A Tractable Mechanism for Time Dependent Markets

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Abstract

Markets with time dependent goods are special cases of multi commodity markets.The design of large flexible markets with time dependent goods is a computational challenge. In this article we present a computationally tractable mechanism for time dependent markets. By a number of predefined bid types, it offers useful flexibility to the bidders.

We present the market mechanism and the corresponding matching algorithm together with some analysis of its behaviour. With s_1 and s_2 the size of the search space in volume and prices, respectively (in a proper resolution), and bids on h hours simultaneously, the computational complexity of the algorithm is $\mathcal{O}\left(h \log s_1 \log^2 s_2\right)$.

Keywords: multi commodity markets, electronic markets, computational markets, equilibrium markets, resource allocation, power markets, bandwidth markets, computational complexity.

1 Introduction

Markets with time dependent goods are special cases of multi commodity markets. In time dependent markets, such as power markets, a set of consecutive time slots are often traded simultaneously. However, although the participants may have various types of dependencies and constraints between the time slots, there is typically no, or very weak, support for the expression of such dependencies. In this article, we propose a market mechanism[1] that

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[1]We use the notion of a *market mechanism* to denote the rules of the market, whereas *market protocol* includes the behaviour of the actors on the market.

allows for a fairly large flexibility in expressing time dependencies, at the same time as it is computationally tractable. The computational aspects are important since the most general way to allow the expression of time dependencies is by a combinatorial auction, which presents us with an NP-hard computational problem. Therefore, it is interesting to have a trading mechanism which (i) allows for sufficient flexibility, (ii) is natural and understandable for participants, and (iii) has a low computational complexity.

Our mechanism has some carefully selected combinatorial features that increase the market flexibility compared to markets where each time period is treated as an independent commodity. The gain is that it enables participants to express preferences more accurately and hence it possibly gives an improved market outcome.

An application area of high interest is day-ahead power markets. If these are to be opened for consumer side bidders and local production bidders, the number of actors on the market grows dramatically, and new market mechanisms and algorithms are needed. Another interesting application area with many similarities is bandwidth markets.

The work of this article was presented at the IEEE International Conference on E-Commerce, CEC'03 [6].

2 Main Idea

We consider a market for a set of consecutive time units, here denoted hours. A bid is assumed to be given as a continuous (positive or negative) demand function, expressing one of the following:

1. *hourly bids*: separate bids for each hour,
2. *block bids*: bids on the same volume each hour,
3. *adaptive consumer bids*: bids describing a consumer demand that is not related to any specific hour; the consumer is prepared to buy whenever the price is low enough, the consumption can even be split between hours,and
4. *adaptive producer bids*: Corresponding to the adaptive consumer bids.

On a k hour market each bidder can give $k + 3$ different demand functions, hence we say that there are $k + 3$ *bidding tracks.*

Please note that all demand functions of a track may be aggregated into one function describing the net demand of the track.

The four types of bids allow for a fairly flexible market. Compared to a fully implemented combinatorial market, the hourly bids correspond to

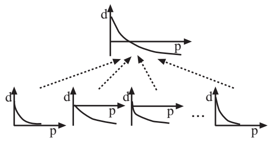

Figure 1: *For each bidding track, the demand functions may be aggregated into one, giving full information on supply and demand of that track. Supply is expressed as negative demand.*

single bids, block bids correspond to traditional combinatorial bids expressing synergies, and the adaptive bids corresponds to XOR bids, expressing substitutability.

If the expressiveness of these bids is not sufficient, for example if there are large synergies in smaller blocks, an after-market could be used to improve further. However, we regard this as outside the scope of this article.

As shown below, although we combine single bids, block bids, and XOR-type of bids, the market can be handled optimally in a computationally efficient way.

3 The Existence of an Equilibrium

We state the problem as follows:

Given one market with the four bid types, compute

- a price for each hour, and
- an allocation of the adaptive bids,

such that supply meets demand.

In the following we show:

1. that there exists a solution, and
2. how to compute it.

As said above, supply and demand bids are assumed to be given as continuous (positive and negative) demand functions. We assume that they are expressed as sample arrays. The bids are aggregated along the separate tracks, giving a set of demand functions, one for each track. These functions give full information on supply and demand and an equilibrium can be calculated without any further communication. An equilibrium is expressed as a set of prices where the excess demand for each hour on the market as a whole, but not necessarily on each bidding track, is zero.

The mechanism relies on following prerequisites; (*i*) demand is continuous and decreasing in price, and (*ii*) the goods traded on the adaptive tracks are divisible.

Before we prove the existence of an equilibrium, we give some definitions starting with the notation on different demand functions is presented. (Note that we need only discuss demand, since supply can be viewed as negative demand.)

Definition. 3.1 *Let d_b be the demand function defined by the aggregated demand for the same volume over all hours of the set of consecutive hours, i.e. of the block track, d_i the aggregated demand of hour i, d_{ac} the aggregated adaptive consumer demand, and d_{ap} the aggregated adaptive producer demand.*

The outcome of the trade depends on a trade or reallocation between on one hand the hourly tracks and on the other hand the block and the adaptive tracks. The resources reallocated between tracks are expressed as follows. (To distinguish the demand for reallocation between tracks from their internal demand we use a notation with a t as in transfer, instead of d as in demand.)

Definition. 3.2 *Let t_b be the resource reallocated from the block track to the hourly tracks (the same resource level for all hours). Let $t_{a,i}$ be the resource reallocated between any one of the adaptive tracks and hour i. Further, let $t_{ac,i}$ be the adaptive consumer demand allocated to hour i, and $t_{ap,i}$ the corresponding producer demand, and let t_c be the resource reallocated between the adaptive tracks.*

The dynamics of the set of prices is expressed as follows:

Definition. 3.3 *Let $p_b(t_b)$ be the equilibrium price of the block track when t_b is reallocated from the block to the hourly tracks, c.f. Figure 2. Let $p_i(t_b, t_{a,i})$ be the equilibrium price of hour i with t_b as before and $t_{a,i}$ traded with one of the adaptive tracks, and let $p_{\forall h}(t_b, t_a)$ be $\sum_{\forall i} p_i(t_b, t_{a,i})$.*

Note that with a demand that is not strictly decreasing, the inverse demand is *an interval valued function*, i.e. for most t_b the inverse demand is an ordinary, distinct function value, but for some[2] it is an interval (and when t_b hits such an interval any price within it may be picked).

3.1 Specification of the Problem

The problem is to determine a price of each hour and an allocation of the adaptive bids as said in the beginning of Section 3, i.e. to determine a price

[2] The demand of some actor(s) is the same for any price within the interval.

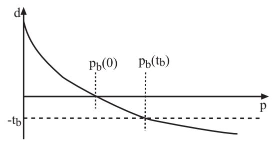

Figure 2: *If the block track is treated independently, the equilibrium price — a price where excess demand on the market is zero — is* $p_b(0)$. *When the sub-markets interact, the equilibrium price on the block is a function of the reallocation between block and hours,* $p_b(t_b)$.

vector p^*:

$$\begin{aligned} & p^* = \{p_1^*, p_2^*, \ldots, p_k^*\} \\ & s.t.\ \forall i : d_i(p_i^*) + t_{ap,i}(p^{max}) + t_{ac,i}(p^{min}) - t_b = 0, \end{aligned} \tag{1}$$

where $p^{min} = min_i\ (p_i^*)$ and $p^{max} = max_i\ (p_i^*)$.

3.2 Hourly Demand and Adaptive Participants

To show that a solution to the problem exists we start with a market with the hourly and the adaptive bid types. The following lemma defines what is needed for an equilibrium in all hours.

Lemma 3.1 *If*

$$d_{ap}(p^{max}) = -\sum_i max\,(d_i(p^{max}), 0) - t_c \tag{2}$$

$$d_{ac}(p^{min}) = -\sum_i min\,\left(d_i(p^{min}), 0\right) + t_c \tag{3}$$

$$p^{max} \geq p^{min} \tag{4}$$

$$t_c \cdot \left(p^{max} - p^{min}\right) = 0 \tag{5}$$

then

$$\forall i : d_i(p_i^*) + t_{ap,i}(p^{max}) + t_{ac,i}(p^{min}) = 0 \tag{6}$$

Proof. Eq. (2), (3), and (5), together with inequality (4) are necessary and sufficient for an equilibrium on a market with hourly actors and adaptive production and consumption.

Necessary: In Eq. (2) we note that $d_{ap}(p^{max})$ is the volume sold by adaptive production at price p^{max}, $\sum_i max\,(d_i(p^{max}), 0)$ is the aggregated positive hourly demand at p^{max}, and t_c is the volume traded between adaptive production and consumption, evenly distributed over the hours. By this we have that equality gives a balance between buying and selling side. Eq. (3) is similar.

Inequality (4) is needed, since if it is not fulfilled, then the adaptive production and consumption would be better off with a larger t_c. Similarly, Eq. (5) is needed, since if it is not fulfilled, the hours would be better off with a smaller t_c.

Sufficient: Given that inequality (4) and Eq. (5) hold, Eq. (2) and (3) define the equilibrium expressed by Eq. (6). $\square$

Eq. (2) and (3) rely on the existence of prices p^{max} and p^{min} such that they hold for a given t_c.

Lemma 3.2 *Given* t_c

$$\begin{aligned} \exists p^{max} &: \textit{Eq. 2 holds, and} & (7)\\ \exists p^{min} &: \textit{Eq. 3 holds.} & (8) \end{aligned}$$

Proof. All (positive and negative) demand is decreasing and continuous. Hence, d_{ap} is decreasing in p^{max}, and $-\sum_i d_i(p^{max})$ is increasing in p^{max}. Continuity gives Eq. (7). The proof of Eq. (8) is similar. $\square$

With this, if there exists a t_c such that the equations and the inequality of Lemma 3.1 hold, we conclude that there exists an equilibrium on a market with hourly and adaptive bid types.

Lemma 3.3

$$\exists t_c : \textit{Eq. } 2, \textit{ Eq. } 3, \textit{ inequality } 4, \textit{ and Eq. } 5 \textit{ hold.} \tag{9}$$

Proof. All (positive and negative) demand is decreasing and continuous. Decreasing demand gives that p^{max} is increasing in t_c, while p^{min} is decreasing in t_c. Continuity give the lemma. $\square$

3.3 All Bid Types

From this we move on to a market with all four bid types (including block bids). Then Eq. (2), (3), and (6) have to be modified to take t_b into account. Furthermore, we need to show that there exists a t_b such that the equations hold.

Theorem 3.1 *Assume that the price $\sum_i p_i^*$ gives a net demand of $-t_b$ from the block. If*

$$d_{ap}(p^{max}) = -\sum_i max\left(d_i(p^{max}) - t_b, 0\right) - t_c \tag{10}$$

$$d_{ac}(p^{min}) = -\sum_i min\left(d_i(p^{min}) - t_b, 0\right) + t_c \tag{11}$$

$$p^{max} \geq p^{min} \tag{12}$$

$$t_c \cdot \left(p^{max} - p^{min}\right) = 0 \tag{13}$$

then

$$\forall i : d_i(p_i^*) + t_{ap,i}(p^{max}) + t_{ac,i}(p^{min}) - t_b = 0 \tag{14}$$

Proof. The proof is similar to the proof of Lemma 3.1, the only difference is the introduction of t_b, present in Eq. (10), (11), and (14). □

The presence of t_b can be viewed as an adjustment of the *material balance line*, compared to what we have in Section 3.2. The material balance line of a single commodity market is zero, i.e. the excess demand at the equilibrium price is zero. In our market construct, the material balance of each hour has to be zero on the market all together, but the material balance line of a single track might well be a non-zero value to balance an excess demand of other tracks.

Note that Eq. (14) is equivalent to the condition of the problem formulation (1).

If there exists a pair $(\sum_i p_i^*,\ t_b)$ with the properties assumed in Theorem 3.1, we have what we need to conclude that the equilibrium exists.

Theorem 3.2 *There exists a price vector p^*, such that all conditions of Theorem 3.1 hold.*

Proof. We note that there is no difference in what is discussed in Lemma 3.1 — 3.3 if the material balance line of the hourly tracks is non-zero, i.e. if they should have a net volume of the commodity to balance a net volume on the block.

Hence, we need only prove further that there exists a $\sum_i p_i*$ such that the block demand over all hours is $-t_b$. This follows from demand being decreasing and continuous. Decreasing demand gives that the equilibrium price on the block is increasing in t_b, and the equilibrium prices of the separate hours are decreasing in t_b. Hence, continuity gives the lemma. □

From this we conclude that there exists an equilibrium in a market with all four bid types and we move on to our algorithm for determination of the equilibrium prices.

4 Algorithm

The following algorithm gives a strategy for the search for an optimum. Details on the computations follows the algorithm description.

Algorithm 4.1 (Determination of prices) *:*

```
{
  pick a value on t_b;                                              (I)
  while (|p_∀h(t_b, t_a) − p_b(t_b)| > ε_1)
  {
    for all i set the material balance to t_b;                      (II)
    compute adaptive demand;                                        (III)
    /*in a binary search*/
    determine p_∀h(t_b, t_a) & p_b(t_b);                            (IV)
    if(p_∀h(t_b, t_a) − p_b(t_b) > ε_1)                              (V)
      raise t_b;
    if(p_∀h(t_b, t_a) − p_b(t_b) < −ε_1)
      lower t_b;
    /*else equilibrium price reached */
  }
  announce prices;
}
```

The process can be viewed as a binary search over t_b. At each search step a number of parallel searches are performed to establish the equilibrium prices of all tracks but the block given t_b.

4.1 Algorithm Details

We give the details on the algorithm step by step:

(I) **Excess block demand level.** The excess demand on the block market, t_b, is the main search variable in the binary search of Algorithm 4.1. Pick a start value on t_b.

(II) **Adjustment of the material balance.** Before the reallocation between the hourly actors and the adaptive ones takes place, the material balance line of the hourly demand is adjusted. This adjustment is to compensate for the volume preliminary reallocated from the block to the hours, see Figure 3. By the adjustment of the material balance line, the equilibrium price changes for the hour under observation (the new price may be determined in a binary search). In Figure 3, $p_i(0, 0)$ equals the equilibrium price with no trade between sub-markets, and $p_i(t_b, 0)$ is the equilibrium price when t_b is traded with the block but no trade has taken place with any adaptive actor.

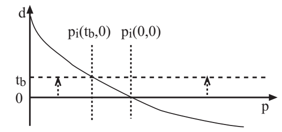

Figure 3: *In the search for a total market equilibrium the hourly demand has to balance an excess demand on the block. Graphically this could be viewed as moving the material balance line in the plot of the demand curve from zero to* t_b.

(III) **Determination of the adaptive demand.** The reallocation between time dependent bidding tracks and time independent tracks requires some special attention. Any reallocation between time dependent actors bound to different time periods has to be avoided. To prevent such a reallocation, the search for an optimal allocation is based on parts of their demand functions only. For each hour we define two new demand functions as follows:

Definition. 4.1 *For all prices* p *we define the function* d_i^+ *and* d_i^- *as follows:* $d_i^+(p) = max(d_i(p) - t_b, 0)$*, and* $d_i^-(p) = min(d_i(p) - t_b, 0)$.

We define two new aggregate demand functions expressing all positive and negative hourly demand, respectively:

Definition. 4.2 *For all prices* p *and hours* i *we define* $d_{\forall h}^+ = \sum_i d_i^+(p)$ *and* $d_{\forall h}^- = \sum_i d_i^-(p)$.

In the reallocation between hours and adaptive consumers $d_{\forall h}^-$ is used together with the adaptive consumer demand, d_{ac}. In a similar way, the reallocation between hours and the adaptive producers is based on $d_{\forall h}^+$ and d_{ap}.

Basically, the equilibrium prices related to the two adaptive tracks are established by aggregation of (i) $d_{\forall h}^+$ and d_{ac}, and of (ii) $d_{\forall h}^-$ and d_{ap}, respectively. An example is given in Section 5. This aggregation is done in a binary search fashion.

In this way a minimum price and a maximum price, p^{min} and p^{max}, over all hours are established. The searches for p^{min} and p^{max} are done independent from each other, hence it is possible that $p^{min} > p^{max}$. To solve this a reallocation between the time independent tracks has to be introduced. The volume reallocated between adaptive tracks, t_c, is determined in an additional binary search.

Note that a reallocation can reduce the price of more than one high demand hour, and raise the price of more than one low demand hour.

(IV) **Determination of prices, including $\mathbf{p_{\forall h}(t_b, t_a)}$.** After this it is possible to determine a set of equilibrium prices covering the hourly tracks, given the value of t_b. That is, the price for hour i, $p_i(t_b, t_{a,i})$, is the equilibrium price taking the trade with the block and at most one of the adaptive tracks into account. (Note that no hour can trade with both adaptive tracks).

From the set of hourly equilibrium prices $p_{\forall h}(t_b, t_a)$ is computed as it is expressed in Definition 3.3[3].

(V) **Breaking condition.** When the difference between $p_{\forall h}(t_b, t_a)$ and $p_b(t_b)$ is sufficiently small to be considered zero the search is ended and the set of prices is fixed.

We conclude that the final equilibrium price within a specific hour is depending on both its own demand and a reallocation between this hour and (*i*) the block and (*ii*) at most one of the adaptive tracks. As shown in the following section, these reallocations are optimal within a finite resolution in prices.

4.2 The Algorithm Determines an Equilibrium

In Section 3, we have shown the existence of an equilibrium on the total market. Above, we have given an algorithm for the problem. What is left is to show that the algorithm determines an optimum (within some finite resolution in prices).

As in the discussion on the existence of an equilibrium, we divide this discussion into two parts, (*i*) given t_b the algorithm determines an equilibrium involving hours and adaptive tracks, and (*ii*) with this and a search over t_b it determines an equilibrium on the full market.

Lemma 4.1 *Given t_b, Algorithm 4.1 determines an equilibrium, within a predefined finite resolution in prices, covering hourly bids and the adaptive bid types.*

Proof. There are two cases (*i*) in the base case the price relation $p^{min} \leq p^{max}$ holds for $t_c = 0$ and (*ii*) the more demanding case with $t_c > 0$ and $p^{min} = p^{max}$ (within a predefined finite resolution).

In the first case, the equilibrium is determined using a standard aggregation technique and nothing else is required. The output is optimal given the input functions.

[3] The prices $p_{\forall}(t_b, t_a)$ and $p_b(t_b)$ could be expressed in two ways, either the price for a resource level during the whole block or on hourly scale. On an hourly scale $p_{\forall h}(t_b, t_a)$ should equal the average hourly price, and the block price, $p_b(t_b)$, should be expressed on an hourly scale.

In the second case, the algorithm performs a binary search over t_c. The existence of an optimum is given by Lemma 3.3. If the breaking condition of the search for this optimum is that $p^{max} - p^{min} \leq epsilon_2$ for some $epsilon_2 > 0$, all hourly prices are within $epsilon_2$ from optimal.

By definition of the involved hourly demand functions $d^-_{\forall h}$ and $d^+_{\forall h}$, we have that the resulting allocation is feasible in both cases, since any hour i that has a non-zero negative demand at p^{min} has a zero positive demand at p^{max}, and the other way around. □

With this, all that is left is to show that the search over t_b gives an equilibrium that includes the block, and to derive the computational complexity. (Note that, once the aggregation of bids is done, the algorithm is independent of the number of actors.)

Theorem 4.1 *Within a predefined finite resolution in prices, Algorithm 4.1 determines an equilibrium covering hourly bids, block bids, and the adaptive bid types.*

With s_1 and s_2 the size of the search space in volume and prices, respectively (in a proper resolution), and bids on h hours simultaneously, the computational complexity of the algorithm is $\mathcal{O}\left(h \log s_1 \log^2 s_2\right)$.

Proof. Theorem 3.2 gives that an optimum exists. By binary search over the material balance line t_b and over minimum and maximum prices, p^{min} and p^{max}, Algorithm 4.1 determines a price vector with the property that $|p_b - \sum_i p_i| \leq \epsilon_1$ for some $\epsilon_1 > 0$. For an outcome that is ϵ-close to optimal, pick ϵ_1, ϵ_2 such that $\epsilon_1 + \epsilon_2 \leq \epsilon$.

The computational complexity is given by the triple-nested binary search strategy; we have a main loop of $\log s_1$ search steps in t_b. Each iteration of this search involves a search for a minimum price, that in turn involves a search for a maximum price.Both the latter involves $\log s_2$ search steps in prices.

Finally, for each iteration in the search of a maximum price, there is a constant time work performed at each hourly node. For constant time work we assume a pre-computation of the inverse demand function of each hour.

All together this gives a complexity $\mathcal{O}\left(h \log s_1 \log^2 s_2\right)$ as stated in the theorem. □

5 Example

To show the behaviour of the algorithm we set up a small example with a two hour block size and walk through one step of the iteration. The enumeration of the example is the same as in the algorithm description.

(I) At some stage of the iteration, the algorithm decides to set the excess demand of the block, t_b, to four (as a guess on the volume to reallocate from the block to the hourly tracks). As a consequence, the equilibrium price of the block market changes from $p_b(0)$ to $p_b(4)$, but there is no evaluation of the price at this stage, see Figure 4.

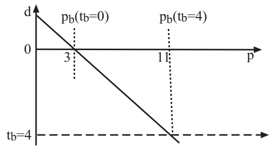

Figure 4: *The demand function of the block. When the excess demand is changed from zero to (minus) $t_b = 4$, the equilibrium price is changed from three to eleven.*

(II) The demand functions of the hourly tracks are affected by t_b in a similar way, and the material balance line and equilibrium price of the hourly demand functions is updated before they are used in the trade with the adaptive actors, Figure 5 and 6.

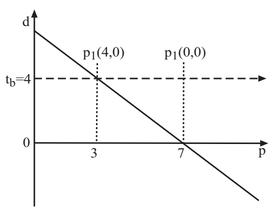

Figure 5: *The demand function of the first hour, h_1. When the material balance line is changed from zero to $t_b = 4$, the equilibrium price changes from seven to three. Step (II).*

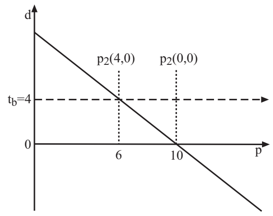

Figure 6: *The demand function of the second hour, h_2. As in the first hour, the equilibrium price changes, in this case from ten to six. Step (II).*

(III) From the hourly demand functions an aggregated positive demand function, $d^+_{\forall h}$, and an aggregated negative demand function, $d^-_{\forall h}$, are constructed, c.f. Definition 4.1 and 4.2 and Figure 7 and 8. The demand function of the adaptive producers, Figure 9, is aggregated with $d^+_{\forall h}$, and the demand of the adaptive consumers, Figure 10, is aggregated with $d^-_{\forall h}$, Figure 11 and 12. This aggregation too is performed as a binary search.

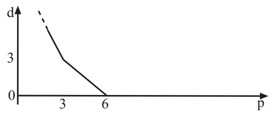

Figure 7: *The positive demand of the hourly sub-markets is aggregated into $d^+_{\forall h}$. The trade with the adaptive producers is based on this function. Step (III).*

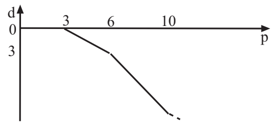

Figure 8: *The negative hourly demand is aggregated into a function, $d^-_{\forall h}$, used for the trade with the adaptive consumers. Step (III).*

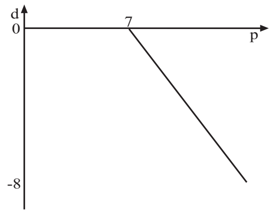

Figure 9: *The demand function of the adaptive producers.*

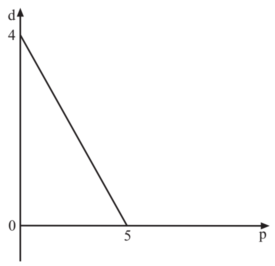

Figure 10: *The demand function of the adaptive consumers.*

In Figure 11 we see that there is a gap between the highest consumption price of the hourly markets and the lowest production price of the adaptive producers, hence no reallocation is performed.

On the other hand, in Figure 12 we see that a reallocation takes place between the aggregated hours and the adaptive consumers (in this case involving h_1 and the adaptive consumers).

(IV) As an effect of the trade with the block and with the adaptive consumers, the equilibrium price of h_1, $p_1(t_b, t_{a,1})$, is changed first from seven to three and then to four.

The price of h_2 is affected by the trade with the block, and changes from ten to six, but it is not affected by any trade with the adaptive actors.

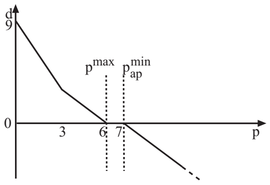

Figure 11: *The demand of the adaptive producers, d_{ap}, is aggregated with $d^{+}_{\forall h}$. There is a gap between the highest buying price, p^{max}, and the lowest selling price, p^{min}_{ap}, and no trade takes place. Step (III).*

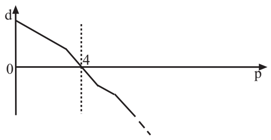

Figure 12: *The demand of the adaptive consumers, d_{ac}, and $d^{-}_{\forall h}$. An equilibrium price of 4 is established in this trade. This affects any hour i with $p_i(t_b, 0) < 4$. Step (III).*

The aggregated equilibrium price of the separate hours with $t_b = 4$, $p_{\forall h}(t_b, t_a)$, of this iteration is $4 + 6 = 10$. The equilibrium price on the block market, $p_b(t_b)$, is 11, Figure 4.

(V) Since $p_{\forall h}(t_b, t_a) < p_b(t_b)$, t_b is to high, and in next iteration step $t_b^{max} \leftarrow t_b$ and $t_b \leftarrow t_b^{min} + (t_b - t_b^{min})/2$.

The search continues until the breaking condition $|p_{\forall h}(t_b, t_a) - p_b(t_b)| < \epsilon_1$ is fulfilled.

6 Concluding Remarks

In this article we present a computationally tractable mechanism for time dependent markets. By a number of predefined bid types, it offers useful flexibility to the bidders. The article presents useful abstractions, holding the combinatorial capabilities on a low level. A reason to keep the combinatorial capabilities of a market mechanism down is to keep it easy to understand and to make it easy to convince oneself that the pricing mechanism is correct. Furthermore, there are complexity reasons — both from a computational and a communicational perspective — to do this.

The main computational (and communicational) task of the mechanism is the aggregation of demand. With the combinatorial capabilities of the mechanism expressed as independent tracks (bids on single hours, block bids and adaptive bids) the computational complexity of this part does not grow more than linear in the number of bidding tracks.

A real world market setup of a large automated market for time dependent goods is most likely a highly distributed market, i.e. most of the information needed for market computation is spread over the network. Since the input to Algorithm 4.1 is aggregated demand, it is natural to distribute a heavy part of the computation — the aggregation — over the network. By this, the communicational load is diminished radically.

In earlier work we have looked into distributed resource allocation and resource allocation with non concave objective functions, [2, 3, 1], e.g. applicable on markets with non-continuous demand [5]. In this article we have assumed continuous demand. Non-continuous demand on time dependent markets is left for future work.

An assumption of ours that may be hard for some adaptive actors is that the allocations are divisible (their allocations might be split between hours). In practice we assume that the number of adaptive actors is large relative the volumes traded, hence the goods can be handled as divisible. The case with non-divisible goods introduces conceptual pricing problems as well as computational problems, and is beyond the scope of this article.

The market mechanism has properties that are highly relevant in e.g. day-ahead power markets [4, 7] and bandwidth markets [8, 9]. In a power setting, the big advantage of the mechanism (compared to the power markets of today) comes with the possibility to set up electronic markets with a huge number of participants. When a direct market participation of a large number of presumably small size actors (formerly represented by distributors) is introduced the market outcome can become considerably more efficient.

The combinatorial possibilities given by the market mechanism enriches the possibilities of the actors. While being easy to understand and computationally feasible, it scales well to markets with a huge number of participants.

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