculated back to the beginning of the anoxic compartments. See further e.g. (Henze *et al.* 1995) and (Olsson and Newell 1999) for a comprehensive description of the different mechanisms behind the biological processes in ASP's.

A.2 The Settler Model

The settler in (Takács *et al.* 1991) is modeled as a one-dimensional layer model, see Figure A.2. It is assumed that each horizontal layer is completely

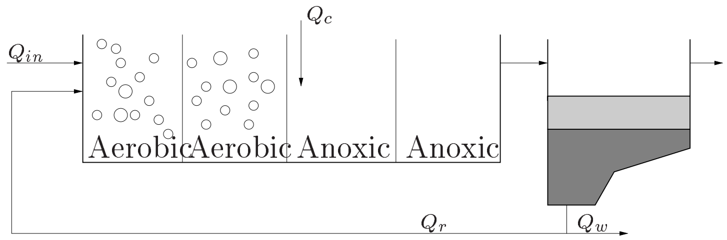

Figure A.1: An activated sludge process configured for nitrogen removal (post-denitrifying). Q_{in} is the influent flow rate, Q_r the recirculated sludge flow rate, Q_w the excess sludge flow rate and Q_c the external carbon flow rate.

mixed. The different flows in and out from the settler are defined in Figure A.1. The solids flux of particles due to gravity sedimentation J depends on

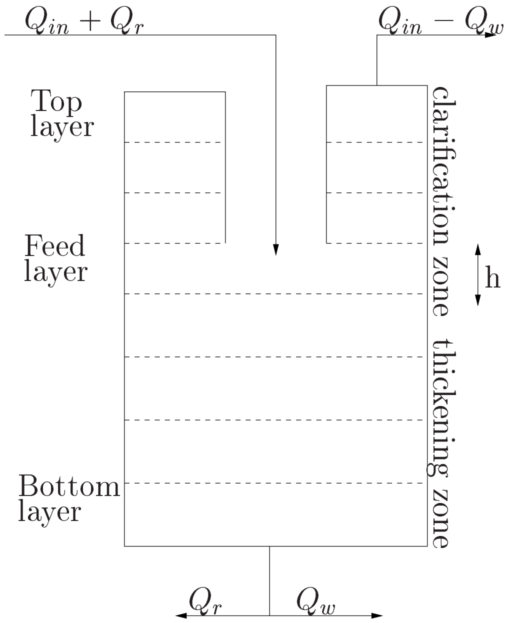

Figure A.2: One-dimensional settler model.

the velocity v_s and the sludge concentration X according to:

$$J = v_s(X)X.$$

For each layer, i, the concentration is then:

$$\frac{dX_i}{dt} = \frac{1}{h}\Delta J_i$$

where ΔJ_i is the difference in the flux in the layer obtained from the solids mass balance around layer i. The settling velocity from (Takács *et al.* 1991) is given by:

$$v_{s,i} = v_0 e^{r_h(X_i - X_{min})} - v_0 e^{-r_p(X_i - X_{min})} \tag{A.1}$$

$$0 \leq v_{s,i} \leq v_0^{'}. \tag{A.2}$$

For an explanation of the different parameters in (A.1)-(A.2), and a derivation of the difference in the flux ΔJ_i, see further (Takács *et al.* 1991).

Appendix B

Appendix to Chapter 3

B.1 The Activated Sludge Model No. 1

In order to evaluate different controllers and control strategies it is important to use a model which realistically simulates a true plant. In the simulations, the IWA Activated Sludge Model No. 1 (ASM1) by (Henze *et al.* 1987) has been used to model each compartment of the implemented process.

B.1.1 Differential Equations

For a completely mixed compartment, the following model structure holds for a general component Z, see (Bastin and Dochain 1990):

$$\frac{dZ}{dt} = R_Z(t) + D(t)(Z_{in}(t) - Z(t)),$$

where $Z_{in}(t)$ is the influent concentration of the actual component, $Z(t)$ is the effluent concentration, which is equal to the concentration in the compartment under the assumption of complete mixing, $D(t)$ is the dilution rate (flow/volume), and $R_Z(t)$ denotes the reaction rate for the component.

The different components used in ASM1 are given in Table B.1. According to ASM1, the reaction rates for the components are:

Table B.1: Components included in the model.

$S_{NH}(t)$	soluble ammonium nitrogen
$S_{NO}(t)$	soluble nitrate nitrogen
$S_{ND}(t)$	soluble biodegradable organic nitrogen
$S_O(t)$	dissolved oxygen
$S_S(t)$	soluble readily biodegradable substrate
$S_I(t)$	soluble inert organic matter
$S_{ALK}(t)$	alkalinity
$X_{B,A}(t)$	active autotrophic biomass
$X_{B,H}(t)$	active heterotrophic biomass
$X_{ND}(t)$	particulate biodegradable organic nitrogen
$X_S(t)$	slowly biodegradable substrate
$X_I(t)$	particulate inert organic matter and products

$$
\begin{aligned}
R_{S_S} &= -\frac{1}{Y_H}\hat{\mu}_H(\frac{S_S}{K_S+S_S})(\frac{S_O}{K_{O,H}+S_O})X_{B,H} \\
&- \frac{1}{Y_H}\hat{\mu}_H(\frac{S_S}{K_S+S_S})(\frac{K_{O,H}}{K_{O,H}+S_O})(\frac{S_{NO}}{K_{NO}+S_{NO}})\eta_g X_{B,H} \\
&+ k_h\frac{\frac{X_S}{X_{B,H}}}{K_X+\frac{X_S}{X_{B,H}}}\left((\frac{S_O}{K_{O,H}+S_O})+\eta_h(\frac{K_{O,H}}{K_{O,H}+S_O})(\frac{S_{NO}}{K_{NO}+S_{NO}})\right)X_{B,H} \\
R_{X_I} &= f_P(b_H X_{B,H}+b_A X_{B,A}) \\
R_{X_S} &= (1-f_P)(b_H X_{B,H}+b_A X_{B,A}) \\
&- k_h\frac{\frac{X_S}{X_{B,H}}}{K_X+\frac{X_S}{X_{B,H}}}\left((\frac{S_O}{K_{O,H}+S_O})+\eta_h(\frac{K_{O,H}}{K_{O,H}+S_O})(\frac{S_{NO}}{K_{NO}+S_{NO}})\right)X_{B,H} \\
R_{X_{B,H}} &= \hat{\mu}_H(\frac{S_S}{K_S+S_S})(\frac{S_O}{K_{O,H}+S_O})X_{B,H} \\
&+ \hat{\mu}_H(\frac{S_S}{K_S+S_S})(\frac{K_{O,H}}{K_{O,H}+S_O})(\frac{S_{NO}}{K_{NO}+S_{NO}})\eta_g X_{B,H} - b_H X_{B,H} \\
R_{X_{B,A}} &= \hat{\mu}_A(\frac{S_{NH}}{K_{NH}+S_{NH}})(\frac{S_O}{K_{O,A}+S_O})X_{B,A} - b_A X_{B,A} \\
R_{S_O} &= -\frac{1-Y_H}{Y_H}\hat{\mu}_H(\frac{S_S}{K_S+S_S})(\frac{S_O}{K_{O,H}+S_O})X_{B,H} \\
&- \frac{4.57-Y_A}{Y_A}\hat{\mu}_A(\frac{S_{NH}}{K_{NH}+S_{NH}})(\frac{S_O}{K_{O,A}+S_O})X_{B,A} + K_L a(S_{O,sat}-S_O)
\end{aligned}
$$

$$
\begin{aligned}
R_{S_{NO}} &= -\frac{1-Y_H}{2.86Y_H}\hat{\mu}_H(\frac{S_S}{K_S+S_S})(\frac{K_{O,H}}{K_{O,H}+S_O})(\frac{S_{NO}}{K_{NO}+S_{NO}})\eta_g X_{B,H} \\
&+ \frac{1}{Y_A}\hat{\mu}_A(\frac{S_{NH}}{K_{NH}+S_{NH}})(\frac{S_O}{K_{O,A}+S_O})X_{B,A} \\
R_{S_{NH}} &= -i_{XB}\hat{\mu}_H(\frac{S_S}{K_S+S_S})(\frac{S_O}{K_{O,H}+S_O})X_{B,H} \\
&- i_{XB}\hat{\mu}_H(\frac{S_S}{K_S+S_S})(\frac{K_{O,H}}{K_{O,H}+S_O})(\frac{S_{NO}}{K_{NO}+S_{NO}})\eta_g X_{B,H} \\
&- (i_{XB}+\frac{1}{Y_A})\hat{\mu}_A(\frac{S_{NH}}{K_{NH}+S_{NH}})(\frac{S_O}{K_{O,A}+S_O})X_{B,A} + k_a S_{ND} X_{B,H} \\
R_{S_{ND}} &= -k_a S_{ND} X_{B,H} \\
&+ \frac{X_{ND}}{X_S}k_h\frac{\frac{X_S}{X_{B,H}}}{K_X+\frac{X_S}{X_{B,H}}}\left((\frac{S_O}{K_{O,H}+S_O}) + \eta_h(\frac{K_{O,H}}{K_{O,H}+S_O})(\frac{S_{NO}}{K_{NO}+S_{NO}})\right)X_{B,H} \\
R_{X_{ND}} &= (i_{XB}-f_P i_{XP})(b_H X_{B,H} + b_A X_{B,A}) \\
&- \frac{X_{ND}}{X_S}k_h\frac{\frac{X_S}{X_{B,H}}}{K_X+\frac{X_S}{X_{B,H}}}\left((\frac{S_O}{K_{O,H}+S_O}) + \eta_h(\frac{K_{O,H}}{K_{O,H}+S_O})(\frac{S_{NO}}{K_{NO}+S_{NO}})\right)X_{B,H}
\end{aligned}
$$

Here the reaction rate of the alkalinity, S_{ALK}, has been omitted, since it does not affect other components and the reaction rate of S_I, because it is inert and, hence, does not react. For an explanation of the different process parameters and their default values, see (Henze *et al.* 1987).

B.2 Taylor Series Expansion of Observations Method with known Y_H

The purpose of this derivation is to find an expression for the reaction rate term $r_{S_{COD}}$, which only depends on measurable quantities. It is assumed that (**A3.6**)-(**A3.9**) hold and that Y_H is known. Differentiating (3.17) results in the following equation:

$$\ddot{S}_{NO}(t) = -D\dot{S}_{NO}(t) + r_{S_{COD}}\dot{X}_{TSS}(t) - r_{S_{COD}}\dot{X}_{COD}(t) + D\dot{S}_{NO,in}(t) \quad \text{(B.1)}$$

From (3.17) and (3.19), where $X_{COD,in}(t) = X_{TSS,in} + S_{COD,in}$, and (B.1) we can formulate an expression for the constant reaction rate term, which only depends on the available state variables and on $S_{COD,in}(t)$. From (B.1) we obtain:

$$\ddot{X}_{COD}(t) = -\frac{1}{r_{S_{COD}}}\ddot{S}_{NO}(t) - \frac{D}{r_{S_{COD}}}\dot{S}_{NO}(t) + \dot{X}_{TSS} + \frac{D}{r_{S_{COD}}}\dot{S}_{NO_{in}}(t). \quad \text{(B.2)}$$

From (3.17) we also have:

$$X_{COD}(t) = -\frac{1}{r_{S_{COD}}}\dot{S}_{NO}(t) - \frac{D}{r_{S_{COD}}}S_{NO}(t) + X_{TSS} + \frac{D}{r_{S_{COD}}}S_{NO_{in}}(t). \quad \text{(B.3)}$$

Combining (B.2) and (3.19) and using (B.3) in that expression results in determination of the constant reaction rate term as:

$$\hat{r}_{S_{COD}} = \frac{D^2\Delta S_{NO}(t) + D\Delta\dot{S}_{NO}(t) - D\dot{S}_{NO}(t)) - \ddot{S}_{NO}(t)}{\Omega(t) + 2.86\dot{S}_{NO}(t) - \dot{X}_{TSS}(t) + DS_{COD,in}(t) + u(t)}, \quad \text{(B.4)}$$

where

$$\begin{aligned} \Delta S_{NO}(t) &= (S_{NO,in}(t) - S_{NO}(t)) \\ \Delta X_{TSS}(t) &= (X_{TSS,in}(t) - X_{TSS}(t)) \\ \Omega(t) &= D(\Delta X_{TSS}(t) - 2.86\Delta S_{NO}(t)). \end{aligned}$$

B.3 Taylor Series Expansion of Observations Method with unknown Y_H

In order to investigate the theoretical identifiability of the unknown Y_H, the same Taylor series expansion of observations method as in Appendix B.2 is used. Differentiating (3.18) and assuming that (**A3.6**)-(**A3.10**) hold, results in the following equation:

$$\ddot{X}_{TSS}(t) = -D\dot{X}_{TSS}(t) + \beta r_{S_{COD}}\dot{X}_{TSS}(t) - \beta r_{S_{COD}}\dot{X}_{COD}(t) + DX_{TSS,in}(t) \quad \text{(B.5)}$$

where $\beta = \alpha Y_H$. Combining (B.5) and (B.1), the constant parameter, β, can be determined as:

$$\beta = \frac{\ddot{X}_{TSS}(t) - D\Delta X_{TSS}(t)}{\ddot{S}_{NO}(t) - D\Delta\dot{S}_{NO}}.$$

Appendix C

Appendix to Chapter 4

C.1 Derivation of a State-Space Model for two Anoxic Compartments

An extension of the model (3.11) for two anoxic compartments is easily derived. The volumes in the two compartments are assumed to be constant and equal, V. Denoting the components in the first and second compartment by the subscript 1 and 2 respectively, the following state-space representation for two anoxic compartments is obtained:

$$\begin{bmatrix} \dot{S}_{NO,1}(t) \\ \dot{S}_{NO,2}(t) \\ \dot{S}_{COD,1}(t) \\ \dot{S}_{COD,2}(t) \end{bmatrix} = A(t) \begin{bmatrix} S_{NO,1}(t) \\ S_{NO,2}(t) \\ S_{COD,1}(t) \\ S_{COD,2}(t) \end{bmatrix} + Bu(t) + G_d d(t) + G_h H_i(t). \quad \text{(C.1)}$$

Here $d(t)$ denotes measurable disturbances and $H_i(t)$ unmeasurable disturbances:

$$d(t) = \begin{bmatrix} D(t)S_{NO,in}(t) \\ D(t)S_{COD,in}(t) \end{bmatrix} \quad \text{(C.2)}$$

$$H_i(t) = \begin{bmatrix} H_1(t) \\ H_2(t) \end{bmatrix}. \quad \text{(C.3)}$$

The system matrices become:

$$A(t) = \begin{pmatrix} F_1(t) & F_2(t) \\ 0 & F_3(t) \end{pmatrix}, \quad \text{(C.4)}$$

where

$$F_1(t) = \begin{pmatrix} -D(t) & 0 \\ D(t) & -D(t) \end{pmatrix}, \quad F_2(t) = \begin{pmatrix} -r_{S_{COD,1}}(t) & 0 \\ 0 & -r_{S_{COD,2}}(t) \end{pmatrix}$$

$$F_3(t) = \begin{pmatrix} \alpha r_{COD,1}(t) - D(t) & 0 \\ D(t) & \alpha r_{S_{COD,2}}(t) - D(t) \end{pmatrix}$$

$$B = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad G_d = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad G_h = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{C.5}$$

where $r_{S_{COD,1}}(t)$ and $r_{S_{COD,2}}(t)$ are transformed reaction rate terms given by:

$$r_{S_{COD,i}}(t) = \frac{R_i(t)}{S_{COD,i}(t)}, \quad i = 1,\ 2. \tag{C.6}$$

C.2 Derivation of an Augmented State-Space Model for two Anoxic Compartments

The system (3.20) can be extended to two anoxic compartments. The resulting state-space representation is of order six and has the structure:

$$\dot{z}_n(t) = F(t)z_n(t) + G(t)g(t), \tag{C.7}$$

where $z_n(t) = \left[\, S_{NO,1}(t) \;\; S_{NO,2}(t) \;\; X_{TSS,1}(t) \;\; X_{TSS,2}(t) \;\; X_{COD,1}(t) \;\; X_{COD,2}(t) \,\right]^T$, and $g(t) = \left[\, u(t) \;\; D(t)S_{NO,in}(t) \;\; D(t)X_{TSS,in}(t) \;\; H_1(t) \;\; H_2(t) \;\; D(t)S_{COD,in}(t) \,\right]^T$. The matrix $F(t)$ contains the transformed reaction rate terms, $r_{S_{COD,1}}(t)$ and $r_{S_{COD,2}}(t)$, given by (C.6). These variables are unknown and time varying and have to be estimated. The system matrices $F(t)$ and $G(t)$ are defined as:

$$F(t) = \begin{pmatrix} F_1(t) & F_2(t) & F_2(t) \\ 0 & F_4(t) & F_5(t) \end{pmatrix}$$

where

$$F_1(t) = \begin{pmatrix} -D(t) & 0 \\ D(t) & -D(t) \end{pmatrix}, \quad F_2(t) = \begin{pmatrix} r_{S_{COD,1}}(t) & 0 \\ 0 & r_{S_{COD,2}}(t) \end{pmatrix}$$

$$F_4(t) = \begin{pmatrix} \alpha Y_H r_{S_{COD,1}}(t) - D(t) & 0 \\ D(t) & \alpha r_{S_{COD,2}}(t) - D(t) \\ 2.86 r_{S_{COD,1}}(t) & 0 \\ 0 & 2.86 r_{S_{COD,2}}(t) \end{pmatrix}$$

$$F_5(t) = \begin{pmatrix} -\alpha Y_H r_{S_{COD,1}}(t) & 0 \\ 0 & -\alpha Y_H r_{S_{COD,2}}(t) \\ -2.86 r_{S_{COD,1}}(t) - D(t) & 0 \\ D(t) & -2.86 r_{S_{COD,2}}(t) - D(t) \end{pmatrix},$$

and

$$G(t) = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

C.3 Basic Discrete-Time EKF Algorithm

Consider the nonlinear stochastic discrete-time model:

$$\begin{aligned} x(t+1) &= f(t, x(t), u(t)) + g(t, x(t))v(t) \\ y(t) &= h(t, x(t)) + e(t), \end{aligned} \tag{C.8}$$

where $y(t)$ is a measured signal, $u(t)$ is the input signal, and $v(t)$ and $e(t)$ are mutually independent Gaussian white noise sequences with zero means and covariances $R_1(t)$ and $R_2(t)$ respectively. The whole EKF algorithm applied to the system (C.8) can be summarized as follows (see (Söderström 1994)):

$$
\begin{aligned}
H(t) =& \left.\frac{\partial h(t,x)}{\partial x}\right|_{x=\hat{x}(t|t-1)} \\
K(t) =& P(t|t-1)H^T(t) \\
& \times \left[H(t)P(t|t-1)H^T(t) + R_2(t)\right]^{-1} \\
\hat{x}(t|t) =& \hat{x}(t|t-1) + K(t)[y(t) - h(t, \hat{x}(t|t-1))] \\
P(t|t) =& P(t|t-1) - K(t)H(t)P(t|t-1) \\
\hat{x}(t+1|t) =& f(t, \hat{x}(t|t), u(t)) \\
F(t) =& \left.\frac{\partial f(t,x,u)}{\partial x}\right|_{x=\hat{x}(t|t)} \\
G(t) =& g(t,x)|_{x=\hat{x}(t|t)} \\
P(t+1|t) =& F(t)P(t|t)F^T(t) + G(t)R_1(t)G^T(t),
\end{aligned}
$$

where $\hat{x}(t|t-1)$ is the predicted state and $\hat{x}(t|t)$ is the most recent estimate.

Appendix D

Appendix to Chapter 5

D.1 Steady-state Values of the Model Parameters

The values of the system parameters in the model (3.35) are examined at a stationary point by running the ASM1 until steady-state has been reached. The constant influent concentrations used in the steady-state simulation are given in Table D.1.

Table D.1: Constant influent concentrations used in the steady-state simulation.

State	$S_{NO,in}$	$S_{COD,in}$	$S_{NH,in}$
$[mg/l]$	1	97	25

The constant flows used in the steady-state simulation are given in Table D.2.

Table D.2: Constant influent concentrations used in the steady-state simulation.

Flow	Q_in	Q_r	Q_a
$[m^3/h]$	400	400	700

A traditional PI controller was used for controlling the level of Dissolved

Oxygen (DO) in the aerobic compartment, and the DO set-point was chosen to be $2mg/l$. The dynamics of the relation between the DO set-point and the concentration of the dissolved oxygen is much faster than the dynamics in the model (3.35). Therefore, the approximation, S_O = DO set-point, is used. The external carbon flow rate was chosen as $u_3 = 0.095\ m^3/h$. The stationary values of the inputs to model (3.35), with $K = 0.2$ are then $u_s = [u_{s1}\ u_{s2}\ u_{s3}]^T = [700\ 0.95\ 0.095]^T$.

The concentrations of the states at steady-state are given in Table D.3.

Table D.3: Concentrations at steady-state.

State	$S_{NO,1}$	$S_{COD,1}$	$S_{NH,1}$	$S_{NO,e}$	$S_{COD,e}$	$S_{NH,e}$
$[mg/l]$	0.025	102.01	8.49	4.57	42.14	2.91

The parameter values of the model (3.35) at steady-state are shown in Table D.4, and the constant values are given in Table D.5.

Table D.4: Parameter values at steady-state.

Parameters	$r_{S_{COD}}$	$\bar{r}_{S_{COD,1}}$	$\bar{r}_{S_{NH}}$	$\bar{r}_{S_{COD,3}}$	$\bar{H}_1$	Am_1	$\bar{H}_e$	Am_e
values	0.05	0.66	3.13	8.03	6.23	2.69	244.2	19.17

Table D.5: Constant values.

Constants	α_1	α_2	α_3	α_1
values	-8.667	-1.0206	0.4994	0.0576

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