

Mereologies in Computing Science

Uppsala: Thursday, 11 November 2010

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1. Abstract

In this talk we solve the following problems:

- we give a formal model of a large class of mereologies,
 - with simple entities modelled as parts
 - and their relations by connectors;
- we show that that class applies to a wide variety of societal infrastructure component domains;
- we show that there is a class of **CSP** channel and process structures that correspond to the class of mereologies where
 - mereology parts become **CSP** processes and
 - connectors become channels;
 - and where simple entity attributes become process states.

1. Abstract

- We have yet to prove to what extent the models satisfy
 - the axiom systems for mereologies of, for example, (Casati&Varzi 1999)
 - and a calculus of individuals (Bowman&Clarke 1981).
- Mereology is the study, knowledge and practice of part-hood relations:
 - of the relations of part to whole and
 - the relations of part to part within a whole.
- By parts we shall here understand simple entities — of the kind illustrated in this talk.

1. Abstract

- Manifest simple entities of domains
 - are either continuous (fluid, gaseous)
 - or discrete (solid, fixed), and if the latter, then
 - * either atomic
 - * or composite.
 - It is how the sub-entities of a composite entity
 - * are “put together”
 - * that “makes up” a mereology of that composite entity
 - at least such as we shall study the mereology concept.
- In this talk we shall study some ways of modelling the mereology of composite entities.

1. Abstract

- One way of modelling mereologies is using
 - **sorts**,
 - **observer functions** and
 - **axioms** (McCarthy style),
- another is using **CSP**.

2. Introduction

2.1. Physics and Societal Infrastructures

2.1.1. Physics

- **Physicists** study that of nature which can be measured
 - within us,
 - around us and
 - between ‘within’ and ‘around’!
- To make mathematical models of physics phenomena,
 - physics has helped develop and uses mathematics,
 - notably calculus and statistics.

- **Domain engineers** primarily studies societal infrastructure components which can be
 - reasoned about,
 - built and
 - manipulated by humans.
- To make domain models of infrastructure components, domain engineering makes use of
 - formal specification languages,
 - their reasoning systems: formal testing, model checking and verification, and
 - their tools.

2.1.2. In Nature

- **Physicists** turns to algebra in order to handle structures in nature.
 - Algebra appears to be useful in a number of applications, to wit:
 - * the abstract modelling of chemical compounds.
 - But there seems to be many structures in nature
 - * that cannot be captured in a satisfactory way by mathematics, including algebra
 - * and when captured in discrete mathematical disciplines such as sets, graph theory and combinatorics
 - the “integration” of these mathematically represented — structures
 - with calculus (etc.) — becomes awkward;
 - well, I know of no successful attempts.

- **Domain engineers** turns to discrete mathematics —
 - as embodied in formal specification languages
 - and as “implementable” in programming languages —in order to handle structures in societal infrastructure components.
- These languages allow
 - (a) the expression of arbitrarily complicated structures,
 - (b) the evaluation of properties over such structures,
 - (c) the “building & demolition” of such structures, and
 - (d) the reasoning over such structures.
- They also allow the expression of dynamically varying structures —
 - something mathematics is “not so good at” !

- But the specification languages have two problems:
 - (i) they do not easily, if at all,
 - * handle continuity, that is, they do not embody calculus,
 - * or, for example, statistical concepts, etc.,and
 - (ii) they handle
 - * actual structures of societal infrastructure components
 - * and attributes of atomic and composite entities of these –
 - usually by identical techniques
 - thereby blurring what we think is an important distinction.

2.2. Structure of This Talk

- The rest of the talk is organised as follows.
- First we give a first main, a meta-example,
 - of syntactic aspects of a class of mereologies.
- We informally show that the assembly/unit structures indeed model structures of a variety of infrastructure components.
- Then we discuss concepts of atomic and composite simple entities.

- We then “perform”
 - the ontological trick of mapping the assembly and unit entities
 - and their connections
 - exemplified in the first main meta-example
 - into CSP processes and channels, respectively —
 - the second and last main — meta-example and now
 - * of semantic aspects of a class of mereologies.

3. A Syntactic Model of a Class of Mereologies

3.1. Systems, Assemblies, Units

- We speak of systems as assemblies.
- From an assembly we can immediately observe a set of parts.
- Parts are either assemblies or units.
- We do not further define what assemblies and units are.

type

$S = A, A, U, P = A \mid U$

value

$\text{obs_Ps}: (S|A) \rightarrow \text{P-set}$

- Parts observed from an assembly are said to be immediately embedded in, that is, **within**, that assembly.
- Two or more different parts of an assembly are said to be immediately **adjacent** to one another.

3. A Syntactic Model of a Class of Mereologies 3.1. Systems, Assemblies, Units

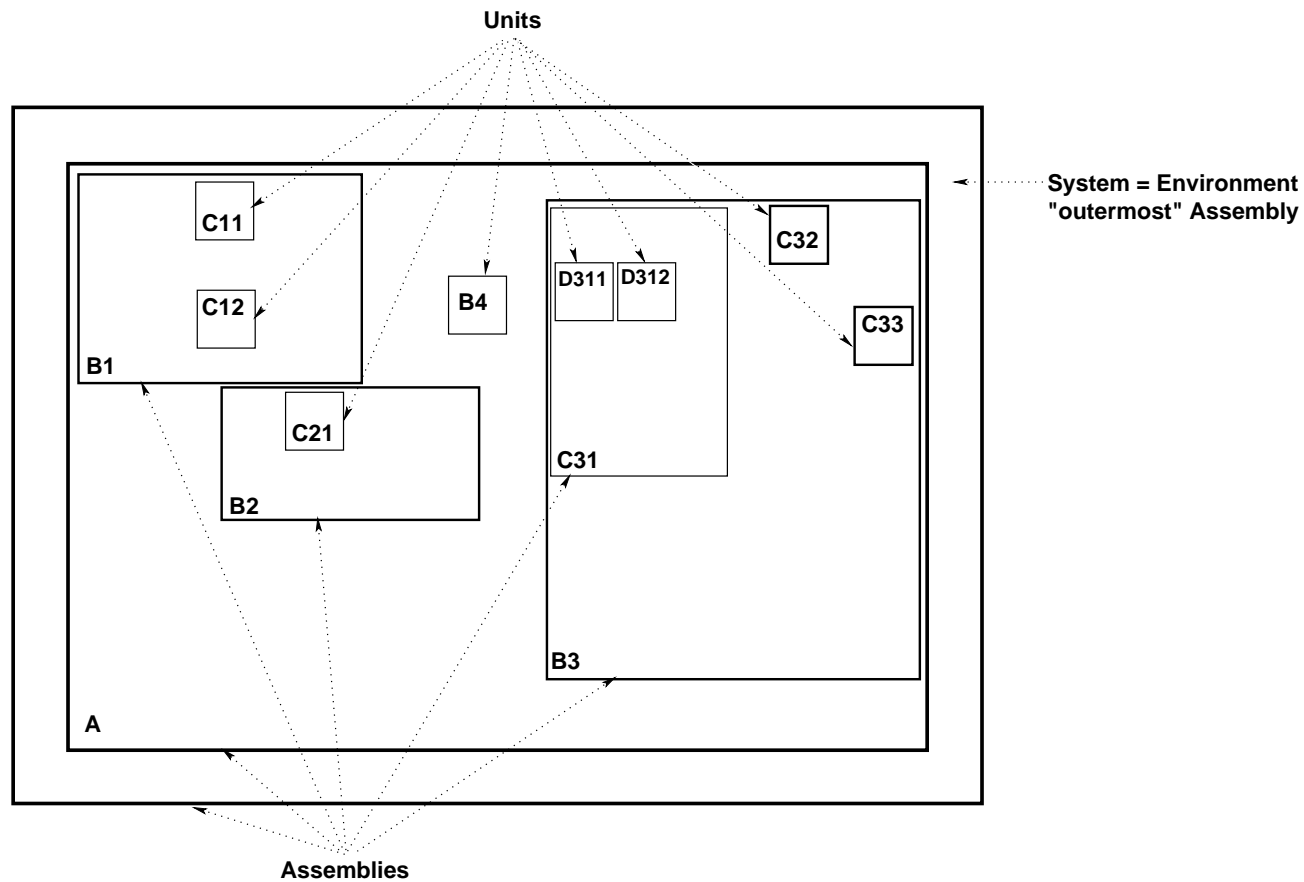


Figure 1: Assemblies and Units “embedded” in an Environment

- A system includes its environment.
- And we do not worry, so far, about the semiotics of all this !

3. A Syntactic Model of a Class of Mereologies 3.1. Systems, Assemblies, Units

- Given obs_Ps we can define a function, xtr_Ps ,
 - which applies to an assembly \mathbf{a} and
 - which extracts all parts embedded in \mathbf{a} and including \mathbf{a} .
- The functions obs_Ps and xtr_Ps define the meaning of embeddedness.

value

$\text{xtr_Ps}: (\mathbf{S}|\mathbf{A}) \rightarrow \mathbf{P}\text{-set}$

$\text{xtr_Ps}(\mathbf{a}) \equiv$

let $\text{ps} = \{\mathbf{a}\} \cup \text{obs_Ps}(\mathbf{a})$ **in** $\text{ps} \cup \mathbf{union}\{\text{xtr_Ps}(\mathbf{a}') \mid \mathbf{a}':\mathbf{A} \cdot \mathbf{a}' \in \text{ps}\}$ **end**

- **union** is the distributed union operator.

- Parts have unique identifiers.
- All parts observable from a system are distinct.

type

AUI

valueobs_AUI: P \rightarrow AUI**axiom** $\forall a:A \cdot$ **let** ps = obs_Ps(a) **in** $\forall p',p'':P \cdot \{p',p''\} \subseteq ps \wedge p' \neq p'' \Rightarrow \text{obs_AUI}(p') \neq \text{obs_AUI}(p'') \wedge$ $\forall a',a'':A \cdot \{a',a''\} \subseteq ps \wedge a' \neq a'' \Rightarrow \text{xtr_Ps}(a') \cap \text{xtr_Ps}(a'') = \{\} \text{ end}$

3.2. 'Adjacency' and 'Within' Relations

- Two parts, p, p' , are said to be *immediately next to*, i.e., $i_next_to(p, p')(a)$, one another in an assembly a
 - if there exists an assembly, a' equal to or embedded in a
 - such that p and p' are observable in that assembly a' .

value

$i_next_to: P \times P \rightarrow A \xrightarrow{\sim} \mathbf{Bool}$, **pre** $i_next_to(p, p')(a): p \neq p'$

$i_next_to(p, p')(a) \equiv \exists a': A \cdot a' = a \vee a' \in xtr_Ps(a) \cdot \{p, p'\} \subseteq obs_Ps(a')$

- One part, p , is said to be *immediately within* another part, p' in an assembly a
 - if there exists an assembly, a' equal to or embedded in a
 - such that p is observable in a' .

value

$$i_within: P \times P \rightarrow A \xrightarrow{\sim} \mathbf{Bool}$$

$$i_within(p,p')(a) \equiv$$

$$\exists a':A \cdot (a=a' \vee a' \in xtr_Ps(a)) \cdot p'=a' \wedge p \in obs_Ps(a')$$

3. A Syntactic Model of a Class of Mereologies 3.2. 'Adjacency' and 'Within' Relations

- We can generalise the immediate 'within' property.
- A part, p , is (transitively) within a part p' , $\text{within}(p,p')(a)$, of an assembly, a ,
 - either if p , is immediately within p' of that assembly, a ,
 - or if there exists a (proper) part p'' of p'
 - such that $\text{within}(p'',p)(a)$.

value

$\text{within}: P \times P \rightarrow A \xrightarrow{\sim} \mathbf{Bool}$

$\text{within}(p,p')(a) \equiv$

$i_within(p,p')(a) \vee \exists p'':P \cdot p'' \in \text{obs_Ps}(p) \wedge \text{within}(p'',p')(a)$

- The function **within** can be defined, alternatively,
- using **xtr_Ps** and **i_within**
- instead of **obs_Ps** and **within** :

value

within': $P \times P \rightarrow A \xrightarrow{\sim} \mathbf{Bool}$

within'(p,p')(a) \equiv

i_within(p,p')(a) $\vee \exists p'' : P \cdot p'' \in \mathbf{xtr_Ps}(p) \wedge \mathbf{i_within}(p'',p')(a)$

lemma: **within** \equiv **within'**

3.2.1. Transitive 'Adjacency'

- We can generalise the immediate 'next to' property.
- Two parts, p , p' of an assembly, a , are adjacent if they are
 - either 'next to' one another
 - or if there are two parts p_o , p'_o
 - * such that p , p' are embedded in respectively p_o and p'_o
 - * and such that p_o , p'_o are immediately next to one another.

value

adjacent: $P \times P \rightarrow A \xrightarrow{\sim} \mathbf{Bool}$

adjacent(p,p')(a) \equiv

i_next_to(p,p')(a) \vee

$\exists p'',p''':P \cdot \{p'',p'''\} \subseteq_{\text{xtr}} Ps(a) \wedge i_next_to(p'',p''')(a) \wedge$
 $((p=p'') \vee within(p,p'')(a)) \wedge ((p'=p''') \vee within(p',p''')(a))$

3.3. Mereology, Part I

- So far we have built a *ground mereology* model, $\mathcal{M}_{\mathcal{G}\text{ground}}$.
- Let \sqsubseteq denote *parthood*, *x is part of y*, $x \sqsubseteq y$.

$$\forall x(x \sqsubseteq x)^1 \quad (1)$$

$$\forall x, y(x \sqsubseteq y) \wedge (y \sqsubseteq x) \Rightarrow (x = y) \quad (2)$$

$$\forall x, y, z(x \sqsubseteq y) \wedge (y \sqsubseteq z) \Rightarrow (x \sqsubseteq z) \quad (3)$$

- Let \sqsubset denote *proper parthood*, *x is part of y*, $x \sqsubset y$.
- Formula 4 defines $x \sqsubset y$. Equivalence 5 can be proven to hold.

$$\forall x \sqsubset y =_{\text{def}} x(x \sqsubseteq y) \wedge \neg(x = y) \quad (4)$$

$$\forall \forall x, y(x \sqsubseteq y) \Leftrightarrow (x \sqsubset y) \vee (x = y) \quad (5)$$

¹Our notation now is not RSL but some conventional first-order predicate logic notation.

[3. A Syntactic Model of a Class of Mereologies, 3.3. Mereology, Part I]

- The *proper part* ($x \sqsubset y$) relation is a strict partial ordering:

$$\forall x \neg(x \sqsubset x) \quad (6)$$

$$\forall x, y (x \sqsubset y) \Rightarrow \neg(y \sqsubset x) \quad (7)$$

$$\forall x, y, z (x \sqsubset y) \wedge (y \sqsubset z) \Rightarrow (x \sqsubset z) \quad (8)$$

- *Overlap*, \bullet , is also a relation of parts:

– Two individuals overlap if they have parts in common:

$$x \bullet y =_{\text{def}} \exists z (z \sqsubset x) \wedge (z \sqsubset y) \quad (9)$$

$$\forall x (x \bullet x) \quad (10)$$

$$\forall x, y (x \bullet y) \Rightarrow (y \bullet x) \quad (11)$$

- Proper overlap, \circ , can be defined:

$$x \circ y =_{\text{def}} (x \bullet x) \wedge \neg(x \sqsubseteq y) \wedge \neg(y \sqsubseteq x) \quad (12)$$

- Whereas Formulas (1-11) holds of the model of mereology we have shown so far, Formula (12) does not.
- In the next section we shall repair that situation.
- The *proper part* relation, \sqsubset , reflects the *within* relation.
- The *disjoint* relation, \oint , reflects the *adjacency* relation.

$$x \oint y =_{\text{def}} \neg(x \bullet y) \quad (13)$$

- Disjointness is symmetric:

$$\forall x, y (x \not\cap y) \Rightarrow (y \not\cap x) \quad (14)$$

- The *weak supplementation* relation, Formula 15, expresses
 - that if y is a proper part of x
 - then there exists a part z
 - such that z is a proper part of x
 - and z and y are disjoint
- That is, whenever an individual has one proper part then it has more than one.

$$\forall x, y (y \sqsubset x) \Rightarrow \exists z (z \sqsubset x) \wedge (z \not\cap y) \quad (15)$$

3. A Syntactic Model of a Class of Mereologies 3.3. Mereology, Part I

- Formulas 1–3 and 15 together determine the *minimal mereology*, $\mathcal{M}_{\text{Minimal}}$.
- Formula 15 does not hold of the model of mereology we have shown so far.
- We shall comment on this once we have introduced the notion of parts having attributes.

3.4. A Syntactic Model of a Class of Mereologies

3.4.1. Connectors

- So far we have only covered notions of
 - parts being next to other parts or
 - within one another.
- We shall now add to this a rather general notion of parts being otherwise related.
- That notion is one of connectors.

3. A Syntactic Model of a Class of Mereologies 3.4. A Syntactic Model of a Class of Mereologies 3.4.1. Connectors

- Connectors provide for connections between parts.
- A connector is an ability to be connected.
- A connection is the actual fulfillment of that ability.
- Connections are relations between pairs of parts.
- Connections “cut across” the “classical”
 - *parts being part of the (or a) whole* and
 - *parts being related by embeddedness or adjacency.*

3. A Syntactic Model of a Class of Mereologies 3.4. A Syntactic Model of a Class of Mereologies 3.4.1. Connectors

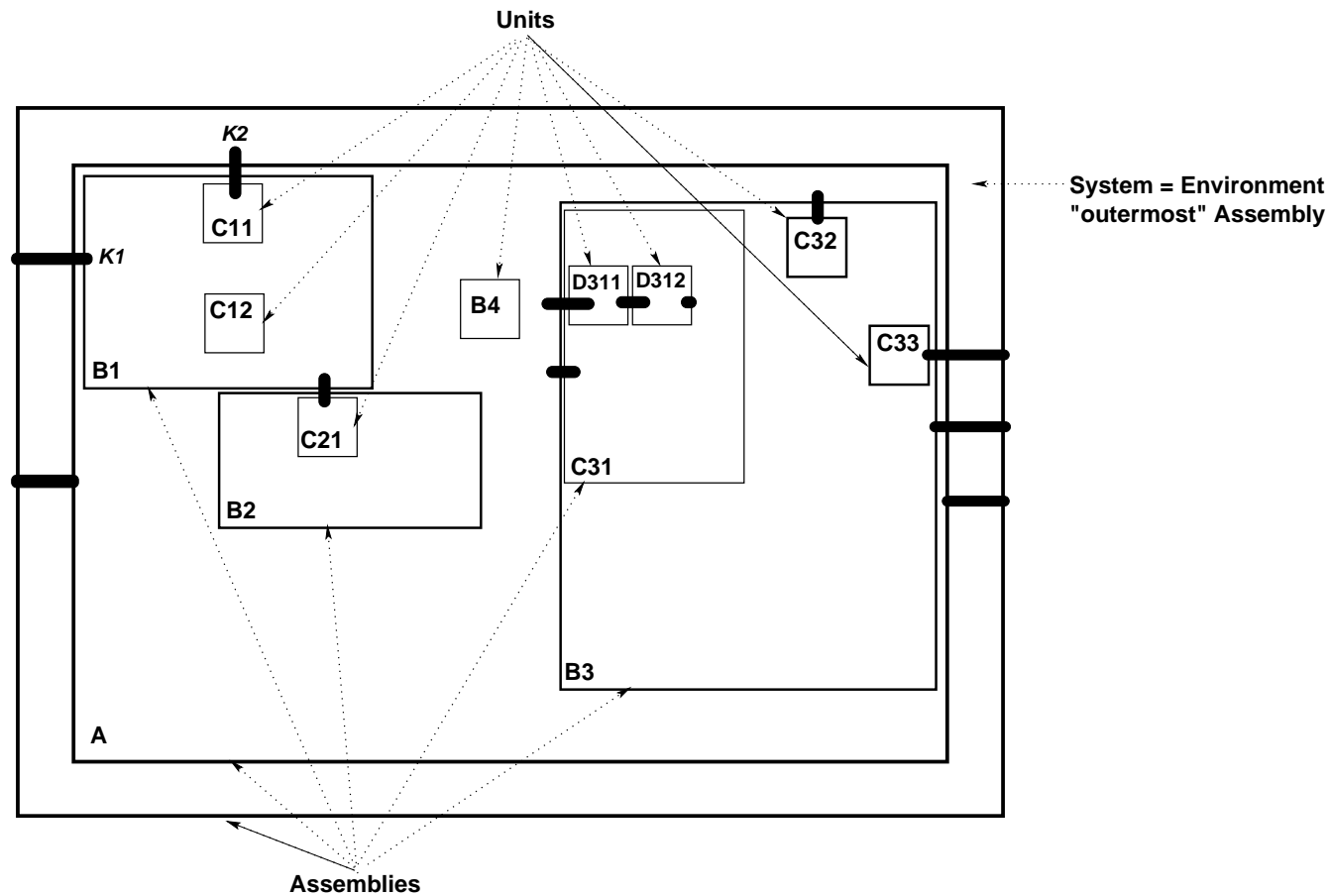


Figure 2: Assembly and Unit Connectors: Internal and External

- For now, we do not “ask” for the meaning of connectors !

- Figure 2 on the facing page “adds” connectors to Fig. 1 on page 14.
- The idea is that connectors
 - allow an assembly to be connected to any embedded part, and
 - allow two adjacent parts to be connected.
- In Fig. 2 on the facing page
 - the environment is connected, by *K2*, to part **C11**;
 - the “external world” is connected, by **K1**, to **B1**;
 - etcetera.

- From a system we can observe all its connectors.
- From a connector we can observe
 - its unique connector identifier and
 - the set of part identifiers of the parts that the connector connects.
- All part identifiers of system connectors identify parts of the system.
- All observable connector identifiers of parts identify connectors of the system.

type

K

value $\text{obs_Ks}: S \rightarrow \mathbf{K\text{-set}}$ $\text{obs_KI}: K \rightarrow \text{KI}$ $\text{obs_Is}: K \rightarrow \mathbf{AUI\text{-set}}$ $\text{obs_KIs}: P \rightarrow \mathbf{KI\text{-set}}$ **axiom** $\forall k:K \cdot \mathbf{card} \text{obs_Is}(k)=2,$ $\forall s:S, k:K \cdot k \in \text{obs_Ks}(s) \Rightarrow$ $\exists p:P \cdot p \in \text{xtr_Ps}(s) \Rightarrow \text{obs_AUI}(p) \in \text{obs_Is}(k),$ $\forall s:S, p:P \cdot \forall ki:KI \cdot ki \in \text{obs_KIs}(p) \Rightarrow$ $\exists! k:K \cdot k \in \text{obs_Ks}(s) \wedge ki = \text{obs_KI}(k)$

- This model allows for a rather “free-wheeling” notion of connectors
 - one that allows internal connectors to “cut across” embedded and adjacent parts;
 - and one that allows external connectors to “penetrate” from an outside to any embedded part.

- We need define an auxiliary function.
 - $\text{xtr}\forall\text{KIs}(p)$ applies to a system
 - and yields all its connector identifiers.

value

$\text{xtr}\forall\text{KIs}: S \rightarrow \text{KI-set}$

$\text{xtr}\forall\text{Ks}(s) \equiv \{\text{obs_KI}(k) \mid k:K \cdot k \in \text{obs_Ks}(s)\}$

3.5. Mereology, Part II

We shall interpret connections as follows:

- A connection between parts p_i and p_j
 - that enjoy a p_i **adjacent to** p_j relationship, means $p_i \circ p_j$,
 - that is, although parts p_i and p_j are **adjacent**
 - they do *share* “something”, i.e., have something *in common*.
 - What that “something” is we shall comment on later, when we have “mapped” systems onto parallel compositions of **CSP** processes.
- A connection between parts p_i and p_j
 - that enjoy a p_i **within** p_j relationship,
 - does not add other meaning than
 - commented upon later, again when we have “mapped” systems onto parallel compositions of **CSP** processes.

- With the above interpretation we may arrive at the following, perhaps somewhat “awkward-looking” case:
 - a connection connects two adjacent parts p_i and p_j
 - * where part p_i is within part p_{i_o}
 - * and part p_j is within part p_{j_o}
 - * where parts p_{i_o} and p_{j_o} are adjacent
 - * but not otherwise connected.
 - How are we to explain that !
 - * Since we have not otherwise interpreted the meaning of parts,
 - * we can just postulate that “so it is” !
 - * We shall, later, again when we have “mapped” systems onto parallel compositions of **CSP** processes, give a more satisfactory explanation.

- We earlier introduced the following operators:
 - \sqsubseteq , \sqsubset , \bullet , \circ , and \oint
- In some of the mereology literature [BowmanLClarke81, BowmanLClarke85, CasatiVarzi1999] these operators are symbolised with caligraphic letters:
 - \sqsubseteq : \mathcal{P} : part,
 - \sqsubset : \mathcal{PP} : proper part,
 - \bullet : \mathcal{O} : overlap and
 - \oint : \mathcal{U} : underlap.

4. Discussion & Interpretation

- Before a semantic treatment of the concept of mereology
 - let us review what we have done and
 - let us interpret our abstraction
 - * (i.e., relate it to actual societal infrastructure components).

4.1. What We have Done So Far ?

- We have
 - presented a model that is claimed to abstract essential mereological properties of
 - * machine assemblies,
 - * railway nets,
 - * the oil industry,
 - * oil pipelines,
 - * buildings with installations,
 - * hospitals,
 - * etcetera.

4.2. Six Interpretations

- Let us substantiate the claims made in the previous paragraph.
 - We will do so, albeit informally, in the next many paragraphs.
 - Our substantiation is a form of diagrammatic reasoning.
 - Subsets of diagrams will be claimed to represent parts, while
 - Other subsets will be claimed to represent connectors.
- The reasoning is incomplete.

4.2.1. Air Traffic

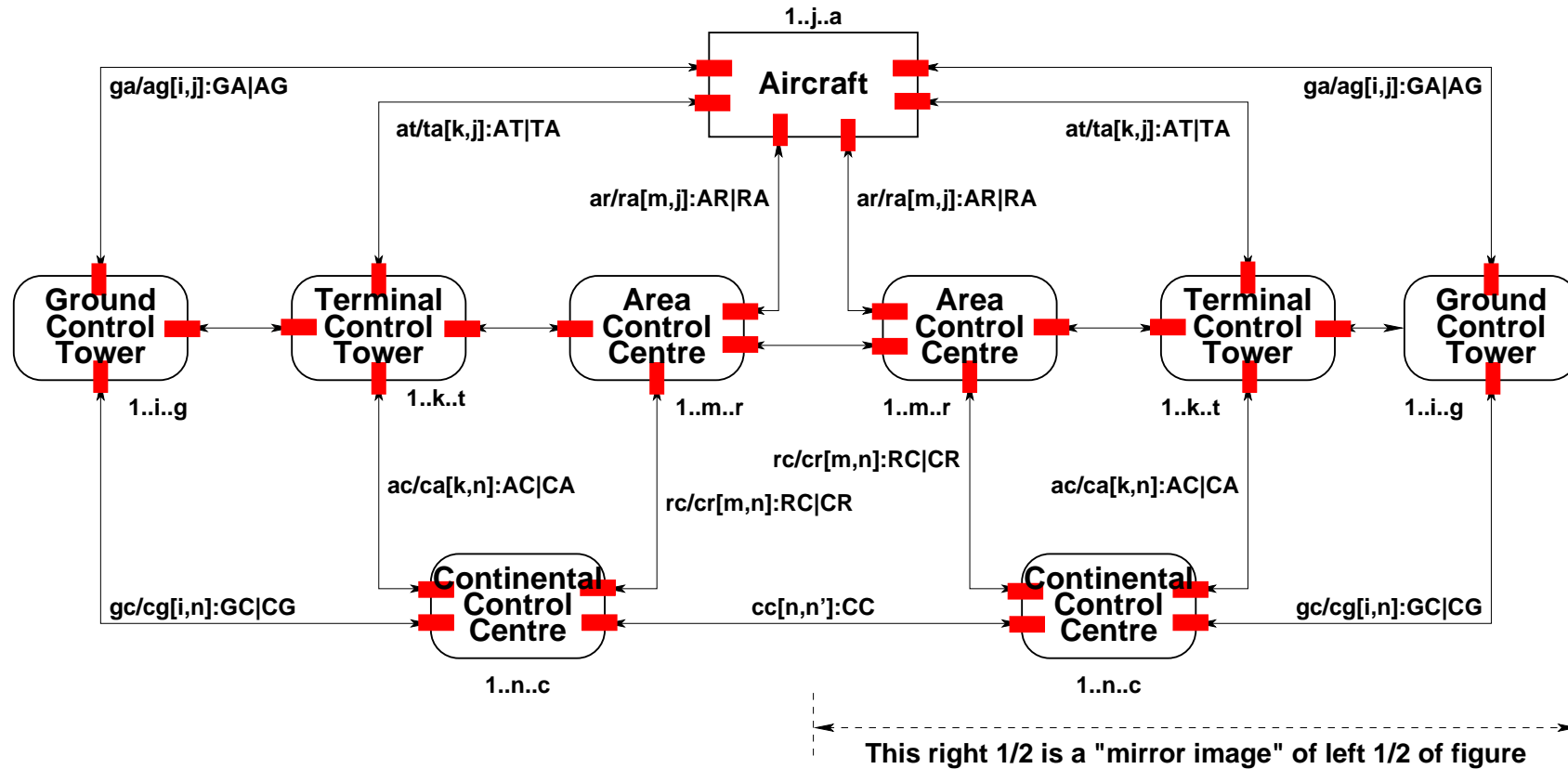


Figure 3: An air traffic system. Black boxes and lines are units; red boxes are connections

4.2.2. Buildings

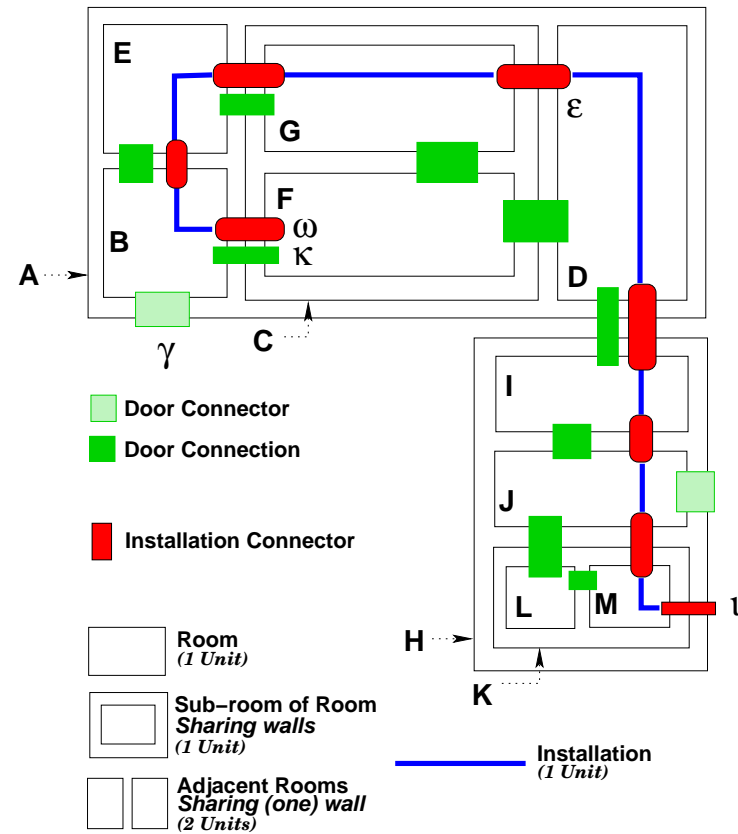


Figure 4: A building plan with installation

4.2.3. Financial Service Industry

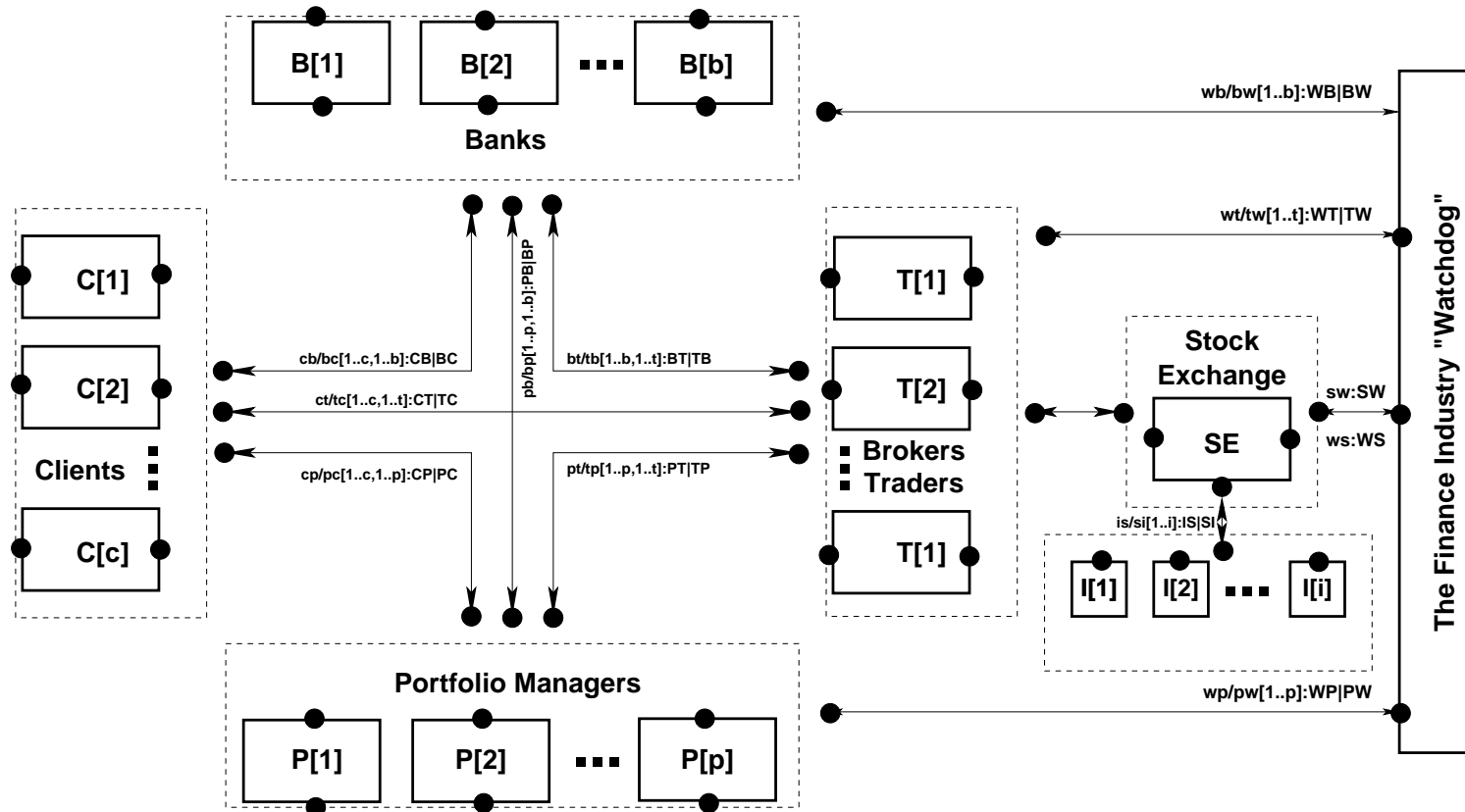


Figure 5: A financial service industry

4.2.4. Machine Assemblies

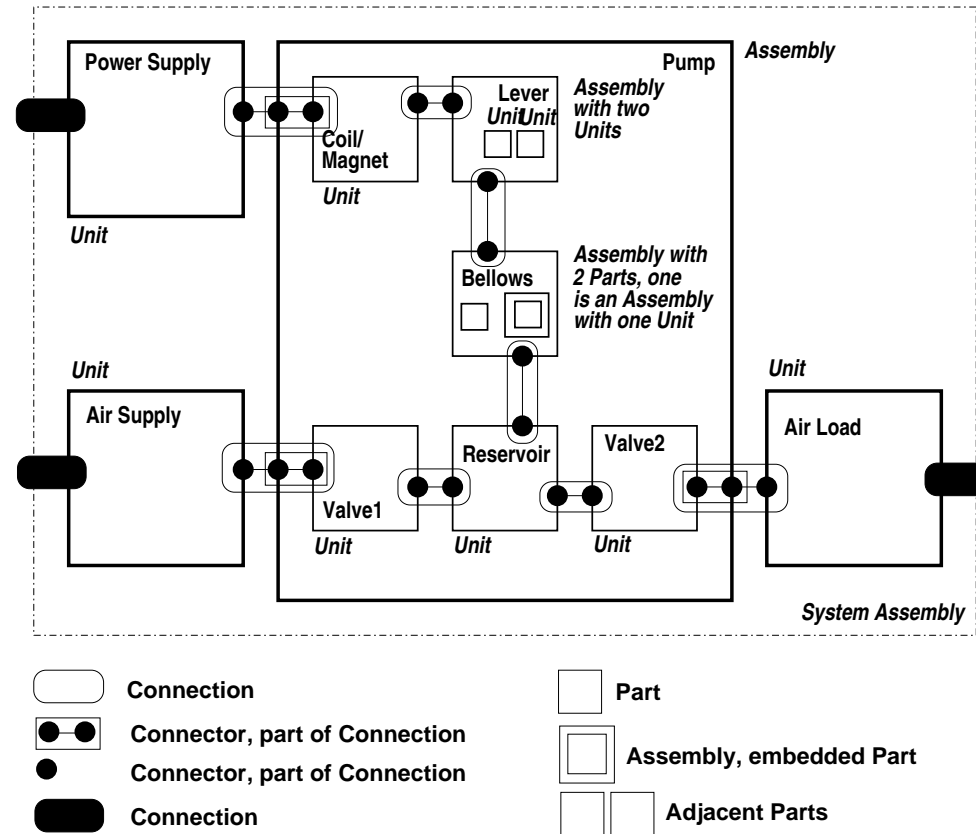


Figure 6: An air pump, i.e., a physical mechanical system

4.2.5. Oil Industry

4.2.5.1. "The" Overall Assembly

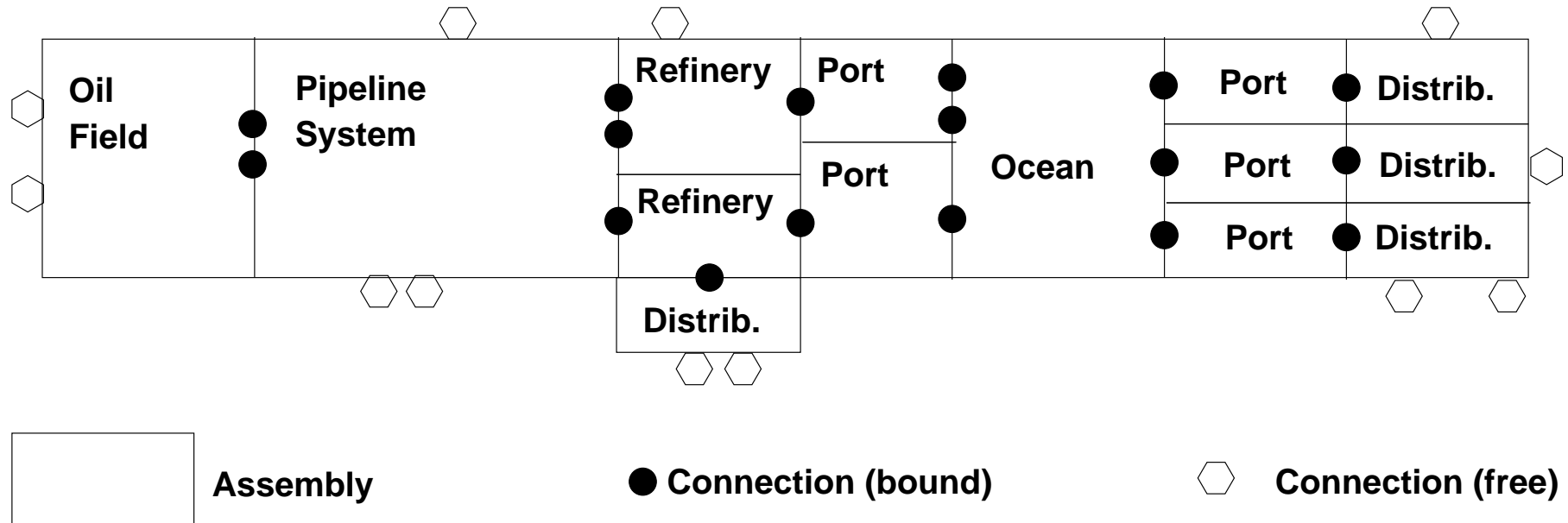


Figure 7: A Schematic of an Oil Industry

4.2.5.2. A Concretised Assembly Unit

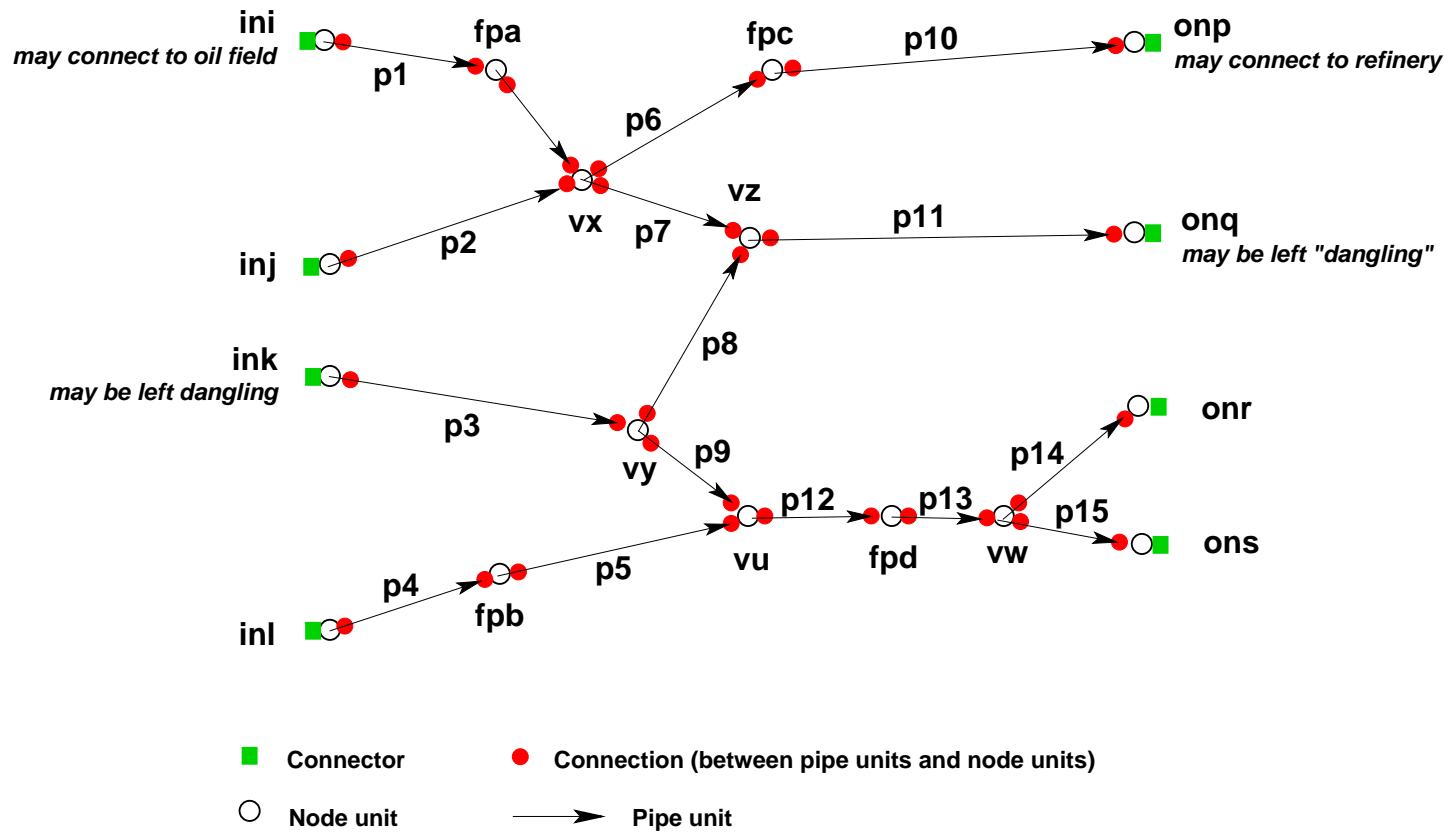


Figure 8: A Pipeline System

4.2.6. Railway Nets

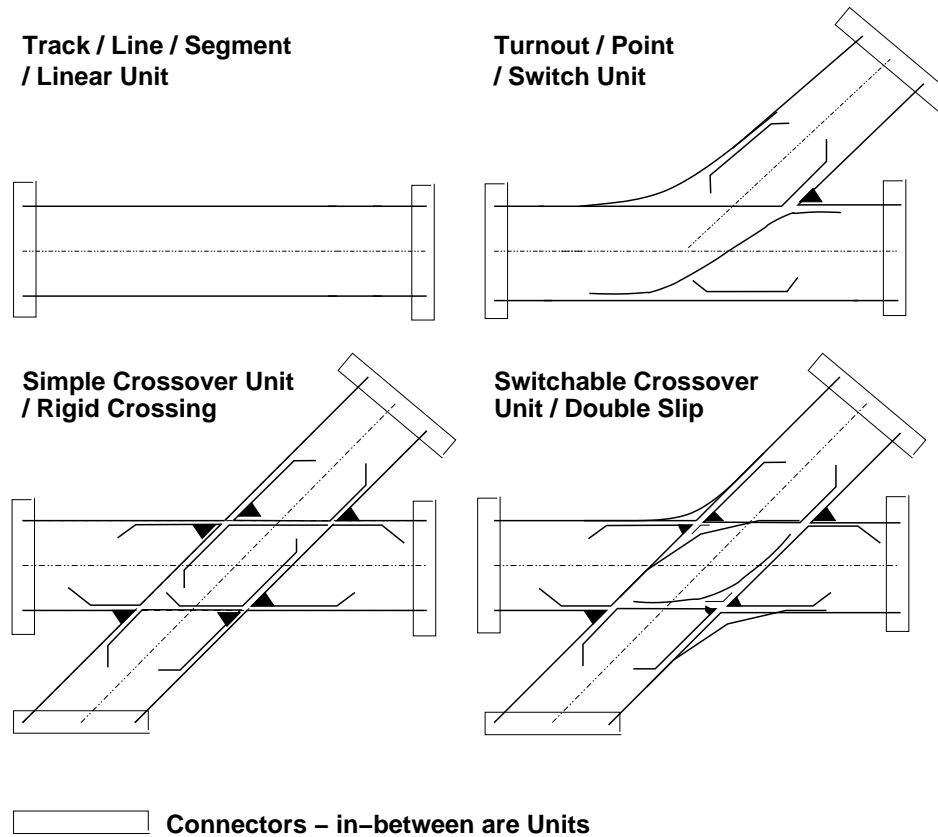


Figure 9: Four example rail units

4. Discussion & Interpretation 4.2. Six Interpretations 4.2.6. Railway Nets

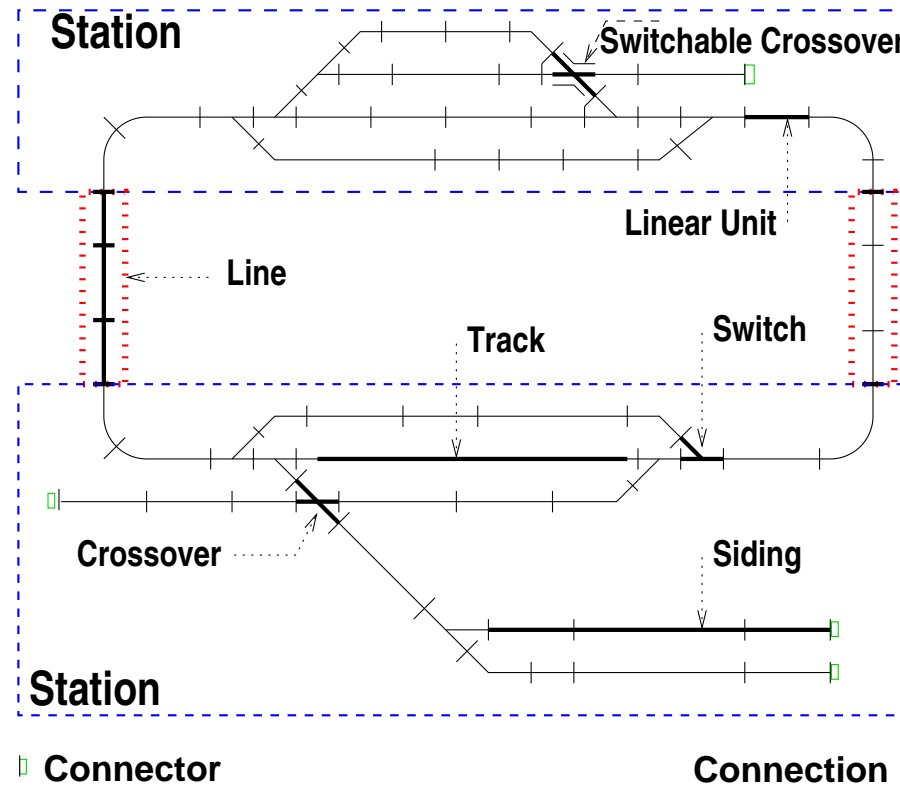


Figure 10: A “model” railway net. An Assembly of four Assemblies:
 Two stations and two lines; Lines here consist of linear rail units;
 stations of all the kinds of units shown in Fig. 9 on the facing page.
 There are 66 connections and four “dangling” connectors

4.3. Discussion

- It requires a somewhat more laborious effort,
 - than just “flashing” and commenting on these diagrams,
 - to show that the modelling of essential aspects of their structures
 - can indeed be done by simple instantiation
 - of the model given in the previous part of the talk.

[4. Discussion & Interpretation, 4.3. Discussion]

- We can refer to a number of documents which give rather detailed domain models of
 - air traffic,
 - container line industry,
 - financial service industry,
 - health-care,
 - IT security,
 - “the market”,
 - “the” oil industry²,
 - transportation nets³,
 - railways, etcetera, etcetera.
- Seen in the perspective of the present paper
 - we claim that much of the modelling work done in those references
 - can now be considerably shortened and
 - trust in these models correspondingly increased.

²<http://www2.imm.dtu.dk/~db/pipeline.pdf>

³<http://www2.imm.dtu.dk/~db/transport.pdf>

4.4. Mereology, Part III

- Formula 15 on page 26 expresses that
 - whenever an individual has one proper part
 - then it has more than one.
- We mentioned there, Slide 27, that we would comment on the fact that our model appears to allow that assemblies may have just one proper part.

- We now do so.
 - We shall still allow assemblies to have just one proper part —
 - in the sense of a sub-assembly or a unit —
 - but we shall interpret the fact that an assembly always have at least one attribute.
 - Therefore we shall “generously” interpret the set of attributes of an assembly to constitute a part.

- In Sect. 5
 - we shall see how attributes of both units and assemblies of the interpreted mereology
 - contribute to the state components of the unit and assembly processes.

[4. Simple Entities]

4.5. Discussion

- In Sect. 3.2 we interpreted the model of mereology in six examples.
- The units of Sect. 2
 - which in that section were left uninterpreted
 - now got individuality —
 - * in the form of

<ul style="list-style-type: none"> · aircraft, · building rooms, 	<ul style="list-style-type: none"> · rail units and · oil pipes.
--	--
 - Similarly for the assemblies of Sect. 2. They became

<ul style="list-style-type: none"> * pipeline systems, * oil refineries, 	<ul style="list-style-type: none"> * train stations, * banks, etc.
--	--

- In conventional modelling
 - the mereology of an infrastructure component,
 - * of the kinds exemplified in Sect. 3.2,
 - was modelled by modelling
 - * that infrastructure component's special mereology
 - * together, “in line”, with the modelling
 - * of unit and assembly attributes.
- With the model of Sect. 2 now available
 - we do not have to model the mereological aspects,
 - but can, instead, instantiate the model of Sect. 2 appropriately.
 - We leave that to be reported upon elsewhere.

5. A Semantic Model of a Class of Mereologies

5.1. The Mereology Entities \equiv Processes

- The model of mereology (Slides 13–38) given earlier focused on the following simple entities (i) the assemblies, (ii) the units and (iii) the connectors.
- To assemblies and units we associate **CSP** processes, and
- to connectors we associate a **CSP** channels,
- one-by-one.
- The connectors form the mereological attributes of the model.

5.1.1. Channels

- The CSP channels,
 - are each “anchored” in two parts:
 - if a part is a unit then in “its corresponding” unit process, and
 - if a part is an assembly then in “its corresponding” assembly process.
- From a system assembly we can extract all connector identifiers.
- They become indexes into an array of channels.
 - Each of the connector channel identifiers is mentioned
 - in exactly two unit or assembly processes.

value $s:S$ $kis:KI\text{-set} = xtr \forall KIs(s)$ **type** $ChMap = AUI \xrightarrow{m} KI\text{-set}$ **value** $cm:ChMap = [obs_AUI(p) \mapsto obs_KIs(p) | p:P \cdot p \in xtr_Ps(s)]$ **channel** $ch[i|i:KI \cdot i \in kis] MSG$

5.2. Process Definitions

value

system: $S \rightarrow \mathbf{Process}$

system(s) \equiv assembly(s)

assembly: $a:A \rightarrow \mathbf{in, out} \{ \text{ch}[\text{cm}(i)] \mid i:KI \cdot i \in \text{cm}(\text{obs_AUI}(a)) \} \mathbf{process}$

assembly(a) \equiv

$\mathcal{M}_{\mathcal{A}}(a)(\text{obs_A}\Sigma(a)) \parallel$

$\parallel \{ \text{assembly}(a') \mid a':A \cdot a' \in \text{obs_Ps}(a) \} \parallel$

$\parallel \{ \text{unit}(u) \mid u:U \cdot u \in \text{obs_Ps}(a) \}$

$\text{obs_A}\Sigma: A \rightarrow A\Sigma$

$\mathcal{M}_{\mathcal{A}}: a:A \rightarrow A\Sigma \rightarrow \mathbf{in, out} \{ \text{ch}[\text{cm}(i)] \mid i:KI \cdot i \in \text{cm}(\text{obs_AUI}(a)) \} \mathbf{process}$

$\mathcal{M}_{\mathcal{A}}(a)(a\sigma) \equiv \mathcal{M}_{\mathcal{A}}(a)(A\mathcal{F}(a)(a\sigma))$

$A\mathcal{F}: a:A \rightarrow A\Sigma \rightarrow \mathbf{in, out} \{ \text{ch}[\text{em}(i)] \mid i:KI \cdot i \in \text{cm}(\text{obs_AUI}(a)) \} \times A\Sigma$

5. A Semantic Model of a Class of Mereologies 5.2. Process Definitions

$\text{unit}: u:U \rightarrow \mathbf{in,out} \{ \text{ch}[\text{cm}(i)] \mid i:KI \cdot i \in \text{cm}(\text{obs_UI}(u)) \} \text{ process}$

$\text{unit}(u) \equiv \mathcal{M}_u(u)(\text{obs_U}\Sigma(u))$

$\text{obs_U}\Sigma: U \rightarrow U\Sigma$

$\mathcal{M}_u: u:U \rightarrow U\Sigma \rightarrow \mathbf{in,out} \{ \text{ch}[\text{cm}(i)] \mid i:KI \cdot i \in \text{cm}(\text{obs_UI}(u)) \} \text{ process}$

$\mathcal{M}_u(u)(u\sigma) \equiv \mathcal{M}_u(u)(U\mathcal{F}(u)(u\sigma))$

$U\mathcal{F}: U \rightarrow U\Sigma \rightarrow \mathbf{in,out} \{ \text{ch}[\text{em}(i)] \mid i:KI \cdot i \in \text{cm}(\text{obs_AUI}(u)) \} U\Sigma$

5.3. Mereology, Part III

- A little more meaning has been added to the notions of parts and connections.
- The **within** and **adjacent to** relations between parts (assemblies and units) reflect a phenomenological world of geometry, and
- the **connected** relation between parts (assemblies and units)
 - reflect both physical and conceptual world understandings:
 - * physical world in that, for example, radio waves cross geometric “boundaries”, and
 - * conceptual world in that ontological classifications typically reflect lattice orderings where *overlaps* likewise cross geometric “boundaries”.

5.4. Discussion

- That completes our ‘contribution’:
 - A mereology of systems has been given
 - a syntactic explanation, Sect. 2,
 - a semantic explanation, Sect. 5 and
 - their relationship to classical mereologies.

5.5. **Acknowledgements**

- I thank Lars-Henrik Eriksson and his colleagues for inviting me to give a two week “PhD School” series of lectures at Uppsala.
- And I also thank for the opportunity to give this seminar.