Simulating Waves on Complex Domains

In Science and Technology many wave-phenomena are of interest, there is a need to accurately and efficiently predict wave propagation. For example, pressure and shear waves propagate through the earth, and can be modeled by a system of partial differential equations (PDEs). To simulate these waves the PDE system must be solved, and this is in general only possible to do numerically. Local properties effect the propagation of the waves, and sudden changes in physical properties within the ground cause reflections. For example, simulating effects of earthquakes require highly efficient numerical methods, which can handle the complex geometry of the different materials in the ground. For many other important wave applications complex geometric features also appear, and in all these settings the geometric features cause numerical difficulties. It is well-known that for wave propagation high order accuracy of numerical methods is essential for efficiency, but when complex geometric features are present standard approaches do not work well.

In this project we will develop new finite element methods, which can handle geometric features, such as boundaries and material interfaces, without aligning the computational grid to these features, see below.

The work builds on the work of former PhD student Simon Sticko, [1,2]. In this project we seek to improve the stability properties, which so far have led to severe time-step restrictions for high order accuracy. We will make use of connections between finite element methods and finite difference methods to design a new finite element method.

The project contains significant elements of numerical analysis, mathematical analysis and of scientific programming. Concepts such as stability and accuracy are central. The programming will be done in the FEM environment deal2, see [3], which is based on C++. Much work will be done in collaboration with the main supervisor, professor Gunilla Kreiss, and her other collaborators. Contacts with scientists from other areas such as geophysics and fluid dynamics, will also be important. There will be possibilities for research visits at other universities, in and out of Sweden.

