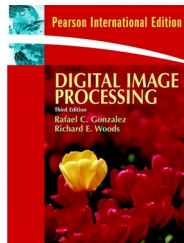
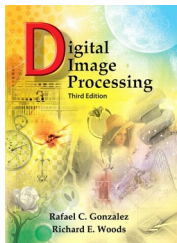


# Lecture 2 – Pointwise processing

Ch. 2.6-2.6.4  
3.1-3.3 in  
Gonzales & Woods



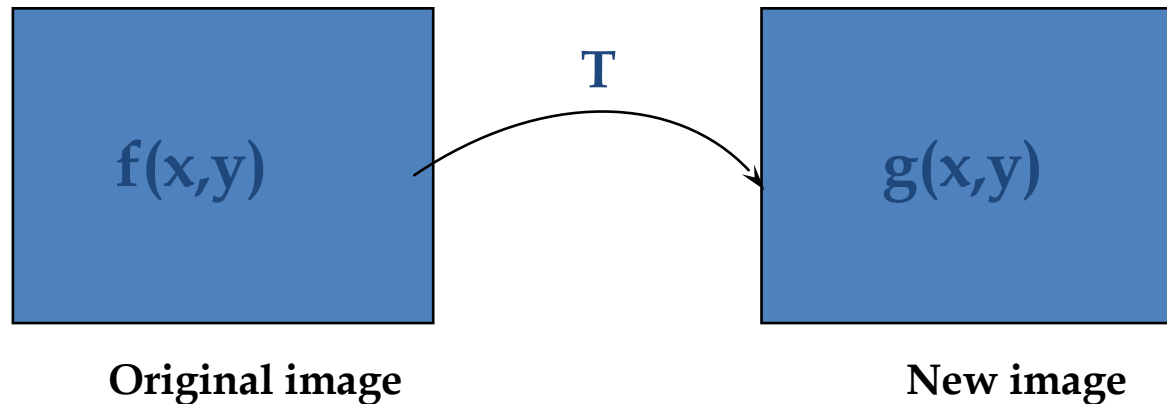
**Filip Malmberg**  
**[filip.malmberg@it.uu.se](mailto:filip.malmberg@it.uu.se)**  
Centre for Image analysis  
Uppsala University

# Image enhancement



- <http://www.youtube.com/watch?v=Vxq9yj2pVWk>
  - an image processing technique to enhance certain features of the image

# Image processing



- We want to create an image which is "better" in some sense.
  - For example
  - Image restoration (reduce noise)
  - Image enhancement (enhance edges, lines etc.)
  - Make the image more suitable for visual interpretation
  - **Image enhancement does NOT increase image information**

# Image processing

- can be performed in the:

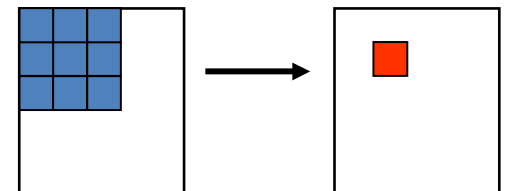
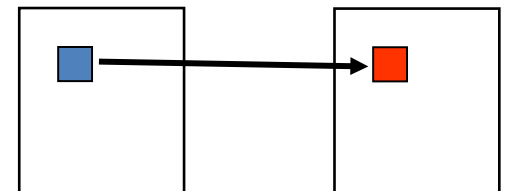
- **Spatial domain**

- Pointwise processing → **Lecture 2**

- Works per pixel

- Spatial filtering → **Lecture 3**

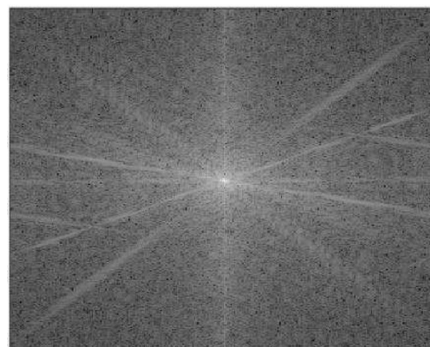
- Works on small neighborhood



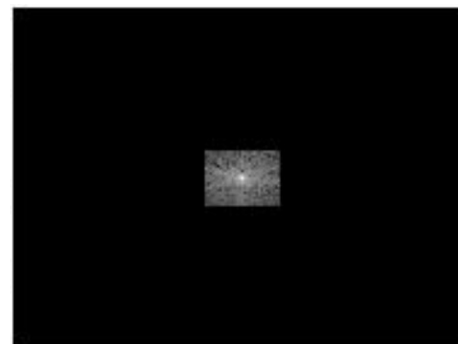
- **Frequency domain** → **Lecture 4**



Original image  
in spatial domain



Original image in  
frequency domain

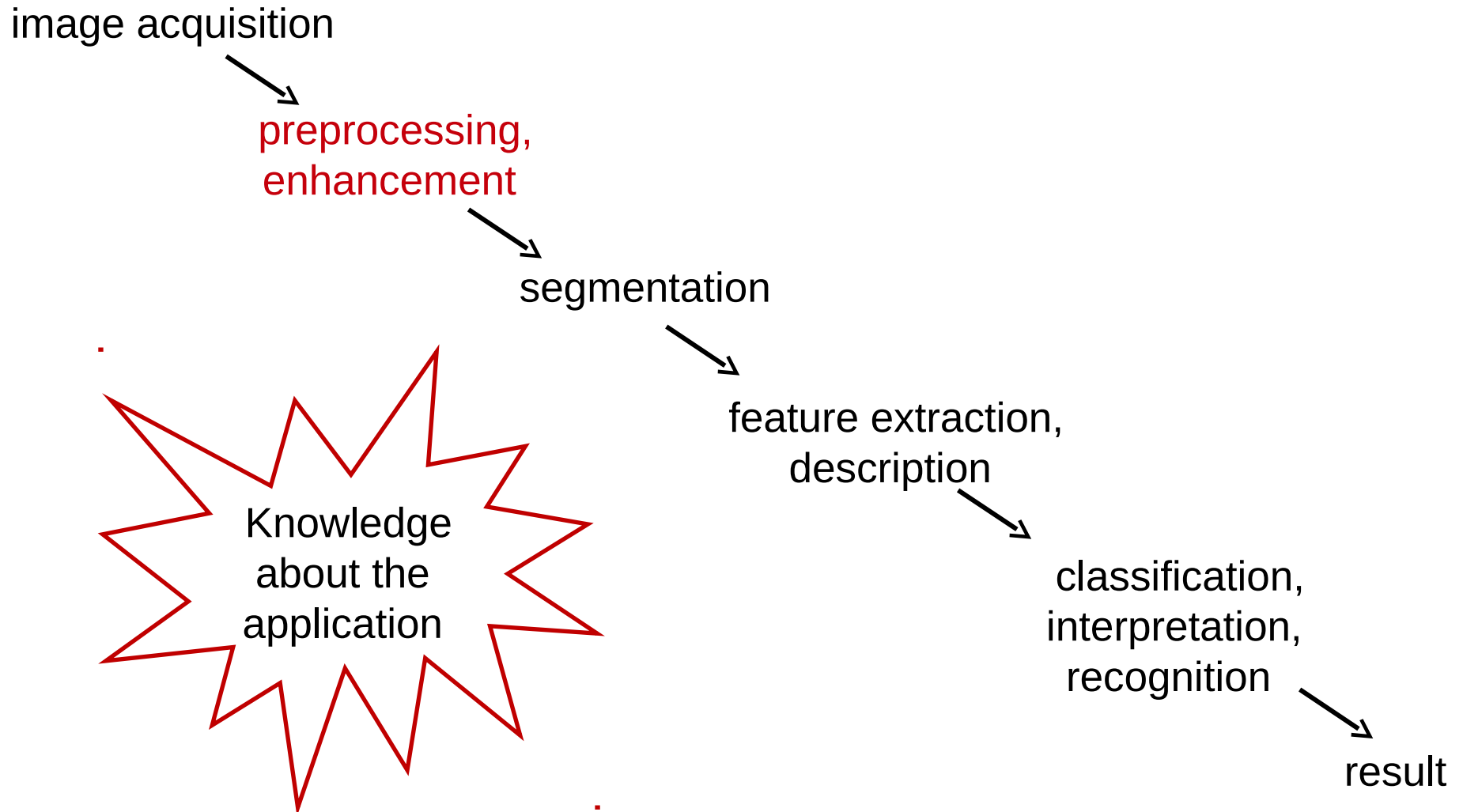


Processed image in  
frequency domain



Processed image  
in spatial domain

# Problem solving using image analysis: fundamental steps



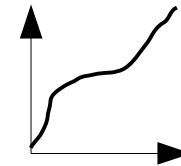
# Overview

i. repetition

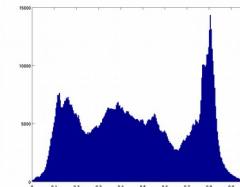
ii. image arithmetics

'+', '-', '\*'

iii. intensity transfer functions



iv. histograms and histogram equalization



# Last lecture

- Digitization
  - Sampling in space (x,y)
  - Sampling in amplitude (intensity)
- Pixel/Voxel
- How often should you sample in space to see details of a certain size?

M



N

# Bit depth

*2 gray levels,  
1bit/pixel*



- Number of bits that are used to store the intensity information
- Images are typically of 8- or 16-bit
  - 1bit =  $2^1 \rightarrow 2$  steps (0,1)
  - 2 bit =  $2^2 \rightarrow 4$  steps
  - 8 bit =  $2^8 \rightarrow 256$  steps
  - 16 bit =  $2^{16} \rightarrow 65\,536$  steps

*64 gray levels,  
6bit/pixel*

AUT01



AUT01

*256 gray levels,  
8bit/pixel*



AUT01



# I. Image arithmetics in the spatial domain

# Image arithmetics

- $A(x,y) = B(x,y) \circ C(x,y)$  for all  $x,y$ .
  - $B, C \rightarrow$  images with the same (spatial) dimensions
    - $\rightarrow$  images + constant value
  - can be
    - Standard arithmetic operation:  $+$ ,  $-$ ,  $*$ .
    - Logical operator (binary images): **AND**, **OR**, **XOR**,...
- Any pitfalls?

# Arithmetics with binary images

■ min value  
□ max value

image1

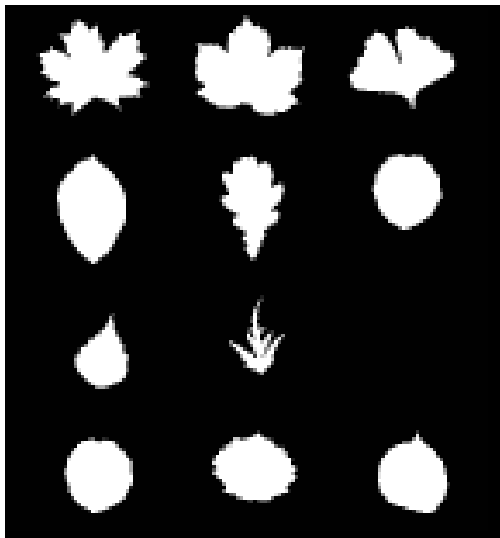


image2

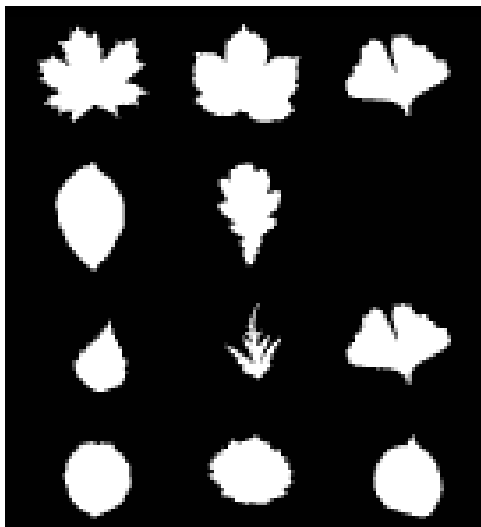


image1-image2

image2-image1

# Arithmetics with binary images

■ min value  
□ max value

image1

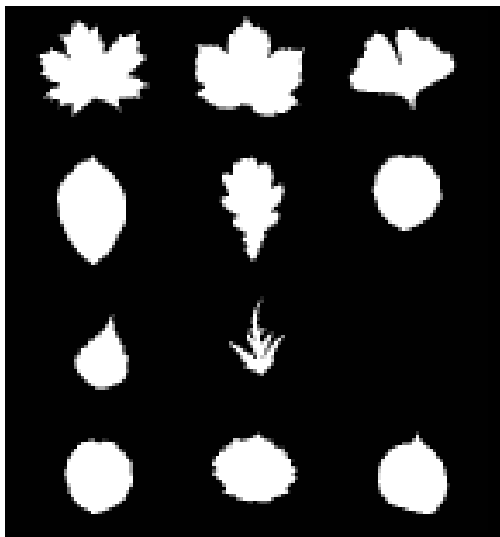


image2

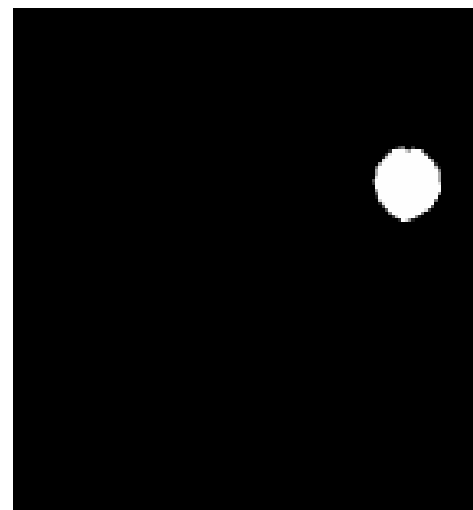
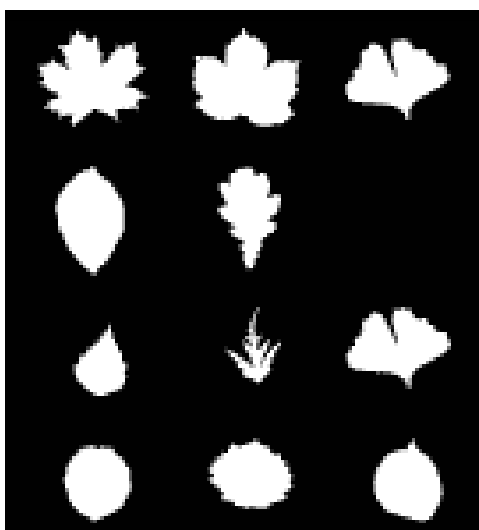


image1-image2

image2-image1

# Arithmetics with binary images

■ min value  
□ max value

image1

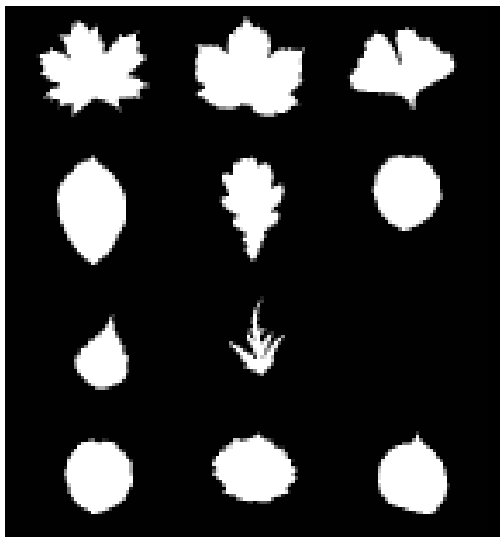


image2

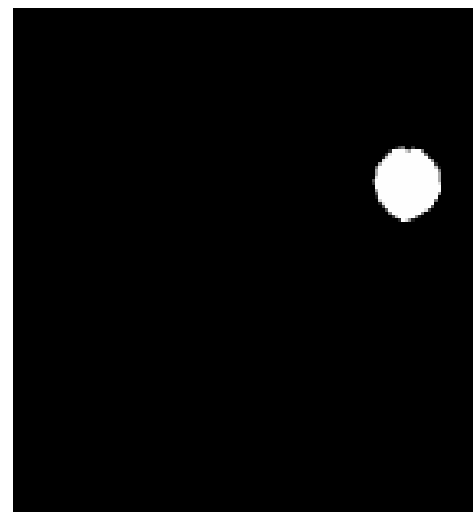
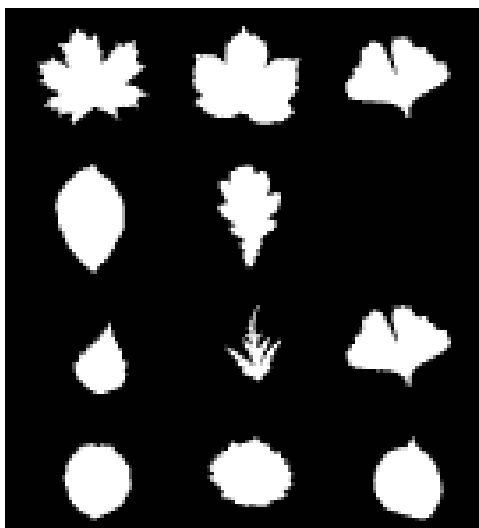
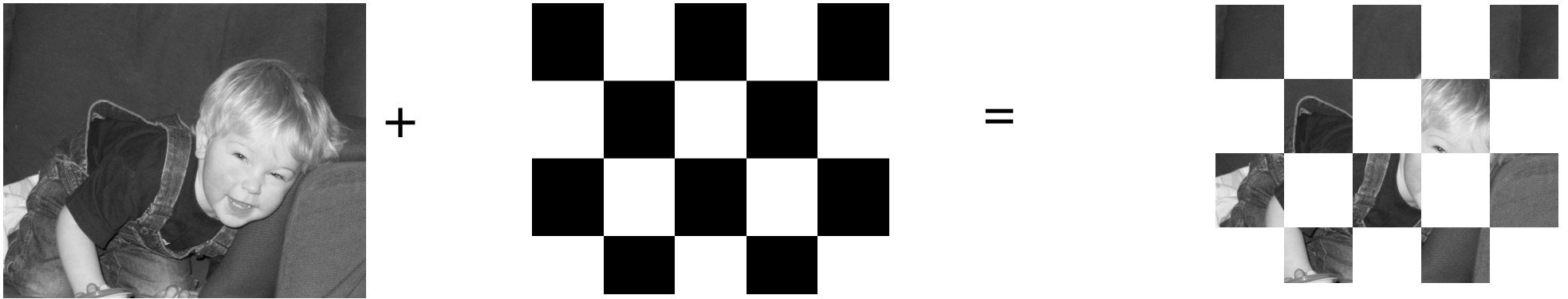


image1-image2



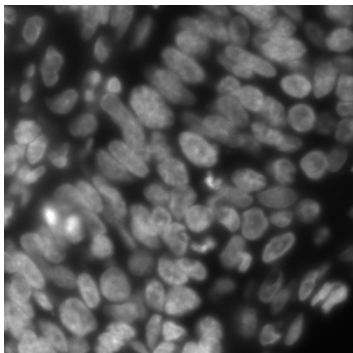
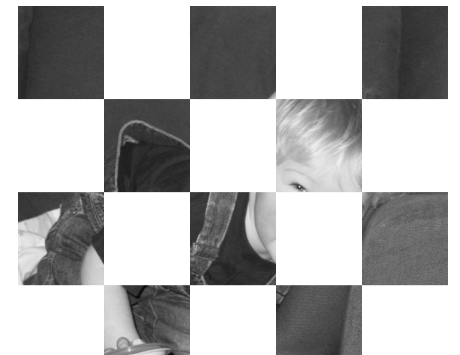
image2-image1

# Arithmetics with greyscale images



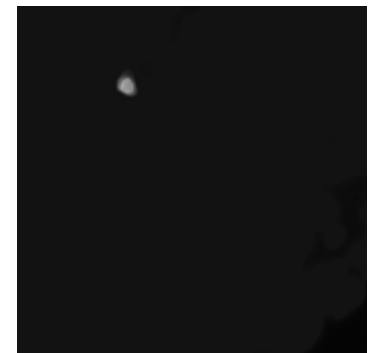
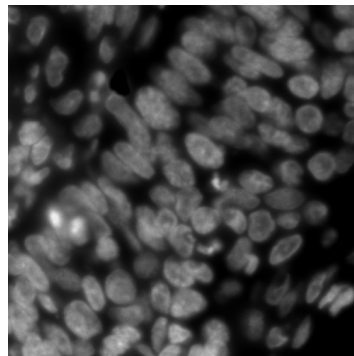
+

=

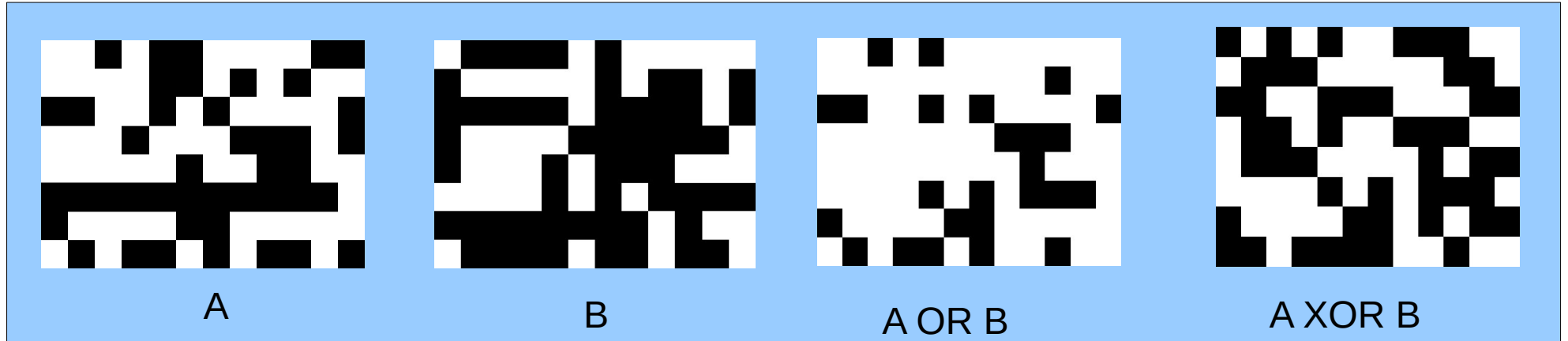


-

=



# Logical operations on binary images



INPUT		OUTPUT
A	B	A OR B
0	0	0
0	1	1
1	0	1
1	1	1

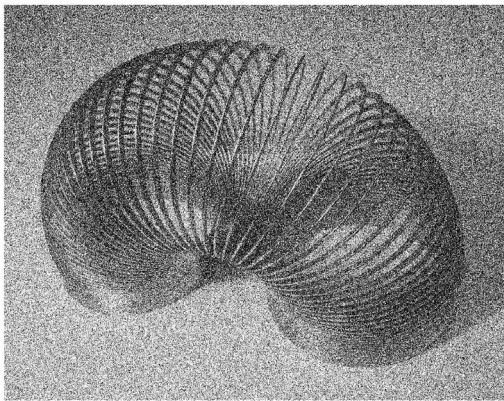
INPUT		OUTPUT
A	B	A XOR B
0	0	0
0	1	1
1	0	1
1	1	0

# Applications

- **Noise reduction** using image mean or median

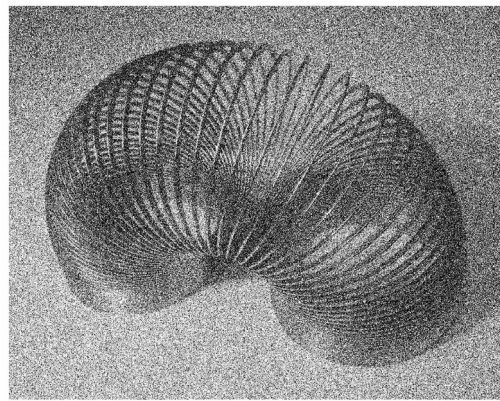
$$I = \frac{1}{n} \sum_{k=1..n} I_k$$

$I_1$



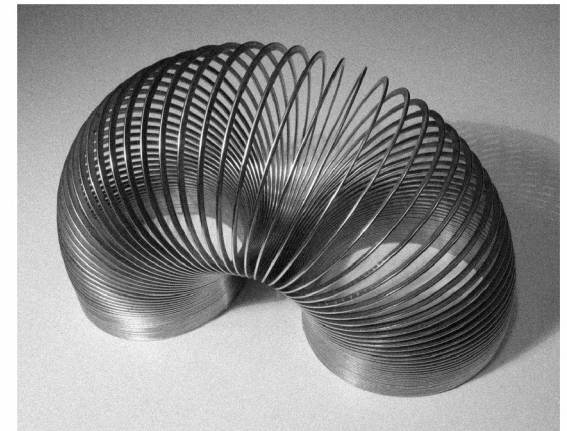
+ ... +

$I_n$



=

$I$



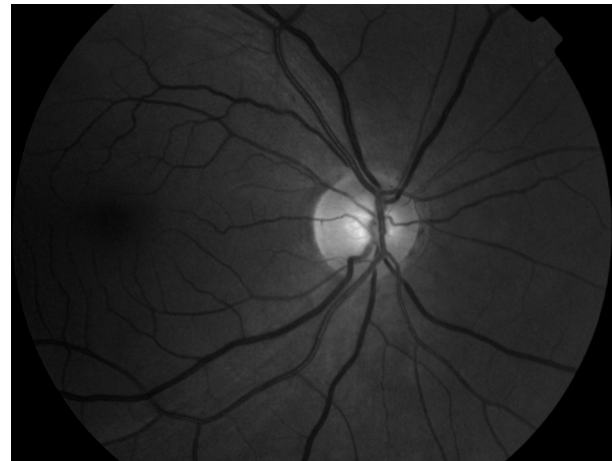


# Applications

- **Change detection using subtraction**

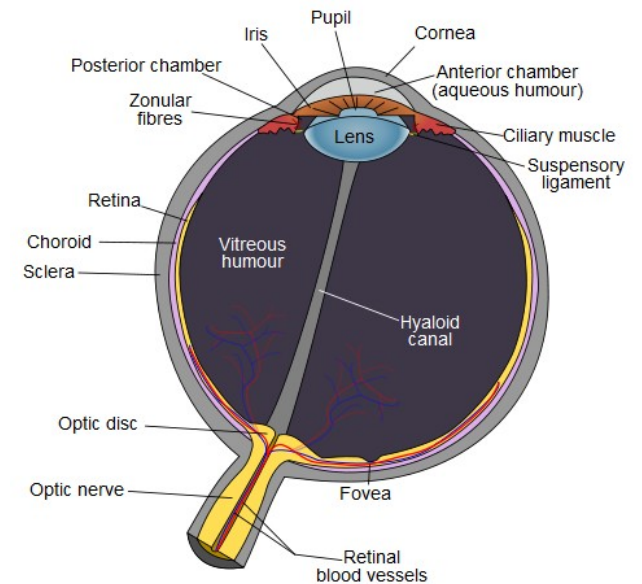


2015



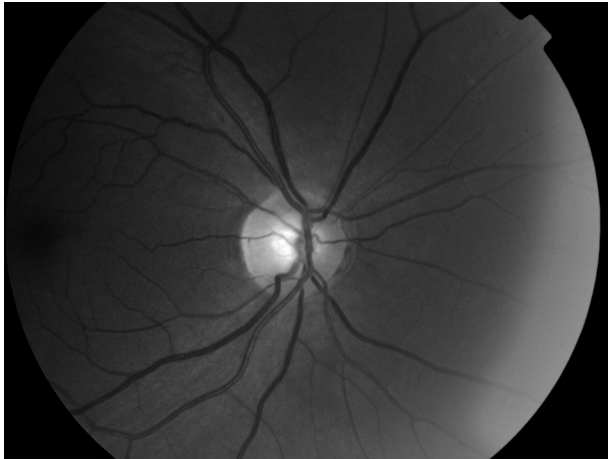
2016

Has anything changed?

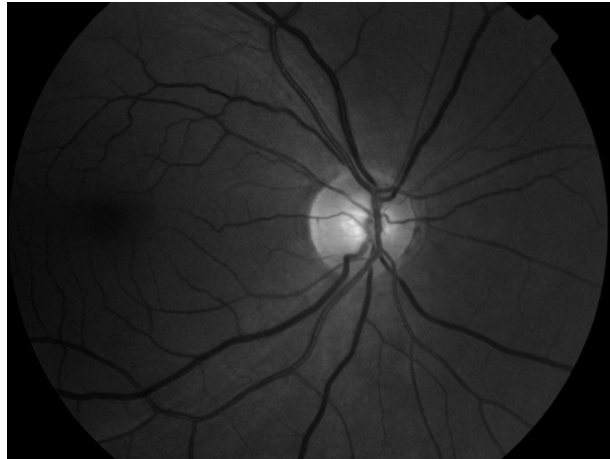


# Applications

- **Change detection** using subtraction



2015



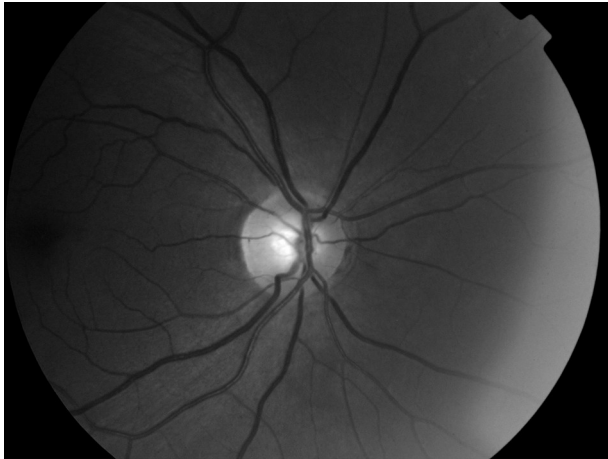
2016



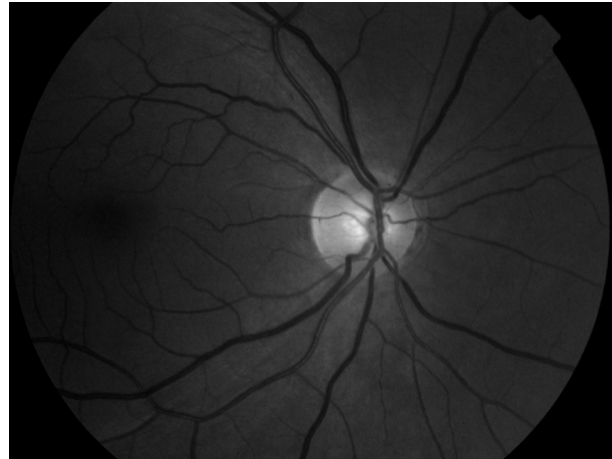
direct difference

# Applications

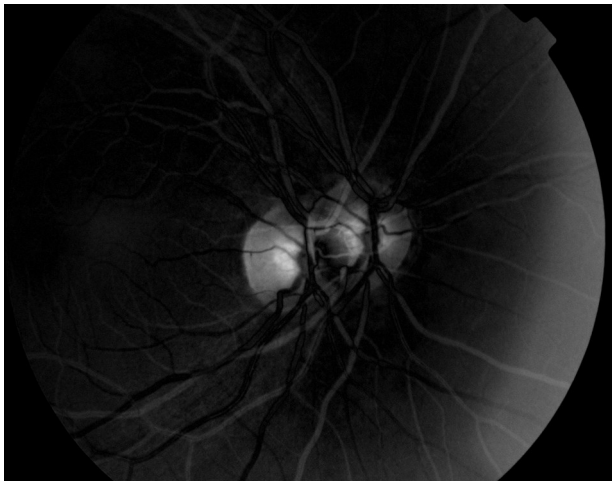
- **Change detection** using subtraction



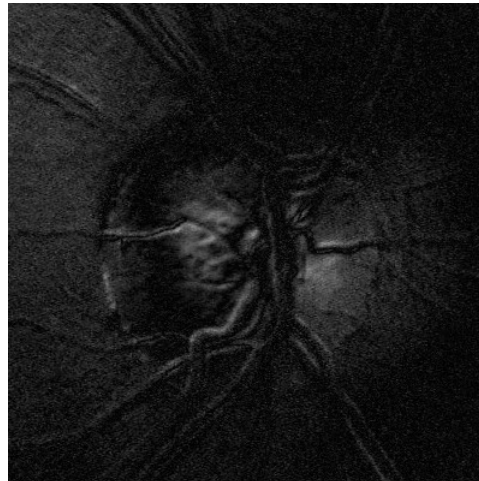
2015



2016



direct difference



difference after registration

# Applications

- **Change/motion detection** using subtraction

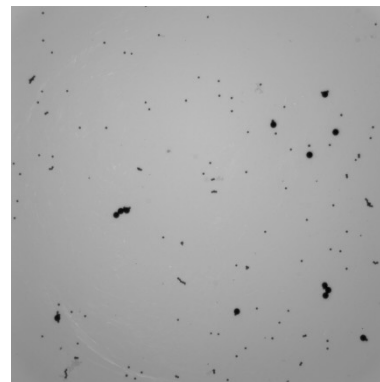
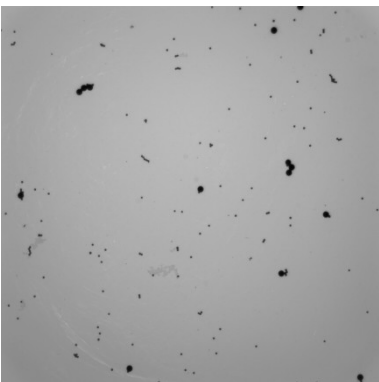
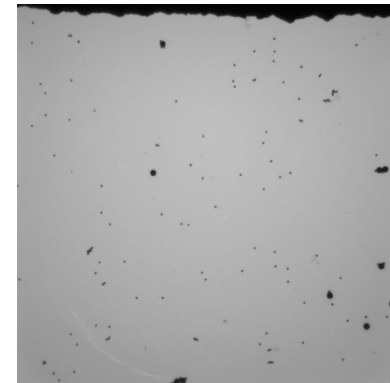
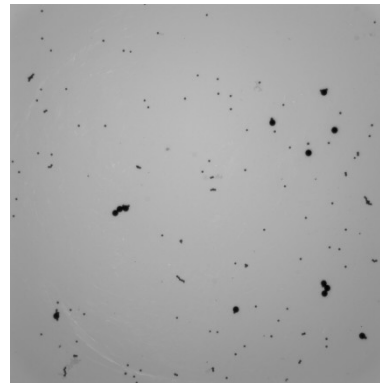
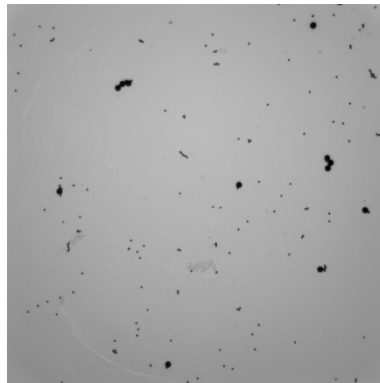
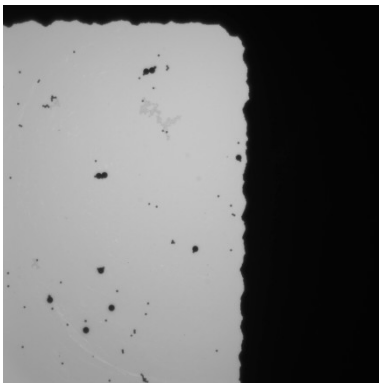


# Applications

- **Background removal**

*image - background image*

- Creating a background image

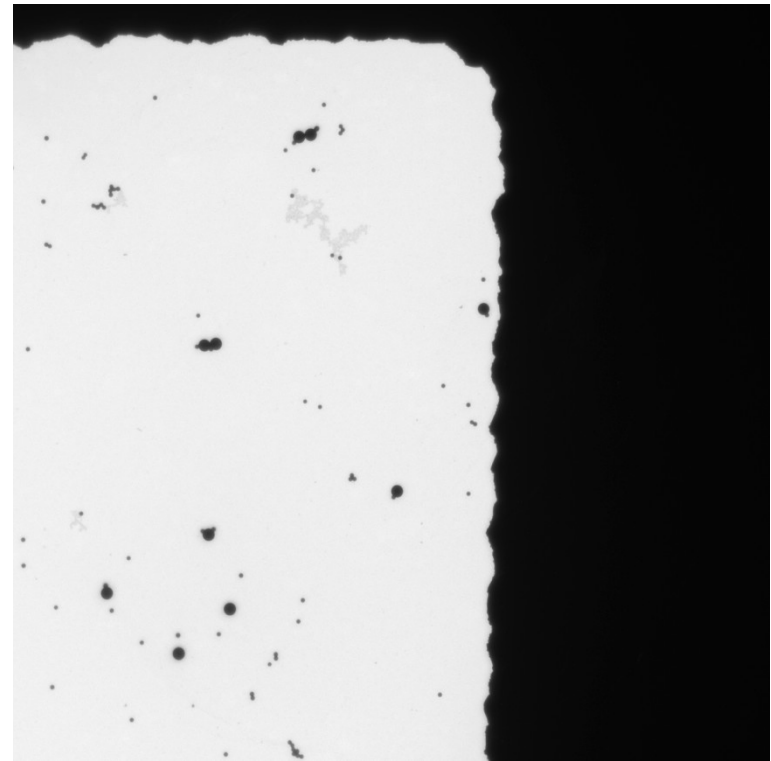
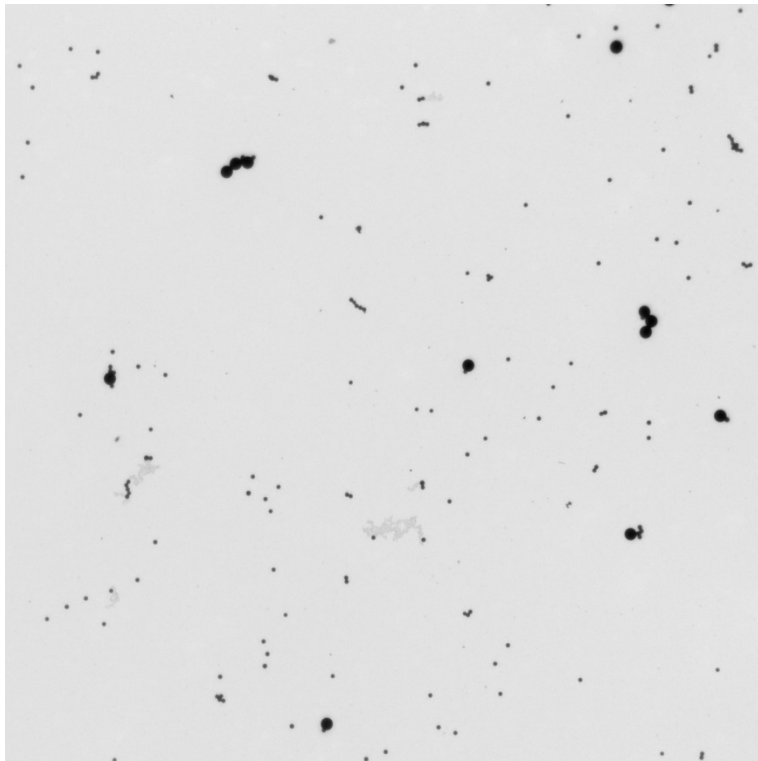


Max or median of  
the pixel intensities  
at all positions.



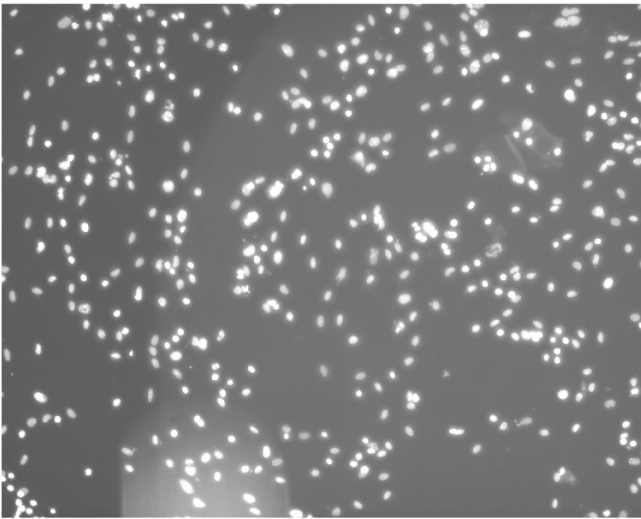
# Applications

- **Background removal - result**



# Applications

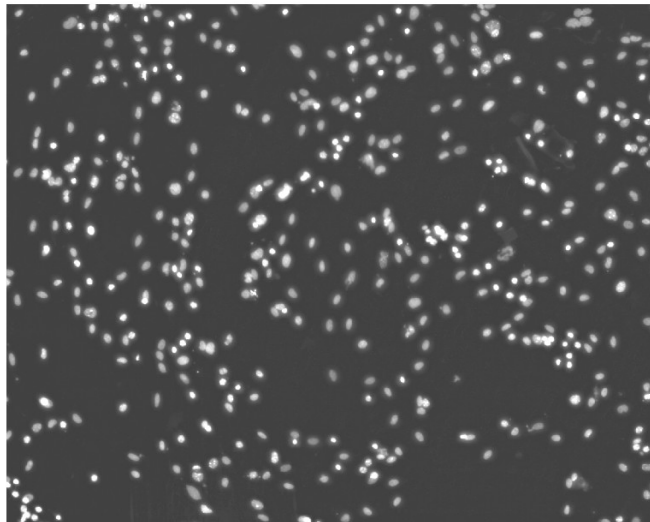
- Subtracting a background image/correcting for uneven illumination



-



=

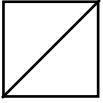


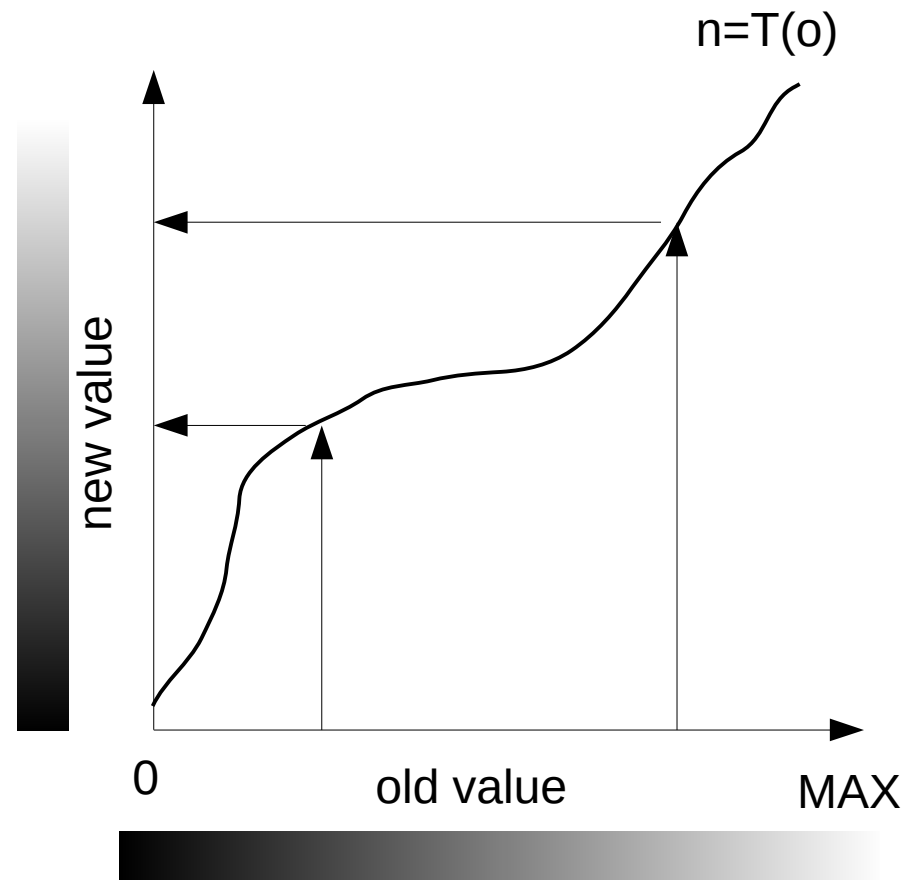
## II. Intensity transfer functions



# Intensity transfer functions

$$g(x, y) = T f(x, y)$$

- i. linear** (neutral  ,  
negative, contrast,  
brightness)
- ii. smooth** (gamma, log)
- iii. arbitrarily**

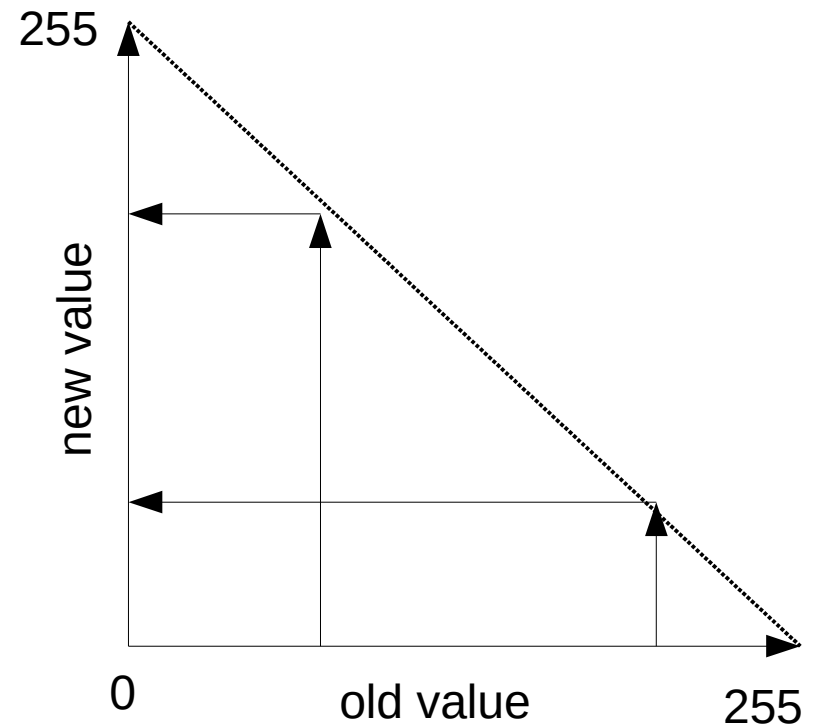


# The negative transformation

$$g(x, y) = \text{max} - f(x, y)$$

- For eight bit image:

$$g(x, y) = 2^8 - 1 - f(x, y)$$



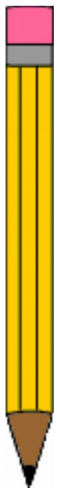
The rules of how to transfer values from the old image to the new one.

# The negative transformation

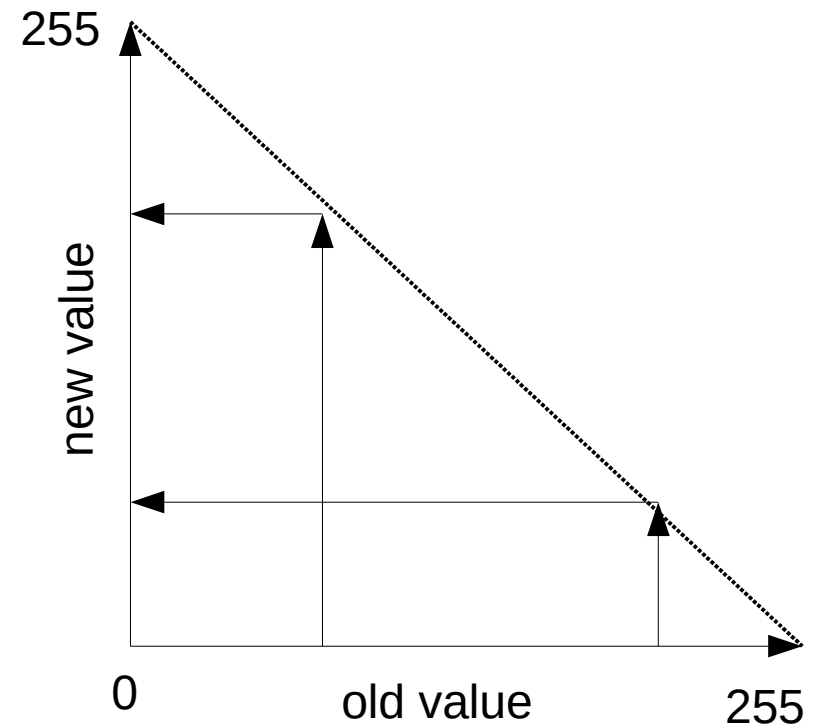
$$g(x, y) = \text{max} - f(x, y)$$

- For eight bit image:

$$g(x, y) = 2^8 - 1 - f(x, y)$$



255	254	253
125	130	110
4	3	0



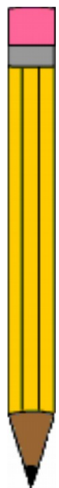
The rules of how to transfer values from the old image to the new one.

# The negative transformation

$$g(x, y) = \text{max} - f(x, y)$$

- For eight bit image:

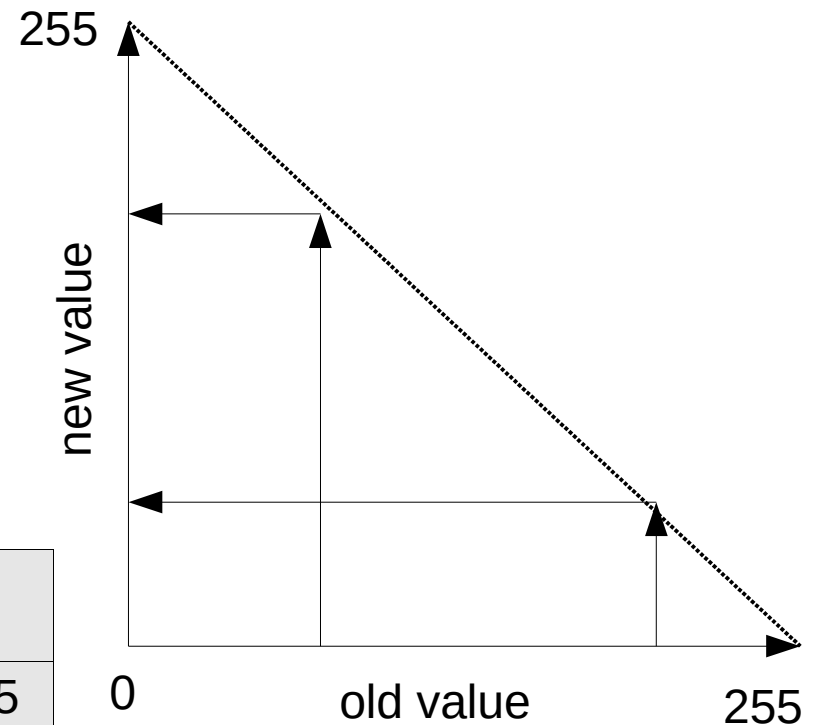
$$g(x, y) = 2^8 - 1 - f(x, y)$$



255	254	253
125	130	110
4	3	0

0	1	2
130	125	145
251	252	255

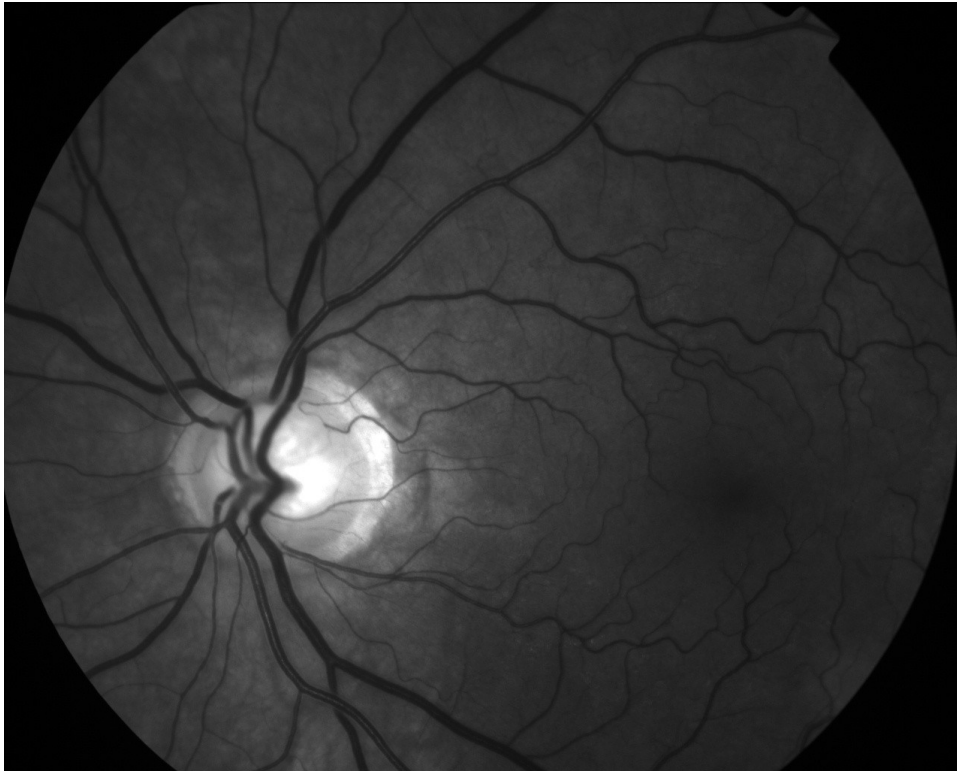
Negative image



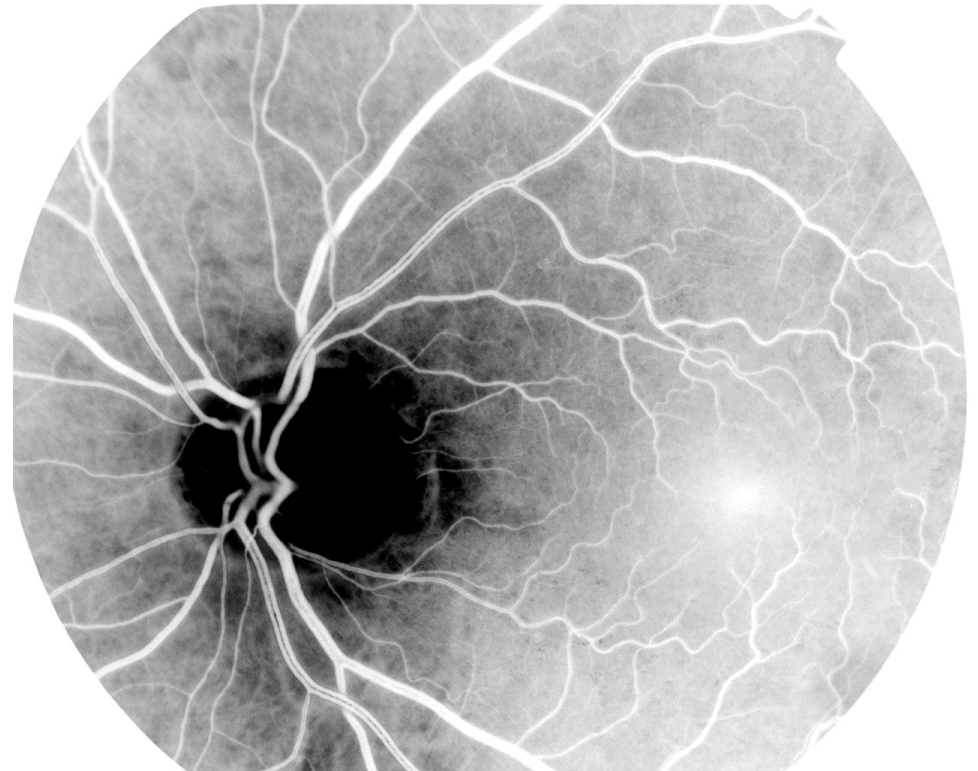
The rules of how to transfer values from the old image to the new one.

# The negative transform

- Example



Original



Negative

# The negative transform



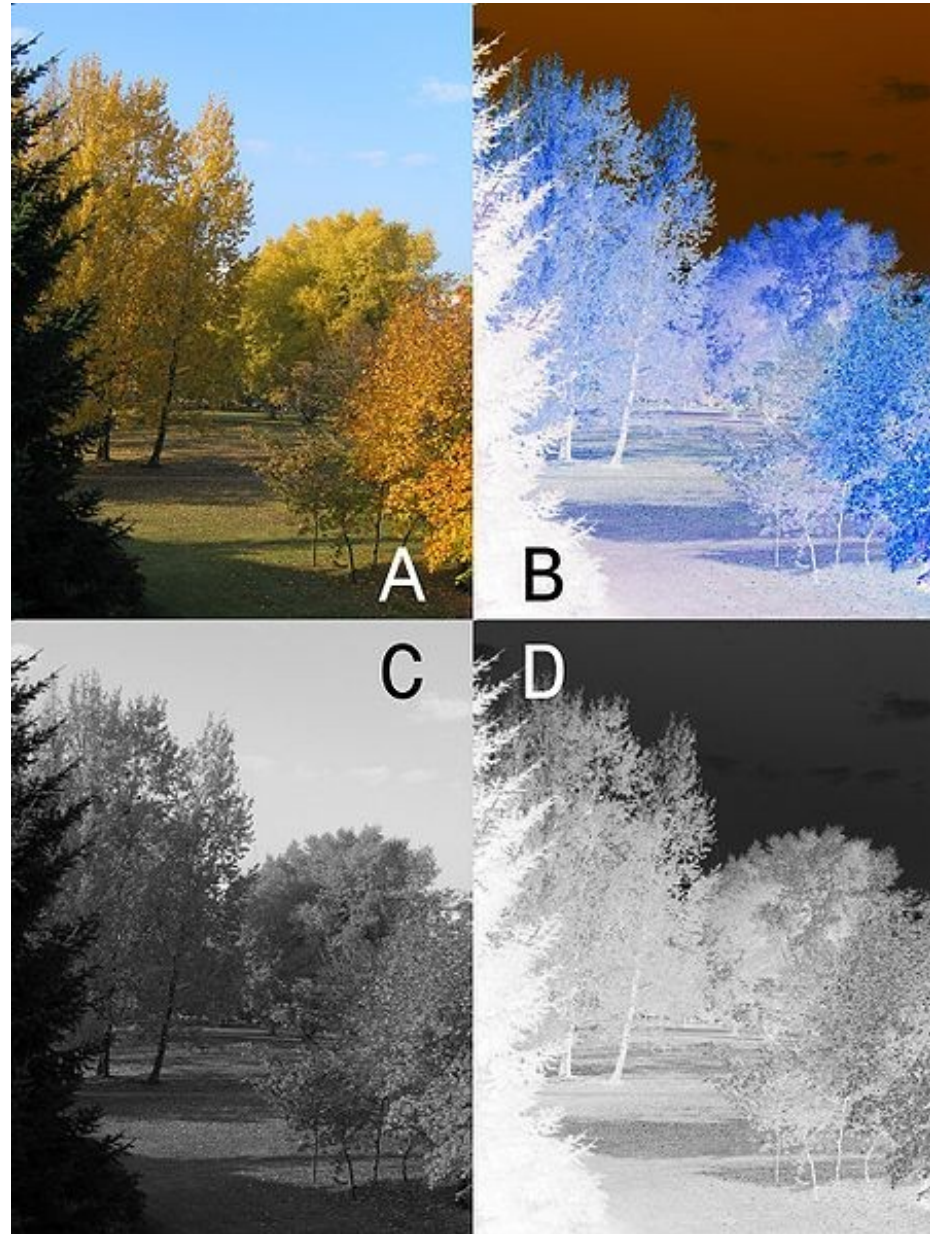
original digital mammogram



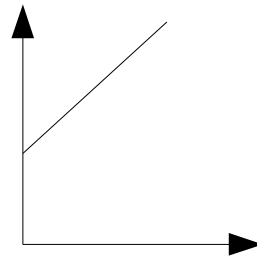
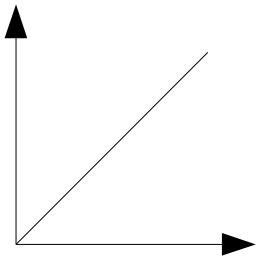
image negative to (visually) enhance  
white or gray details embedded in  
dark regions

# The negative transformation

- Careful with color images



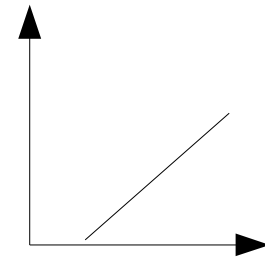
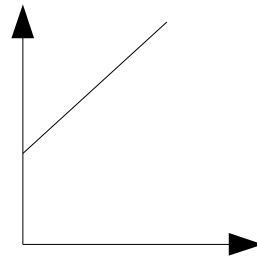
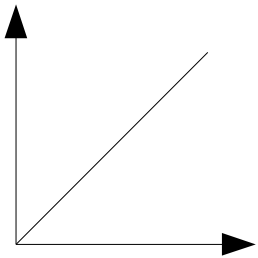
# Brightness



$$g(x, y) = f(x, y) + C$$



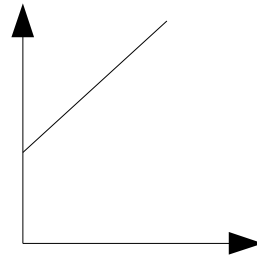
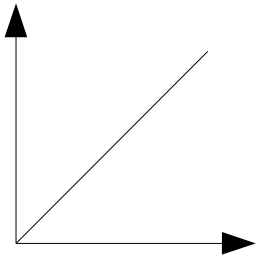
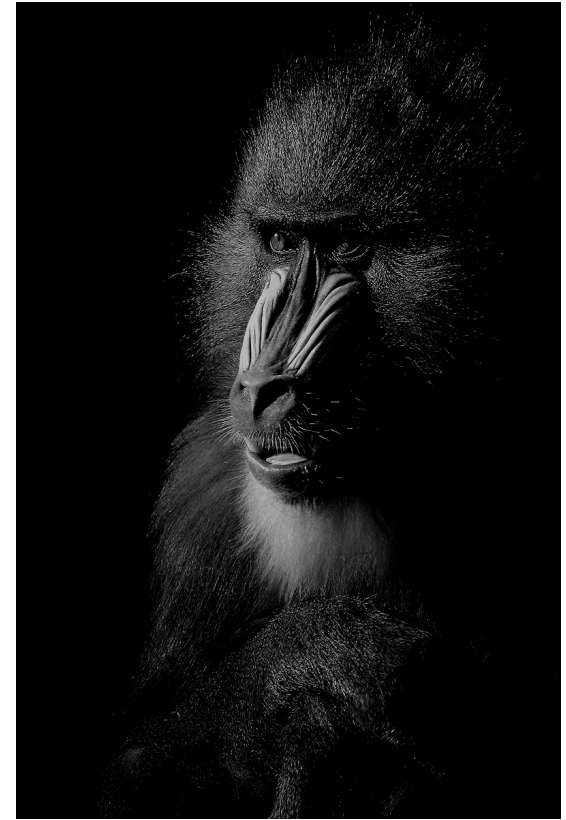
# Brightness



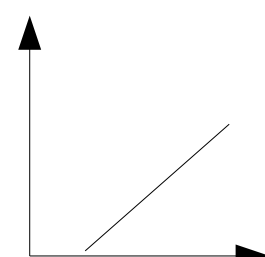
$$g(x, y) = f(x, y) + C$$

$$g(x, y) = f(x, y) - C$$

# Brightness

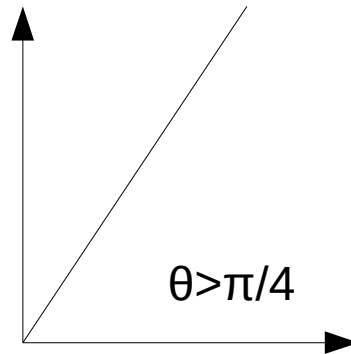
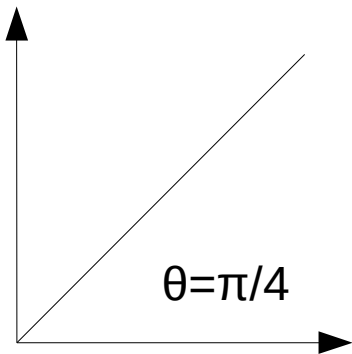


$$g(x, y) = f(x, y) + C$$



$$g(x, y) = f(x, y) - C$$

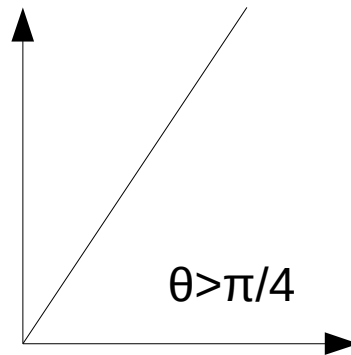
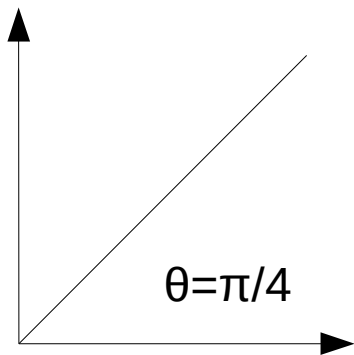
# Contrast



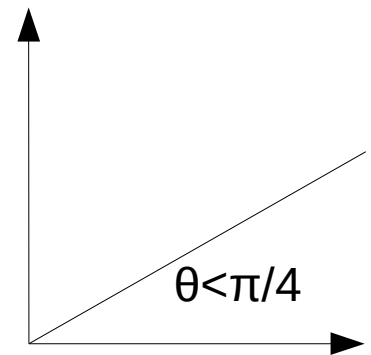
$$g(x, y) = f(x, y) \times C$$

$$C > 1$$

# Contrast

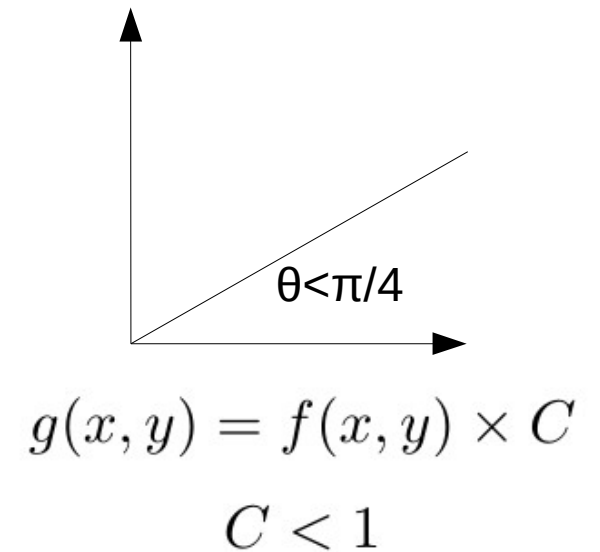
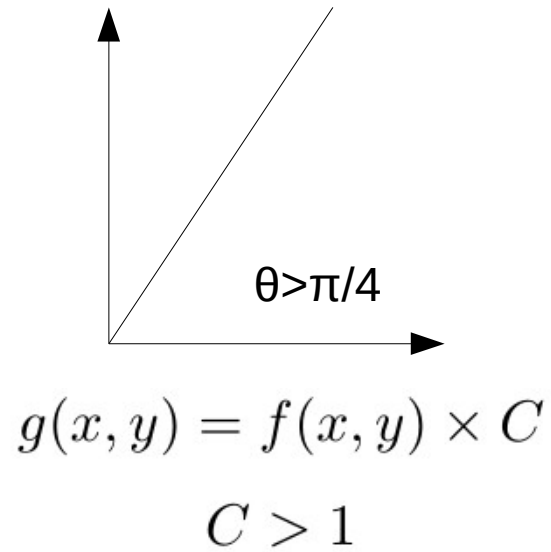
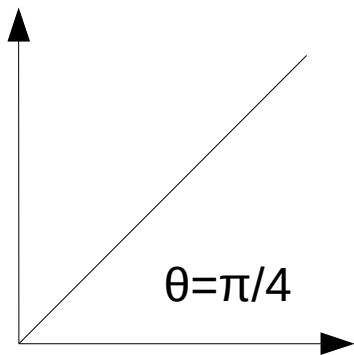


$$g(x, y) = f(x, y) \times C$$
$$C > 1$$



$$g(x, y) = f(x, y) \times C$$
$$C < 1$$

# Contrast



# Examples

- ◆ Decrease the brightness by 10

$$g(x, y) = f(x, y) - 10$$

- ◆ Decrease the contrast by 2

$$g(x, y) = f(x, y) \times 0.5$$



255	254	253
125	130	110
4	3	0

# Examples

- ◆ Decrease the brightness by 10

$$g(x, y) = f(x, y) - 10$$

255	254	253
125	130	110
4	3	0

- ◆ Decrease the contrast by 2

$$g(x, y) = f(x, y) \times 0.5$$

245	244	243
115	120	100
0	0	0

Decreased  
brightness

128	127	127
63	65	55
2	2	0

Decreased  
contrast



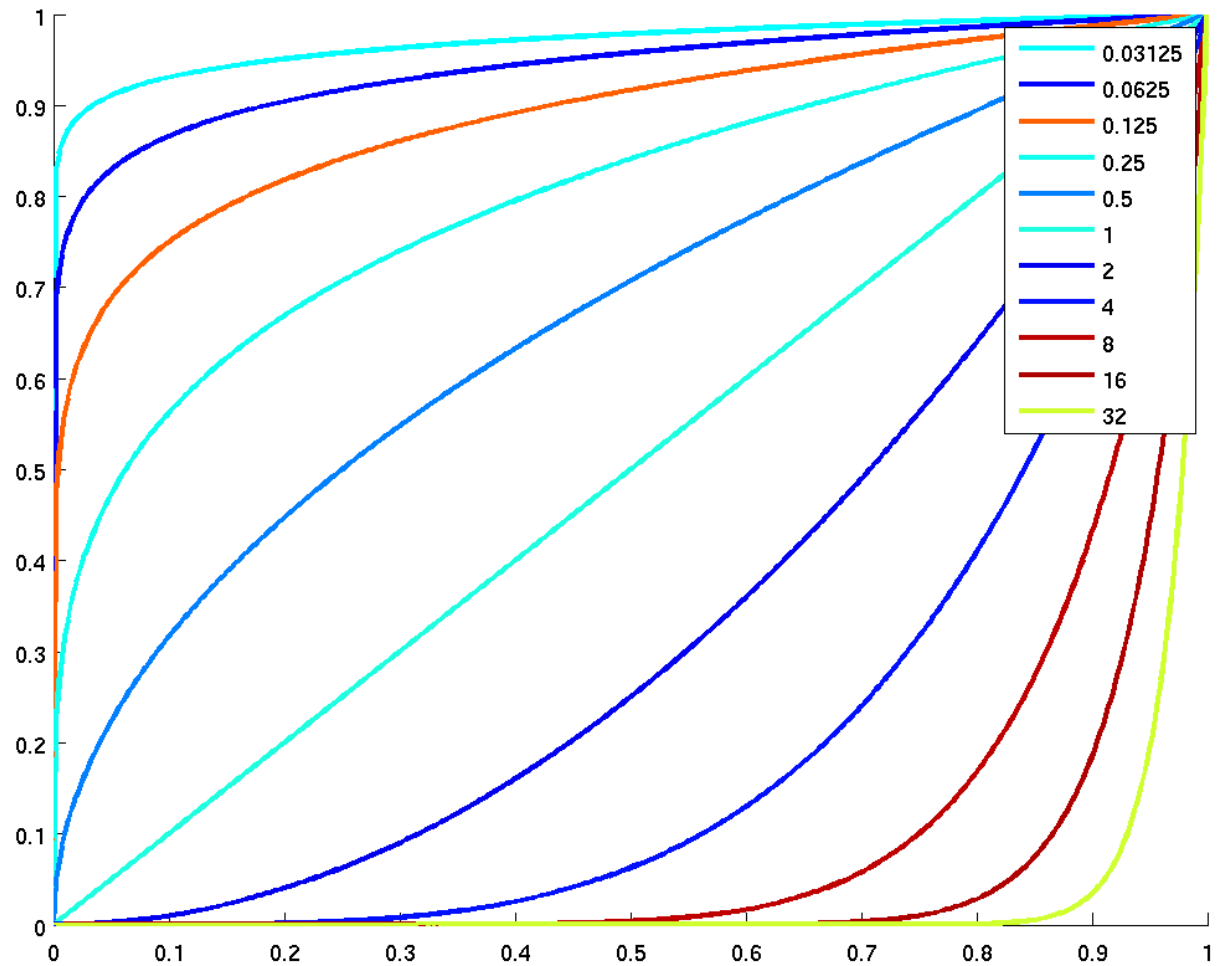
tebe - interesno

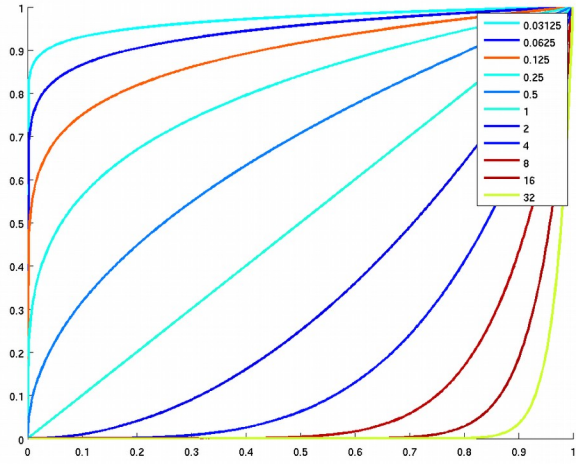


# Gamma transformation

$$g(x, y) = C \times f(x, y)^\gamma$$

- Computer monitors have  $\gamma \sim 2.2$
- Eyes have  $\gamma \sim 0.45$
- Microscopes should have  $\gamma = 1$

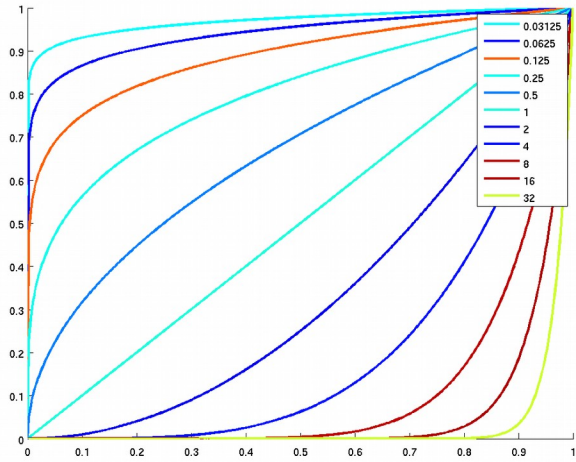




$\gamma=0.25$



$\gamma=1$

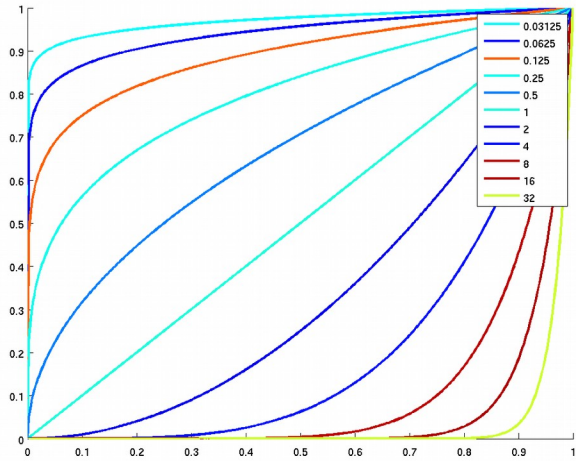


$\gamma=0.25$



$\gamma=1$

$\gamma=4$



$\gamma=0.25$



$\gamma=1$

$\gamma=4$

$\gamma=4$



# Log transformations

- Log transformation to visualize patterns in the dark regions of an image

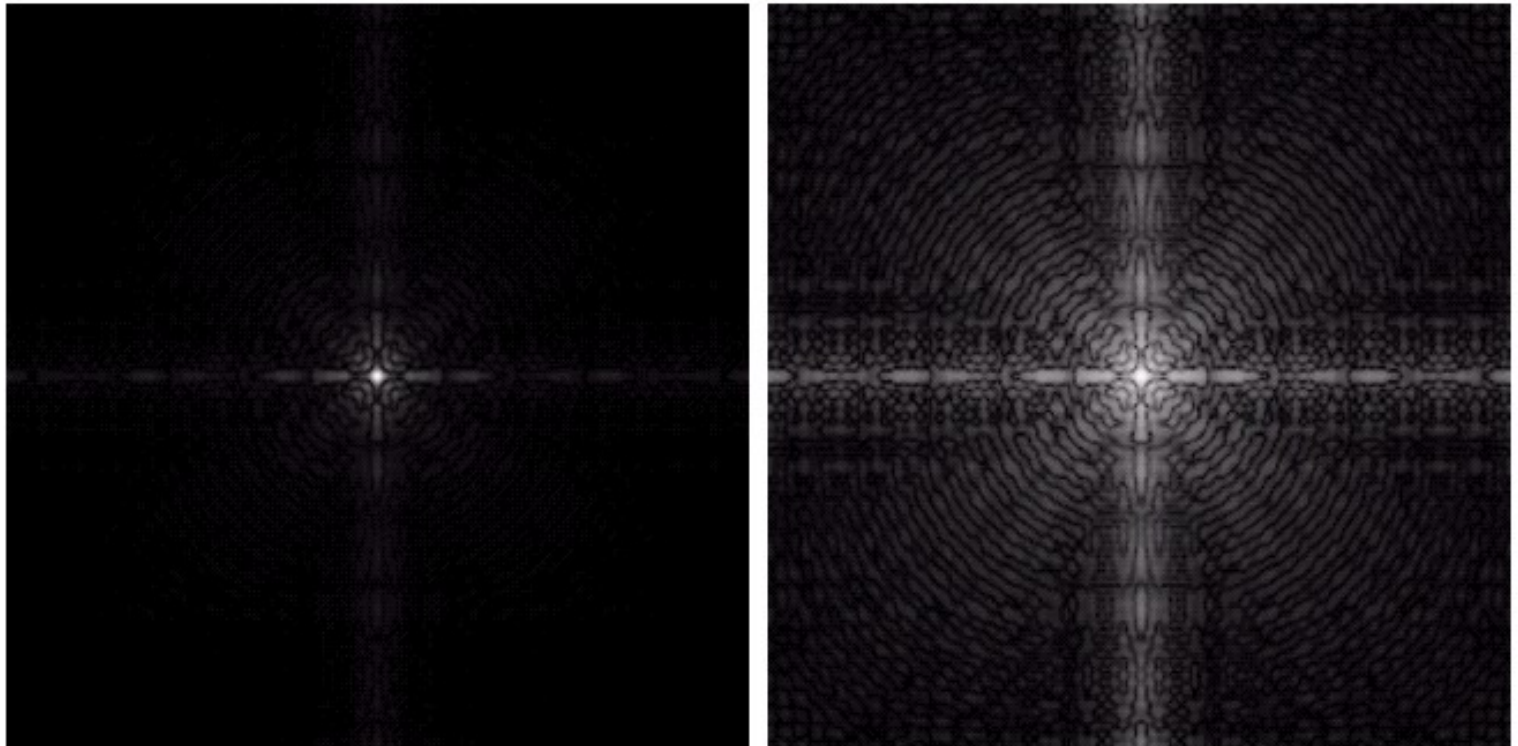
$$g(x, y) = C \log(1 + f(x, y))$$

a b

## FIGURE 3.5

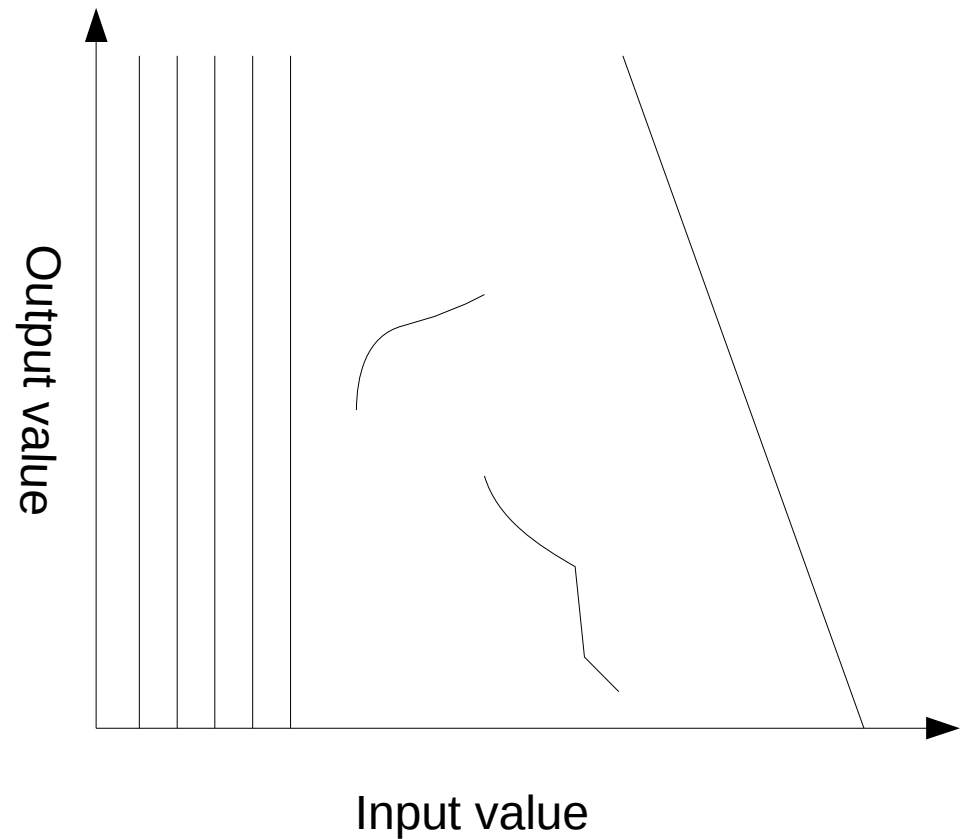
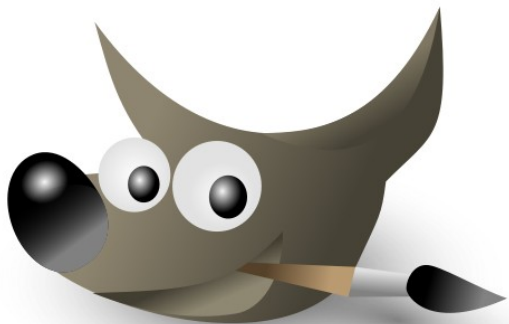
(a) Fourier spectrum.  
(b) Result of applying the log transformation given in Eq. (3.2-2) with  $c = 1$ .

---



# Arbitrary transfer functions

- Only one output per input.
- Possibly non-continuous.
- Usually no inverse



# III. Histograms and histogram equalization

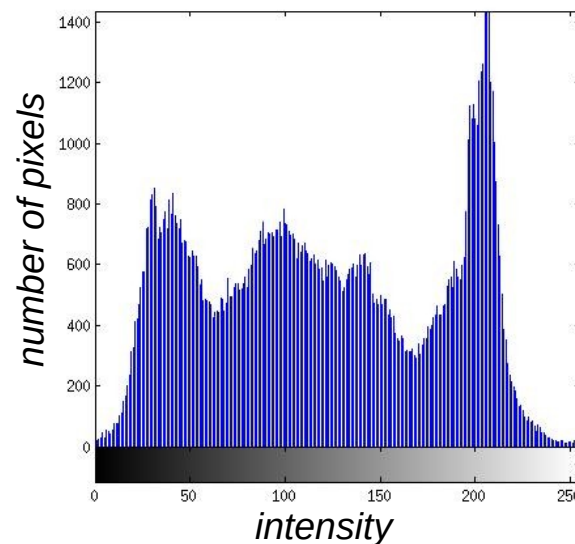
# Image histogram

- A gray scale histogram shows how many pixels there are at each intensity level.

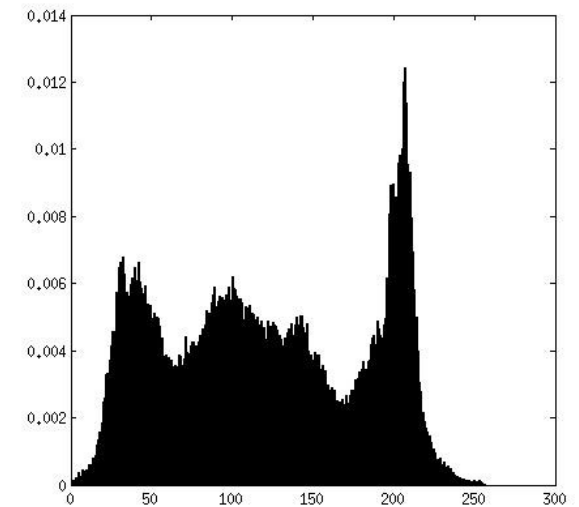


- width = 340 px
- height = 370 px
- bit-depth = 8 bits → 0..255

Histogram

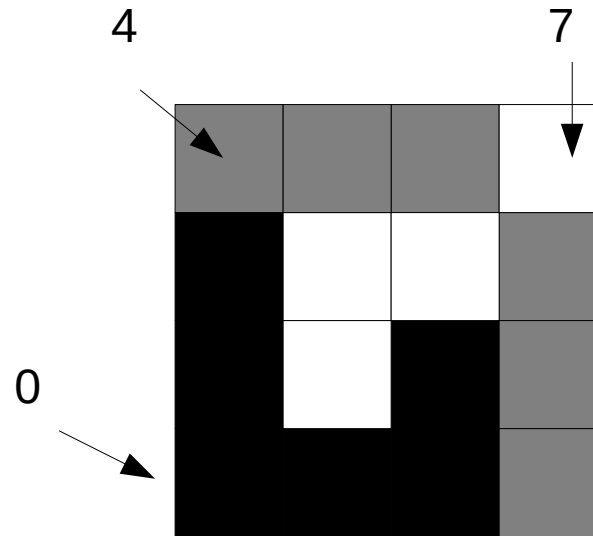
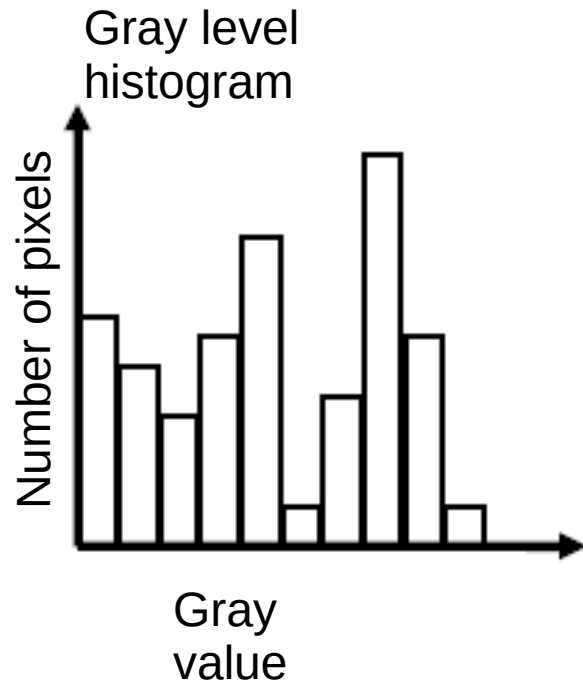


Normalized histogram



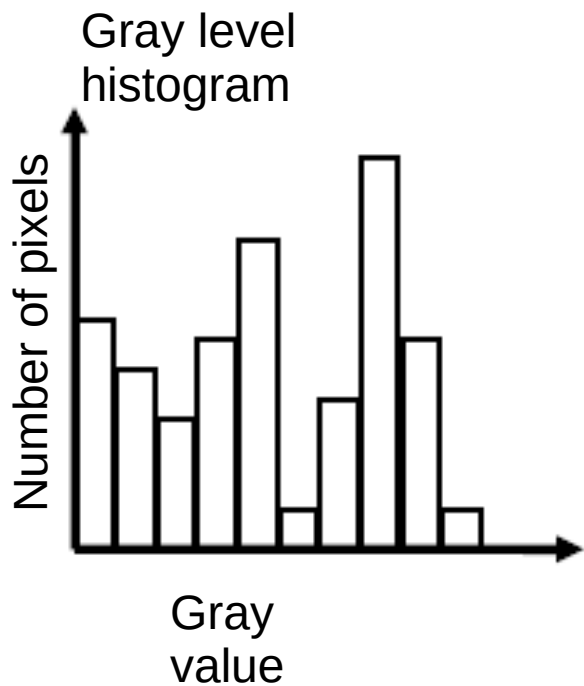


# Exercise

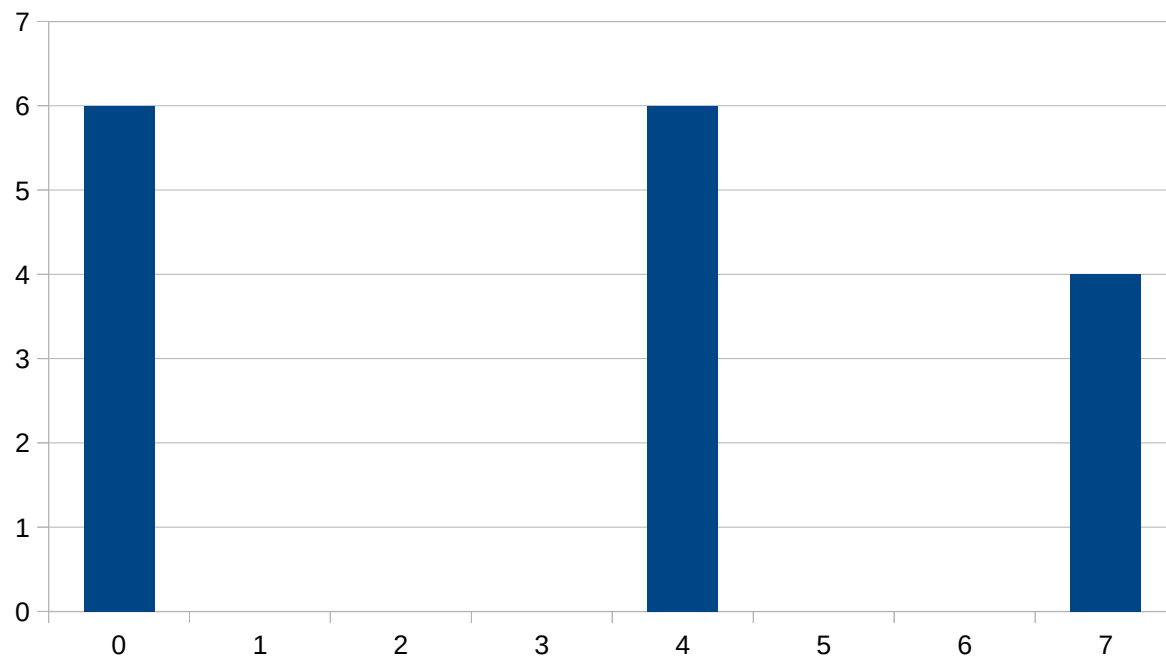
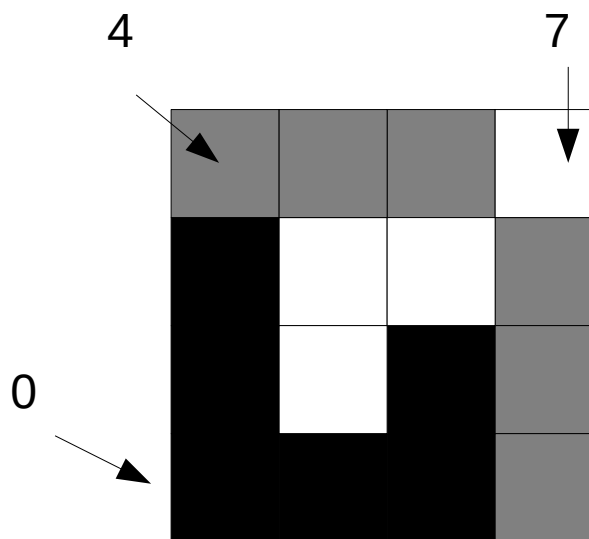


- width = 4px
- height = 4px
- bit-depth = 3 bits

# Exercise

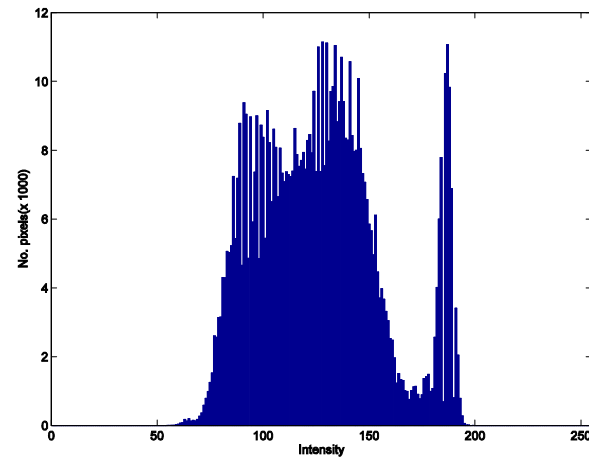
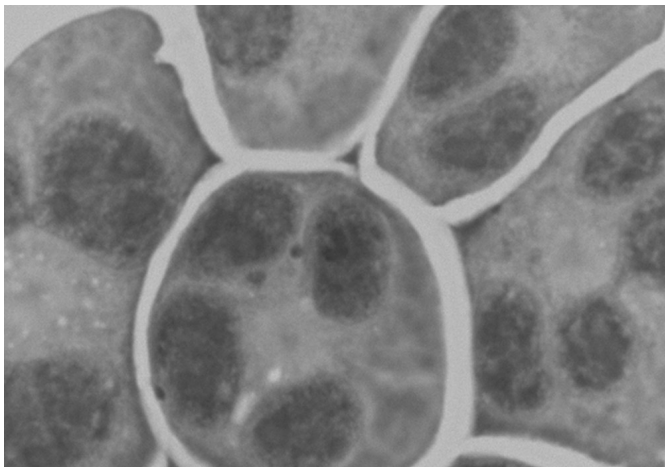
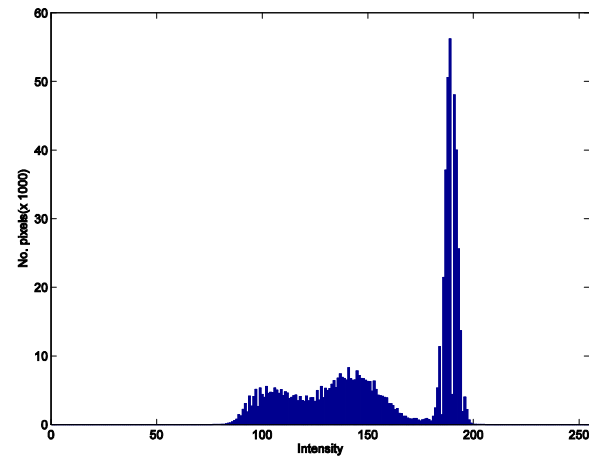
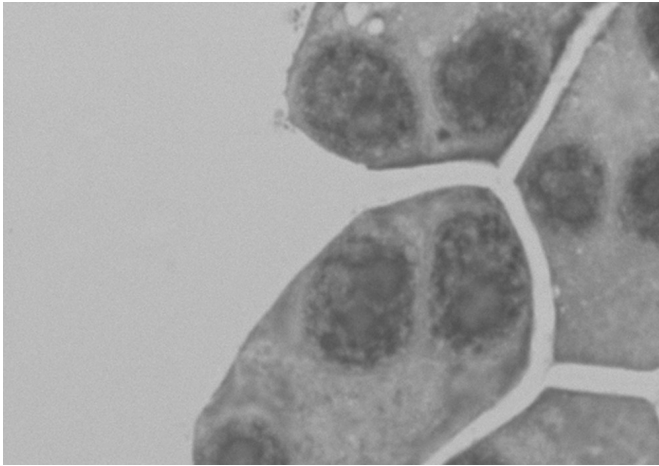


- width = 4px
- height = 4px
- bit-depth = 3 bits



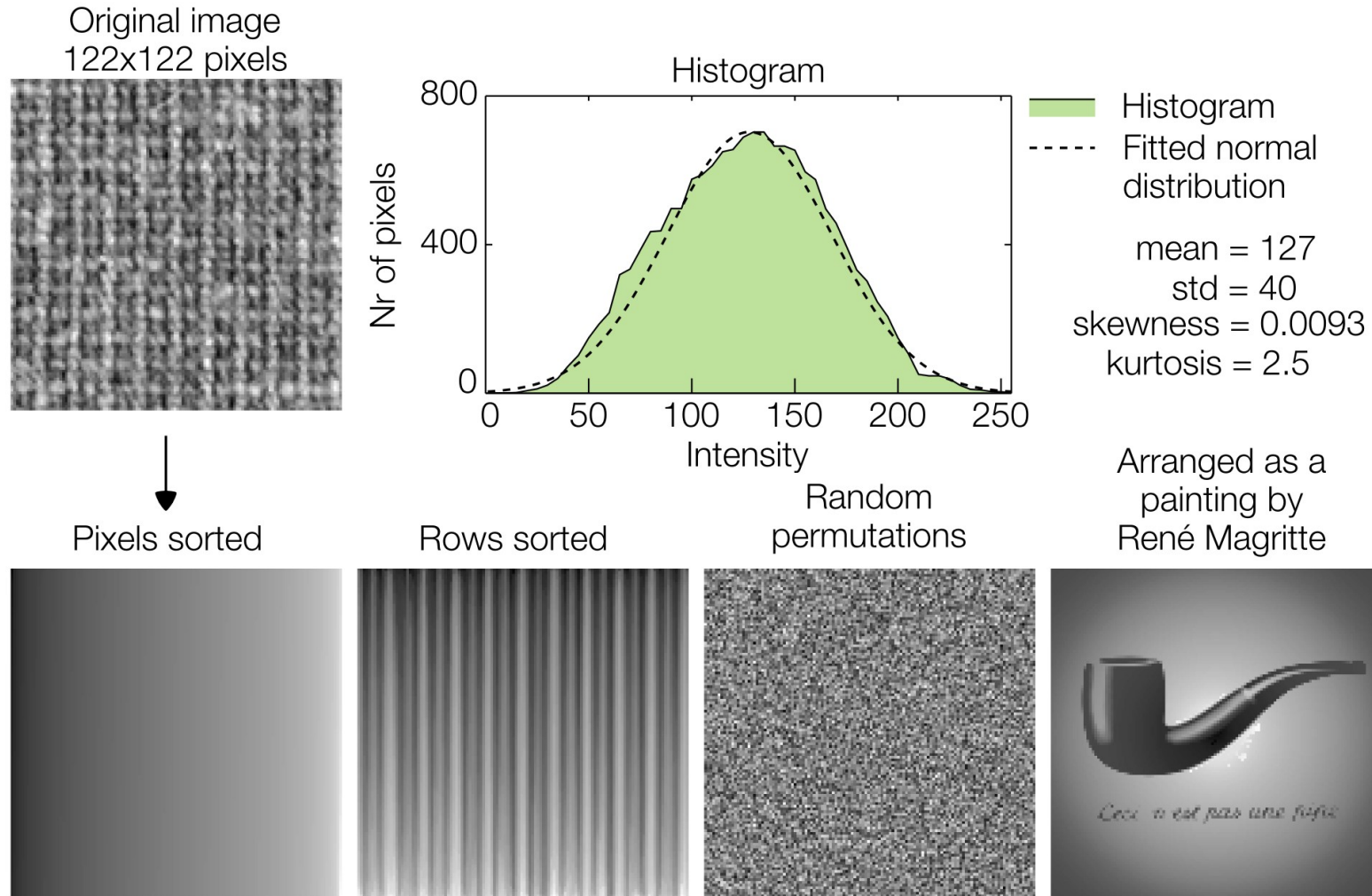
# Image histogram

- Gray-level histogram shows intensity distribution



# Beware

- Intensity histogram says nothing about the spatial distribution of the pixel intensities





A



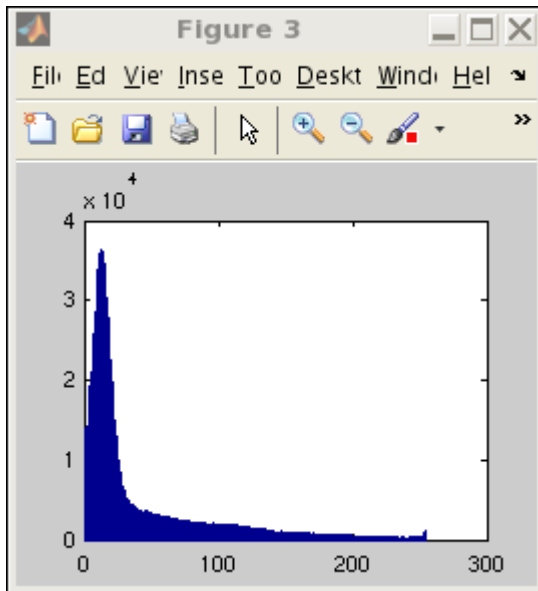
B



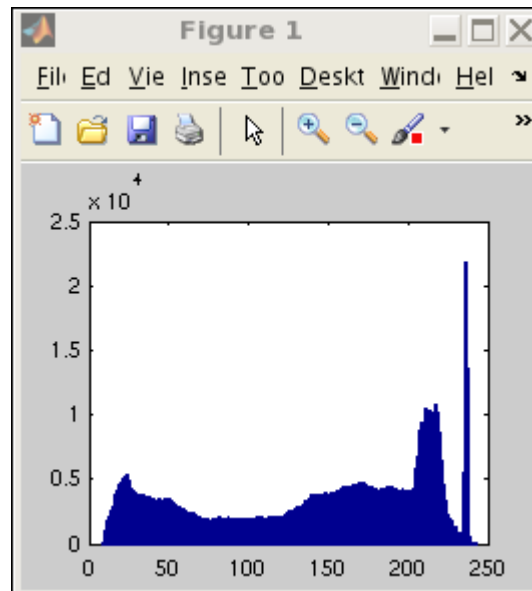
C

Pair images and histograms!

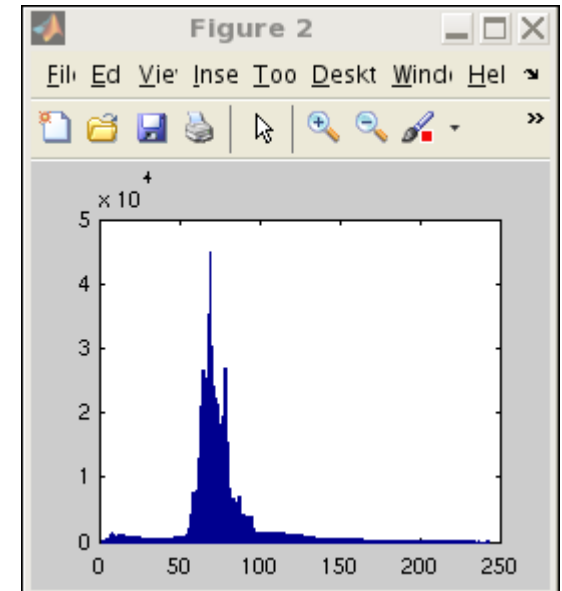
E



F



G





A

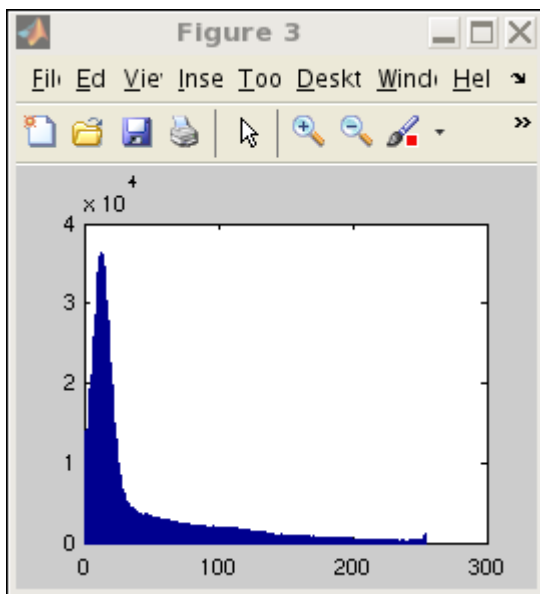


B

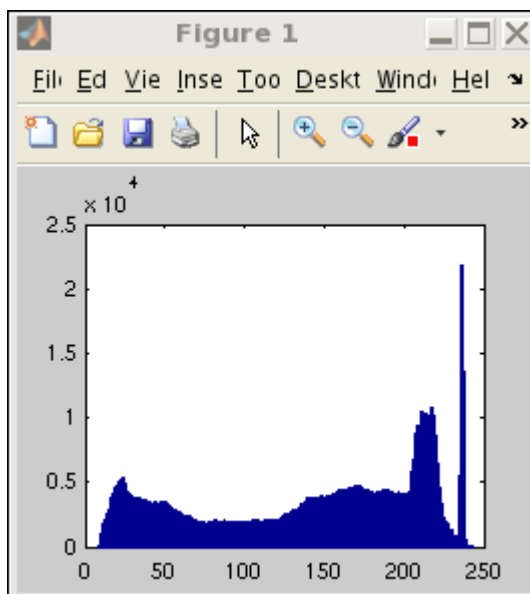


C

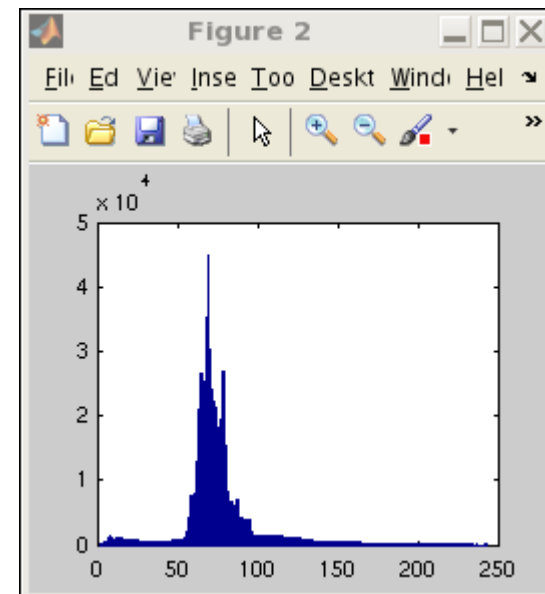
E



F

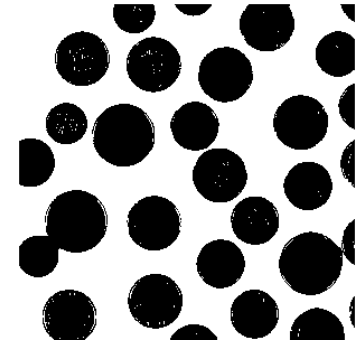
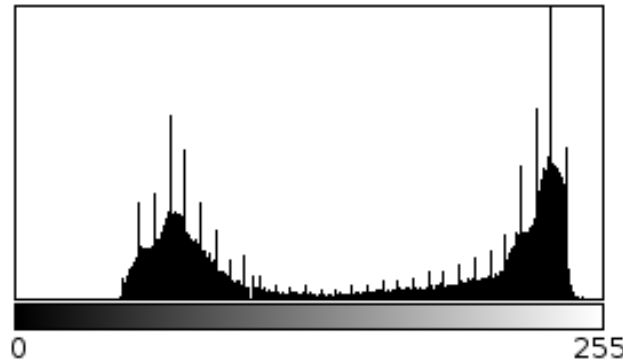
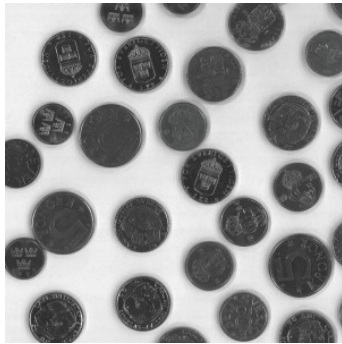


G

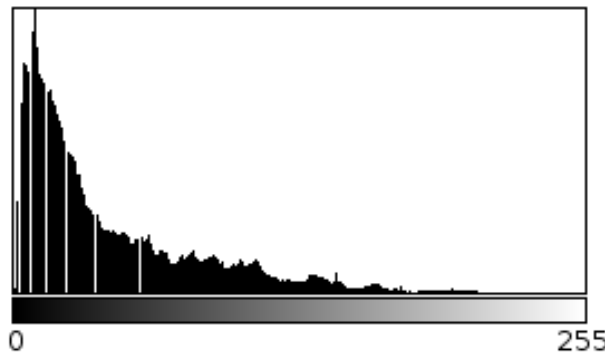
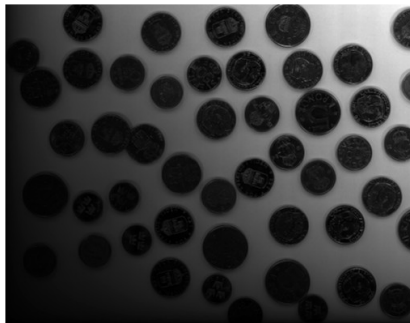


# Use of histogram

- Thresholding → decide the best threshold value
- *works well with bi-modal histograms*

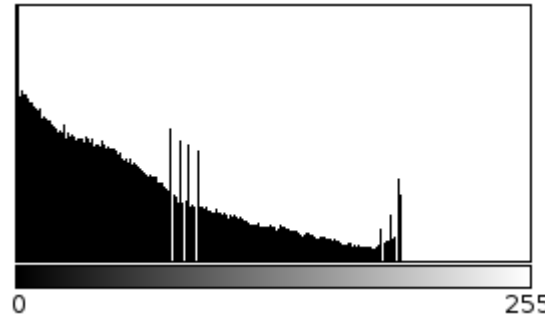
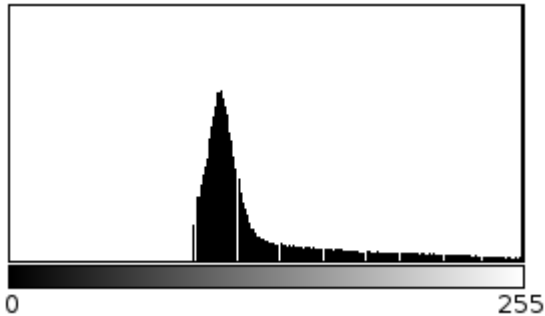


- *does not work with uni-modal histograms*

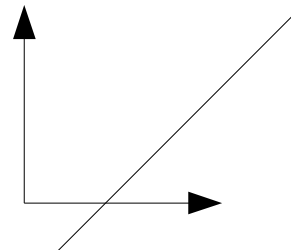
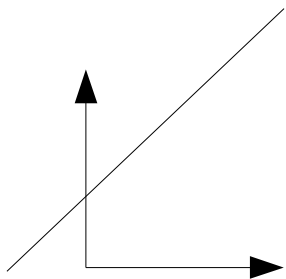


- Analyze the brightness and contrast of an image
- Histogram equalization

# Analyze the brightness



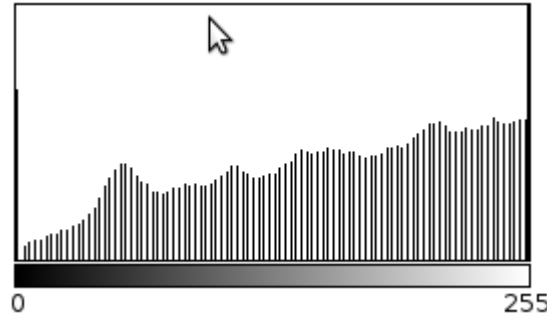
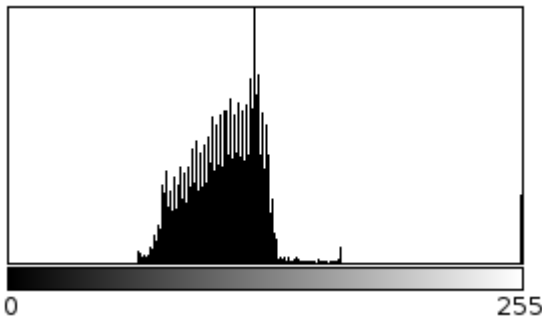
- Increase brightness - shift histogram to the right
- Decrease brightness - shift histogram to the left



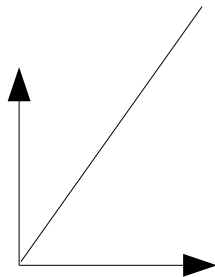
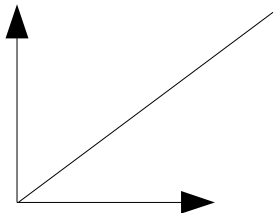
Greylevel transform:  
up → increased brightness  
down → decreased brightness



# Analyze the contrast



- Decreased contrast - compressed histogram.
- When contrast is increased - the histogram is stretched.



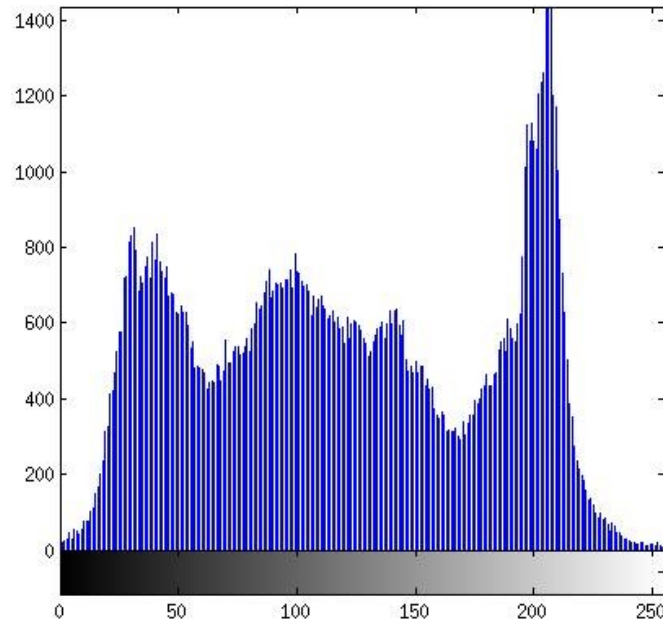
- Greylevel transform:
  - $<45^\circ \rightarrow$  decreased contrast
  - $>45^\circ \rightarrow$  increased contrast

# Cumulative histogram

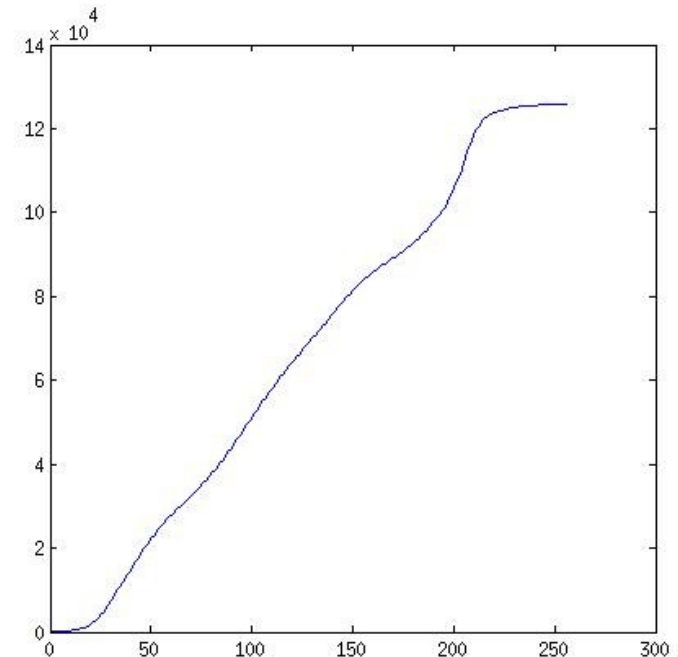
- Easily constructed from the histogram



$$c_j = \sum_{i=0}^j h_i$$



Histogram

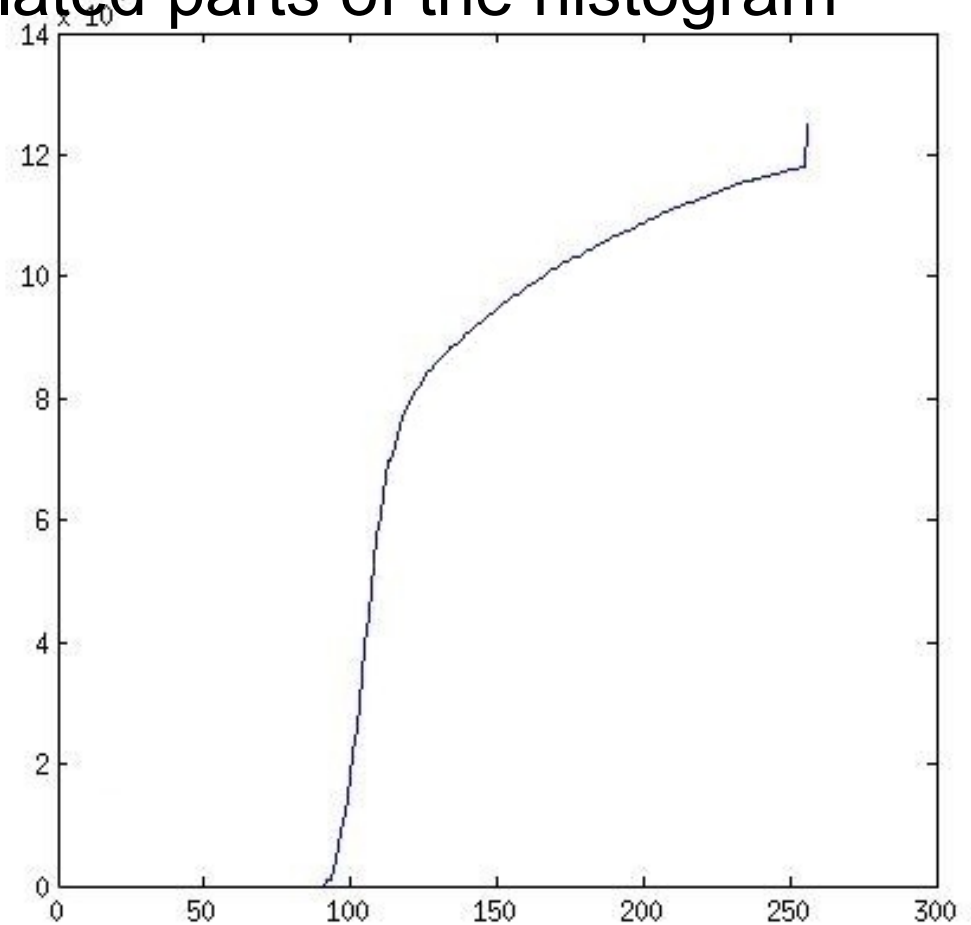
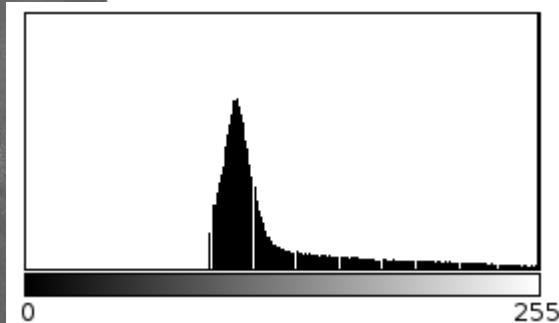
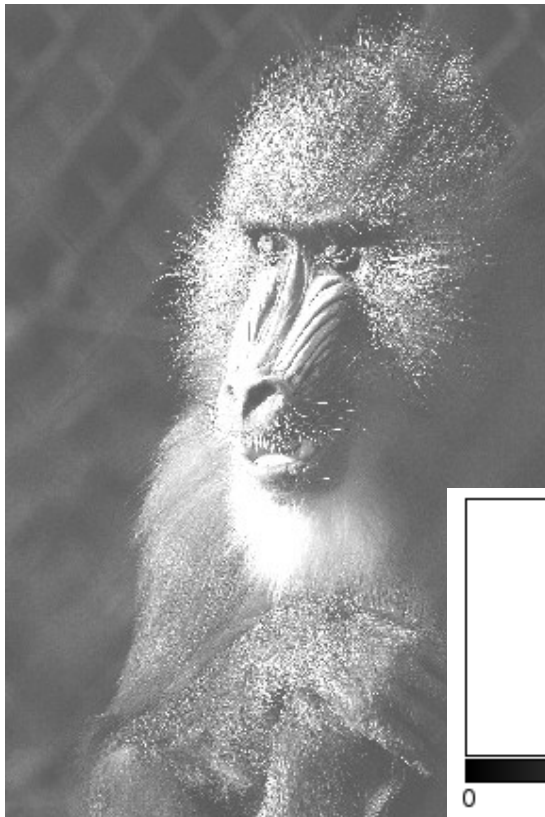


Cumulative histogram

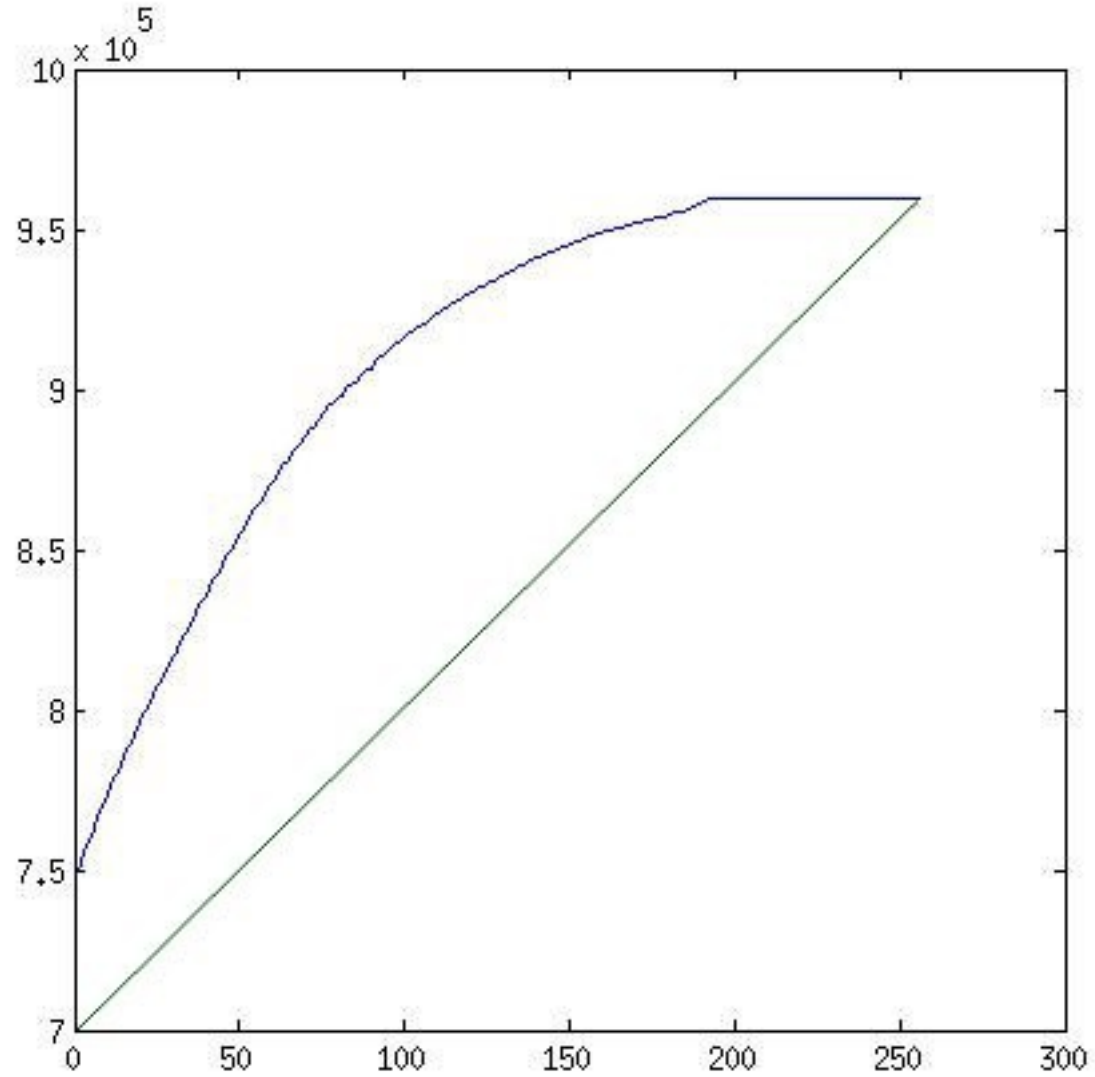
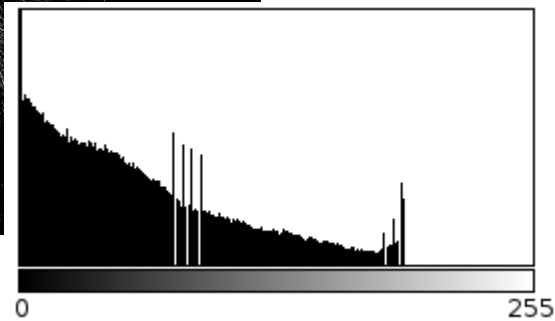
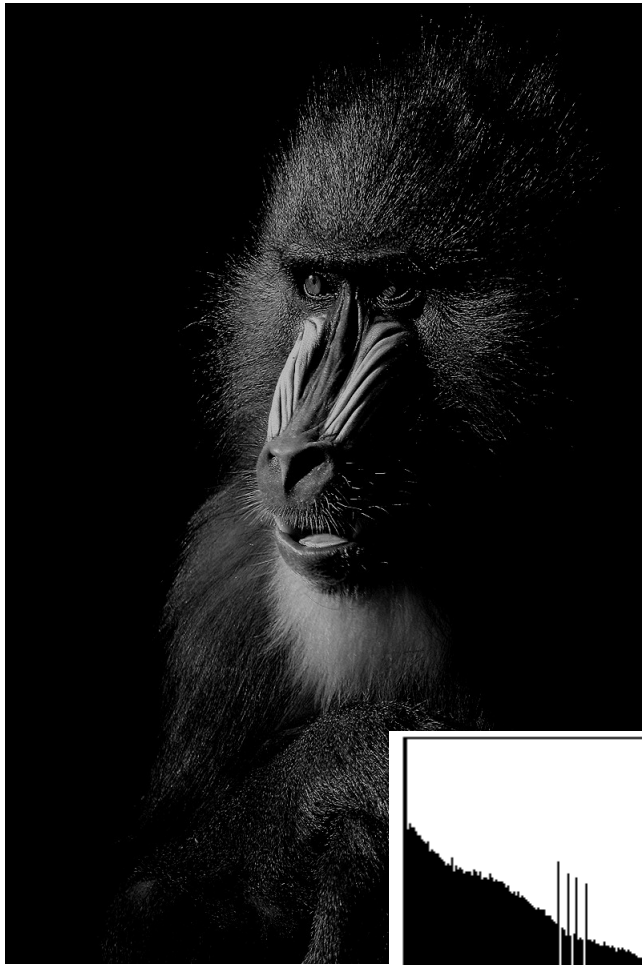
# Cumulative histogram

- Slope

- Steep  $\rightarrow$  intensely populated parts of the histogram
- Gradual  $\rightarrow$  in sparsely populated parts of the histogram



# Cumulative histogram

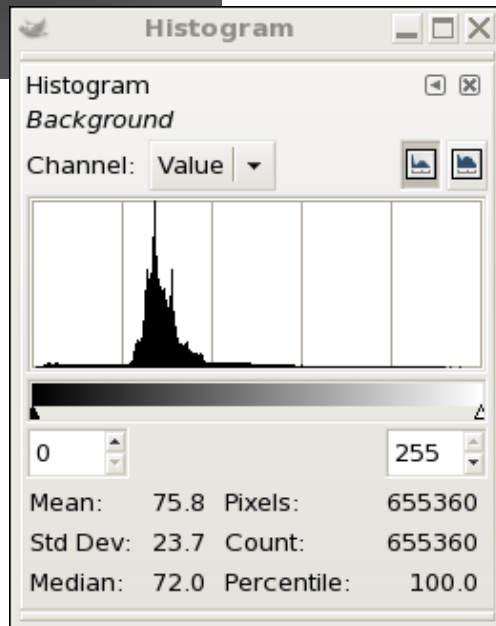
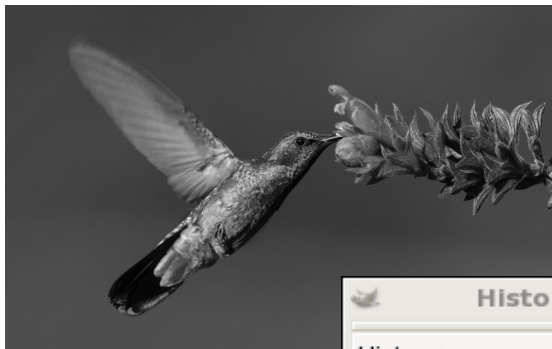


# Histogram equalization

- Idea: Create an image with evenly distributed greylevels, for visual contrast enhancement
- Goal: Find the transformation that produces the most even histogram → cumulative histogram curve
- Equalization flattens the histogram, linearizes cumulative histogram
- Automatic contrast enhancement

# Histogram equalization

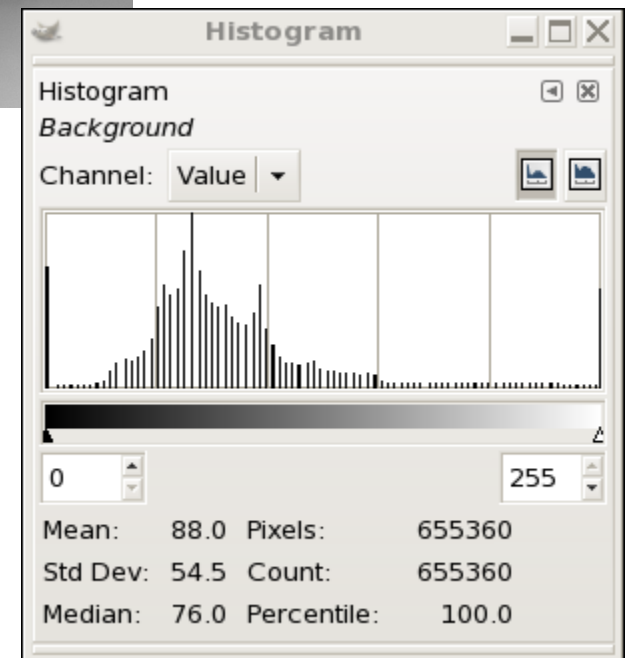
original image



result of histogram equalization



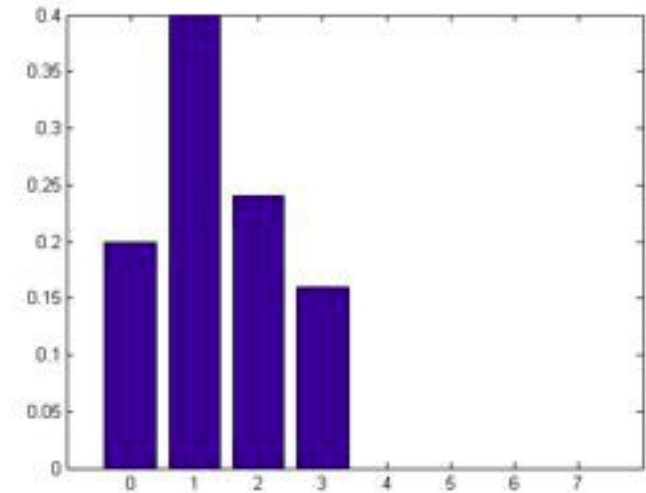
The contrast transform



# Hist eq: small example

- Intensity                    0   1   2   3   4   5   6   7
- Number of pixels 10 20 12 8 0 0 0 0

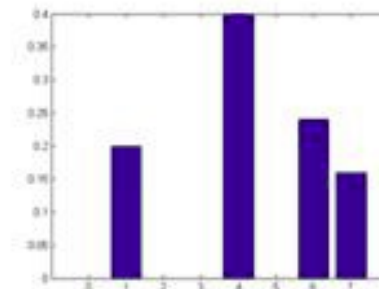
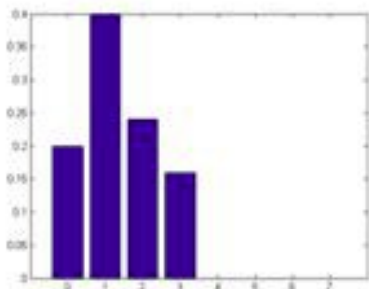
- $p(0) = 10/50 = 0.2$ ,  $\text{cdf}(0)=0.2$
- $p(1) = 20/50 = 0.4$ ,  $\text{cdf}(1)=0.6$
- $p(2) = 12/50 = 0.24$ ,  $\text{cdf}(2)=0.84$
- $p(3) = 8/50 = 0.16$ ,  $\text{cdf}(3)=1$
- $p(r) = 0/50 = 0$ ,  $r = 4, 5, 6, 7$   $\text{cdf}(r)=1$



# Hist eq: small example

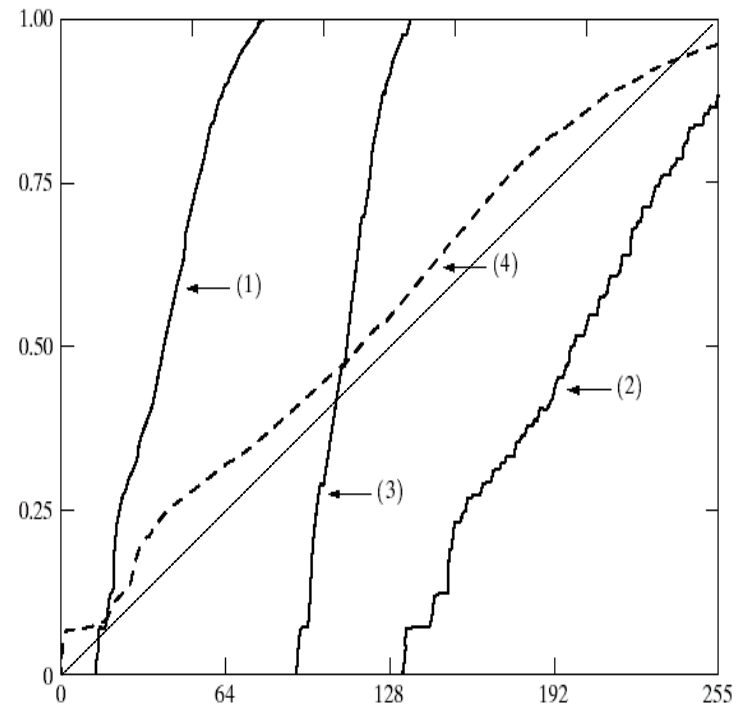
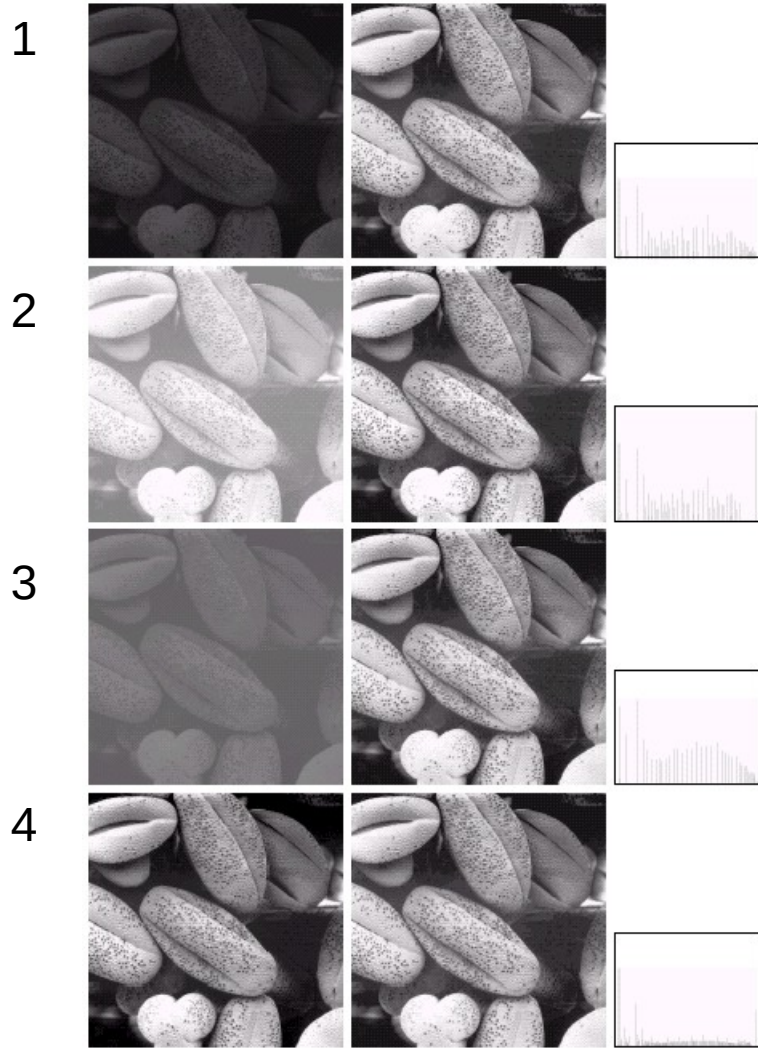
- $T(0) = 7 * (p(0)) = 7 * 0.2 = 1.4 \approx 1$
- $T(1) = 7 * (p(0) + p(1)) = 7 * 0.6 = 3.6 \approx 4$
- $T(2) = 7 * (p(0) + p(1) + p(2)) \approx 6$
- $T(3) = 7 * (p(0) + p(1) + p(2) + p(3)) \approx 7$
- $T(r) = 7, r = 4, 5, 6, 7$

Intensity	0	1	2	3	4	5	6	7
Number of pixels	0	10	0	0	20	0	12	8



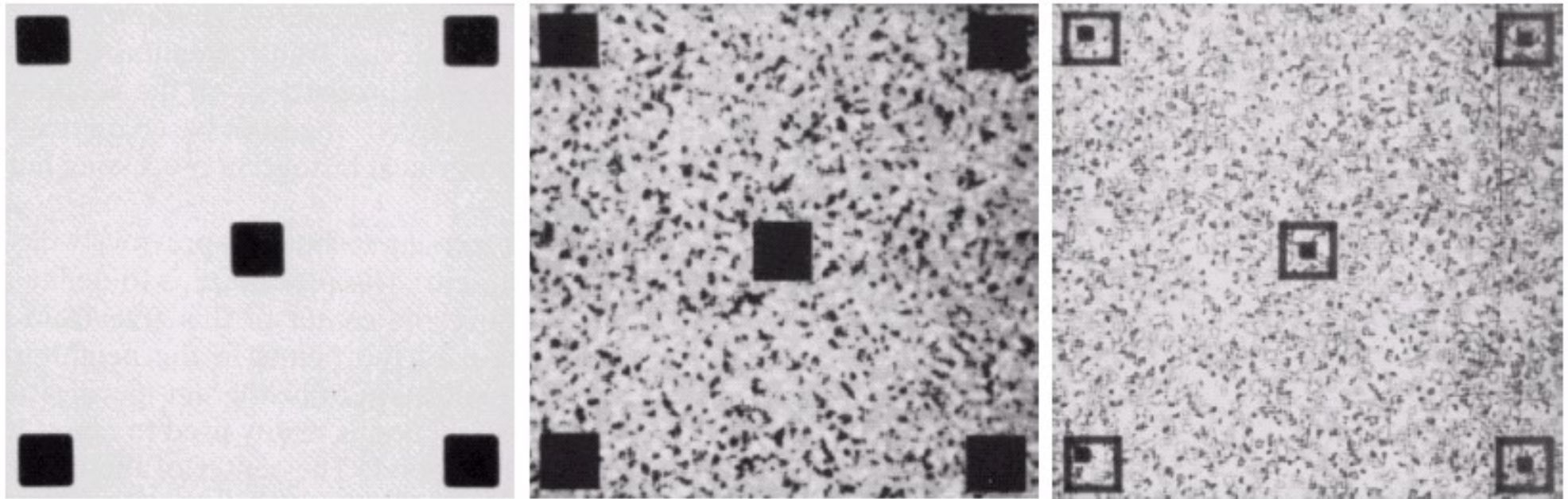


# More examples of hist eq



Transformations for image 1-4.  
Note that the transform for figure 4  
(dashed) is close to the neutral  
transform (thin line).

# Local histogram equalization



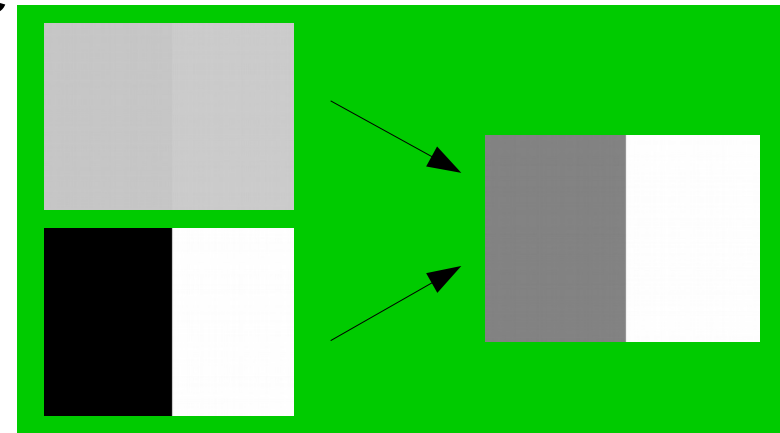
a b c

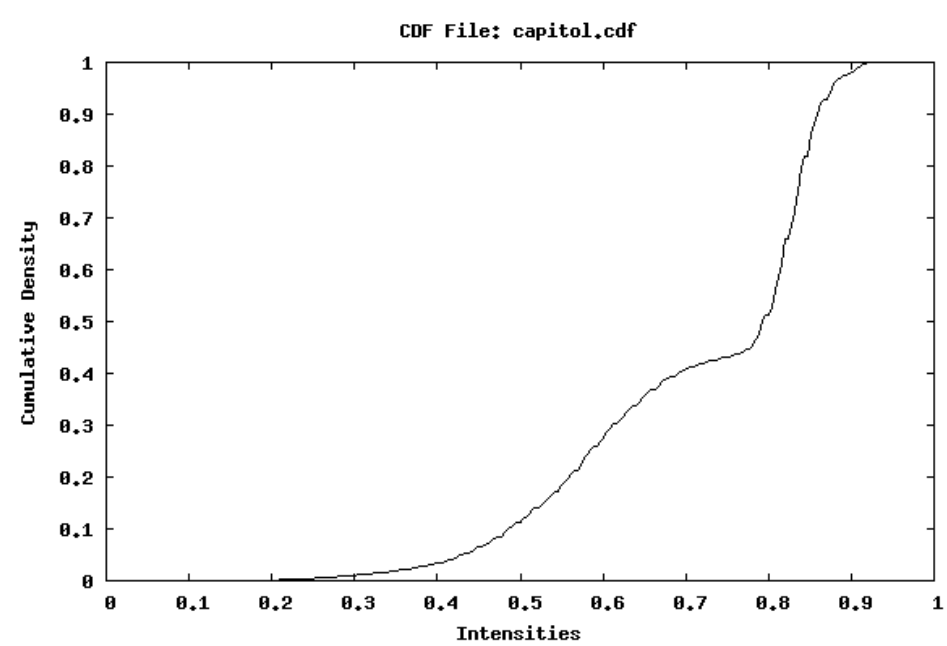
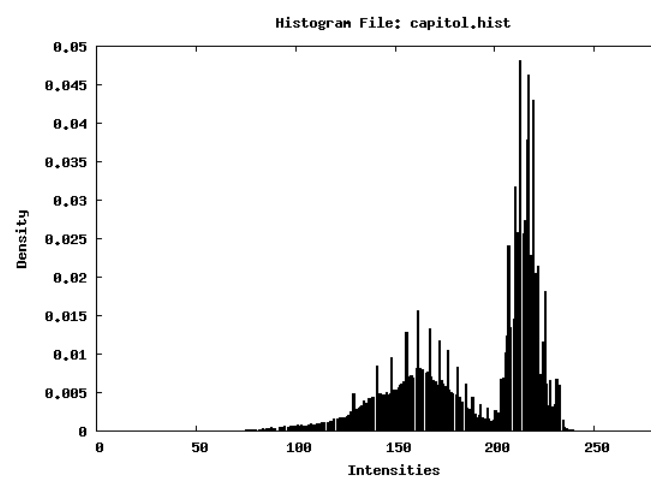
**FIGURE 3.23** (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization using a  $7 \times 7$  neighborhood about each pixel.

---

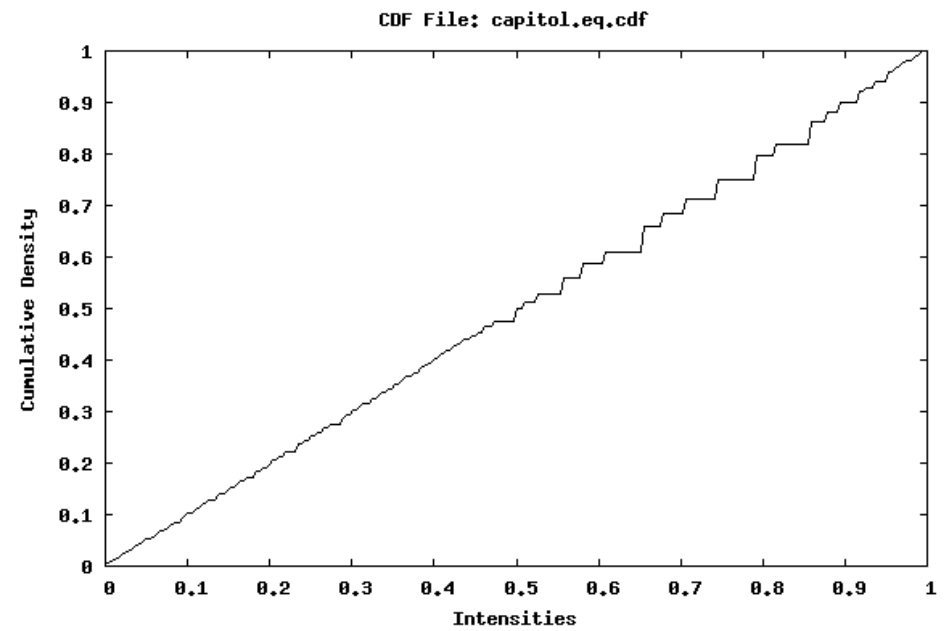
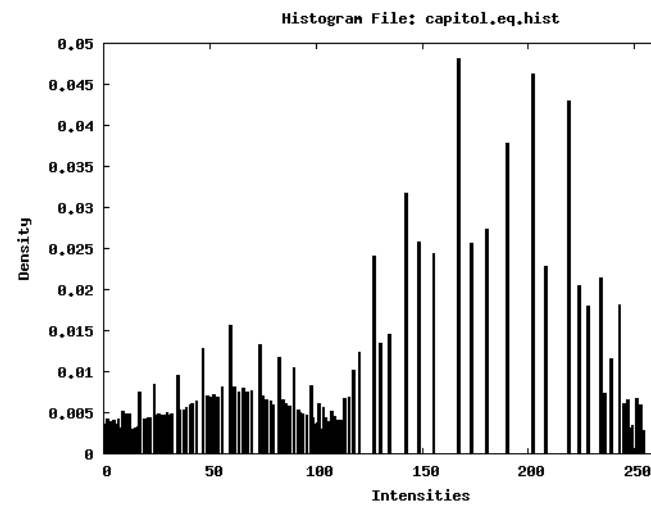
# Histogram equalization

- Useful when much information is in a narrow part of the histogram
- Drawbacks:
  - Amplifies noise in large homogenous areas
  - Can produce unrealistic transformations
  - Information might be lost, no new information is gained
  - Not invertible, usually destructive

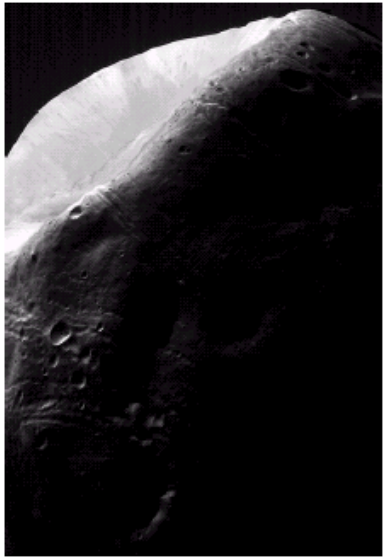




- Does not work well in all cases!



- Histogram equalization is not always “optimal” for visual quality



original image

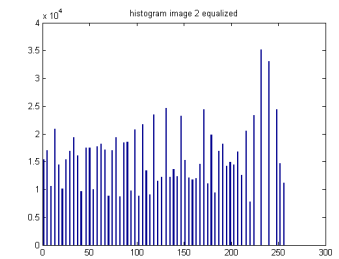
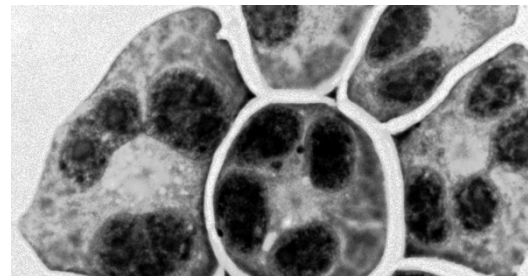
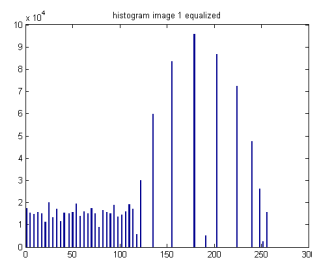
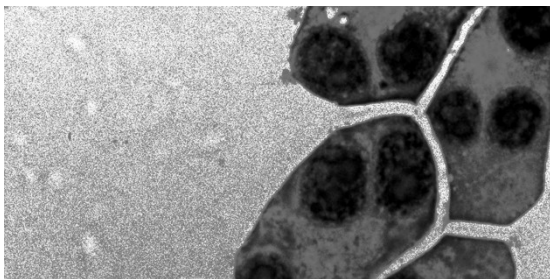
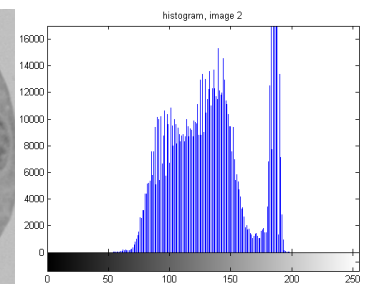
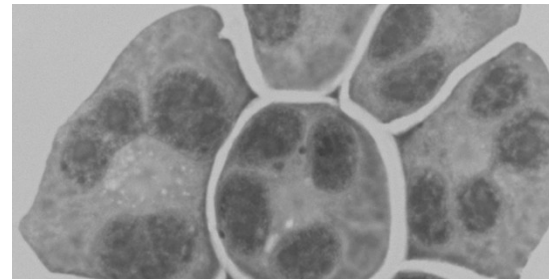
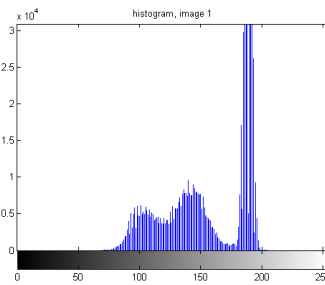
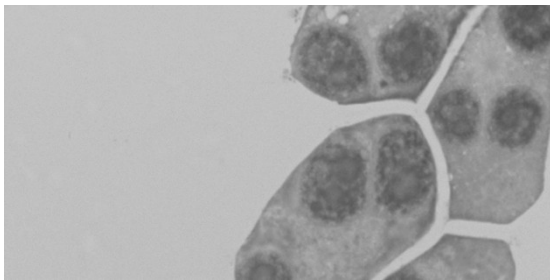


image after histogram  
equalization



image after  
manual choice of  
transform

- Histogram eq: the result depends on the amount of different intensities

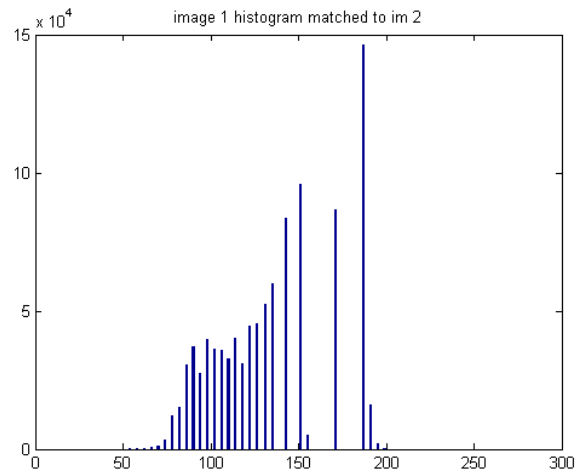
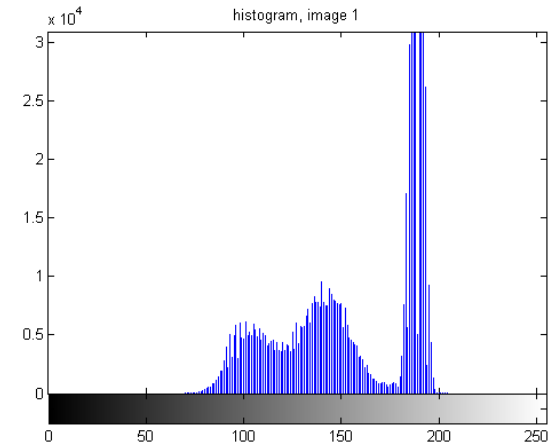
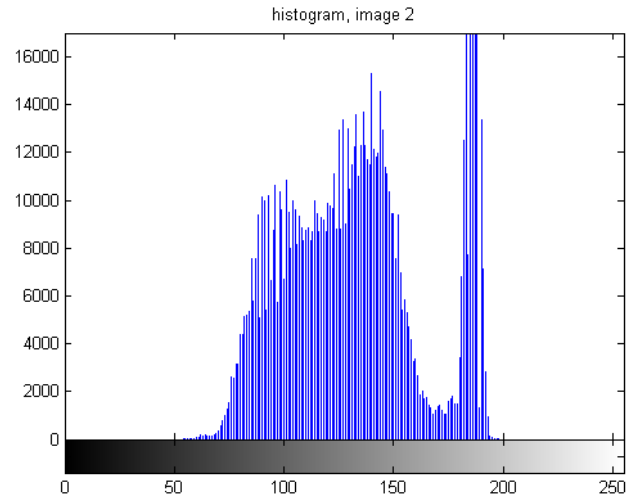
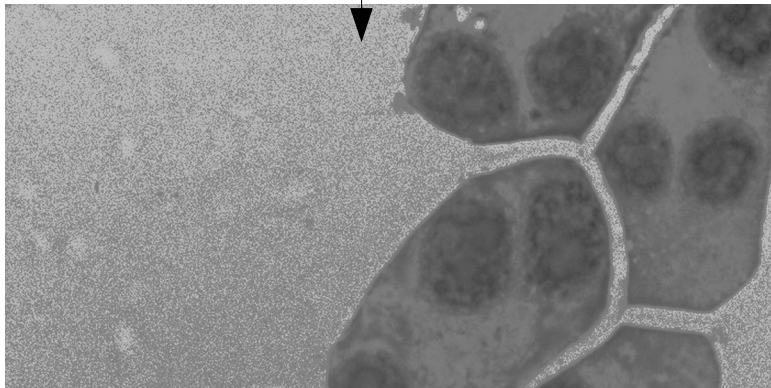
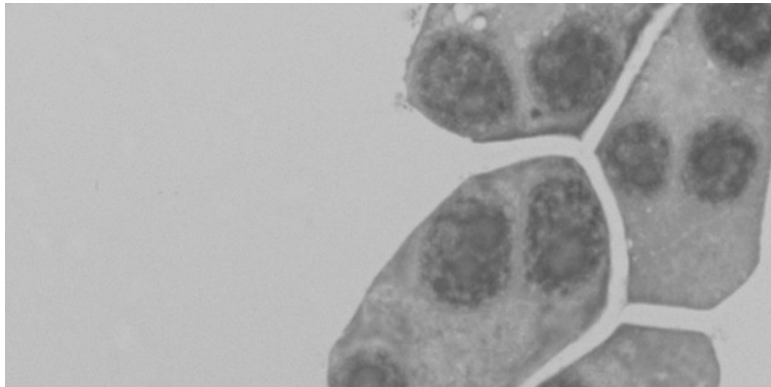
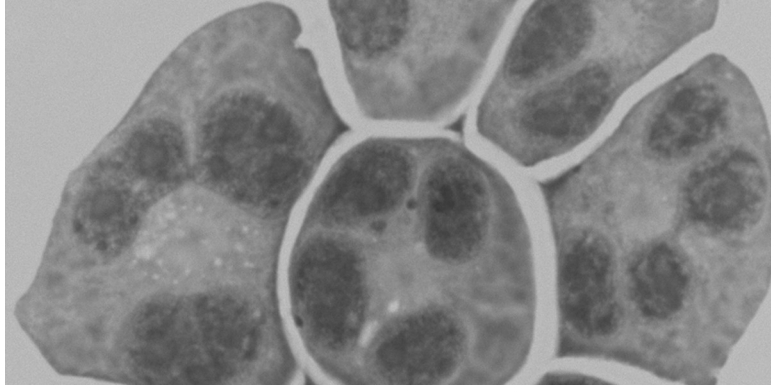


# Histogram matching

- In histogram equalization a **flat distribution** is the goal
- In histogram matching the **distribution of another image** is the goal

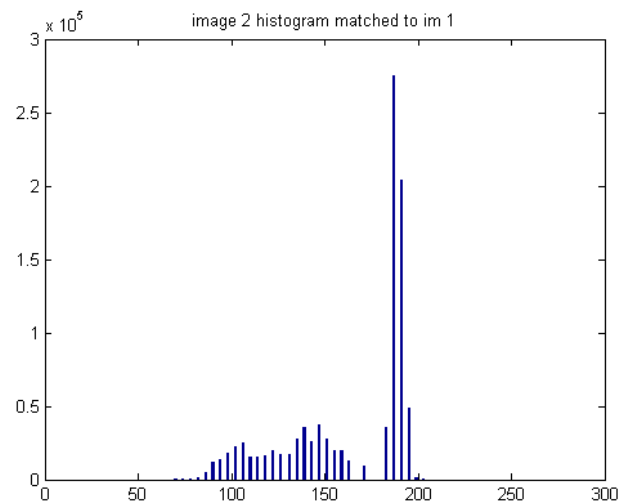
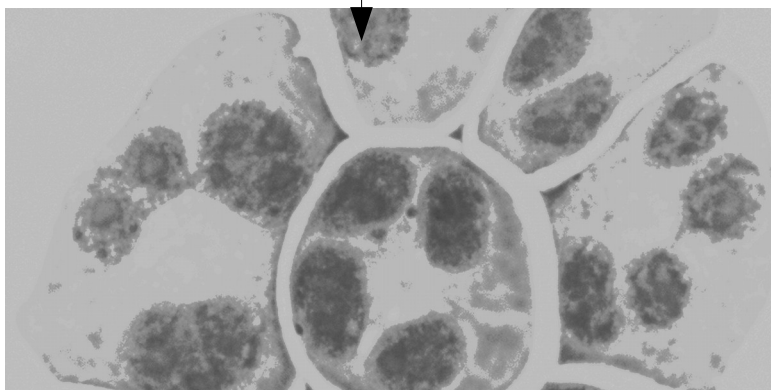
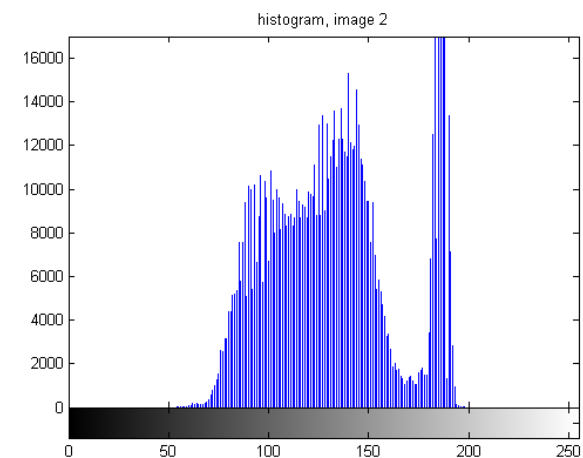
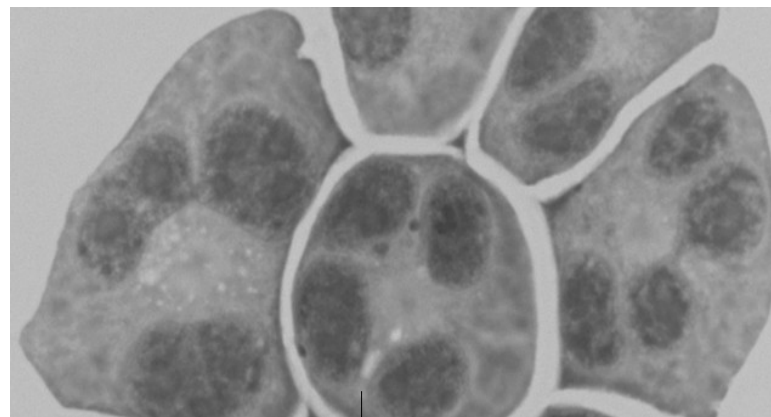
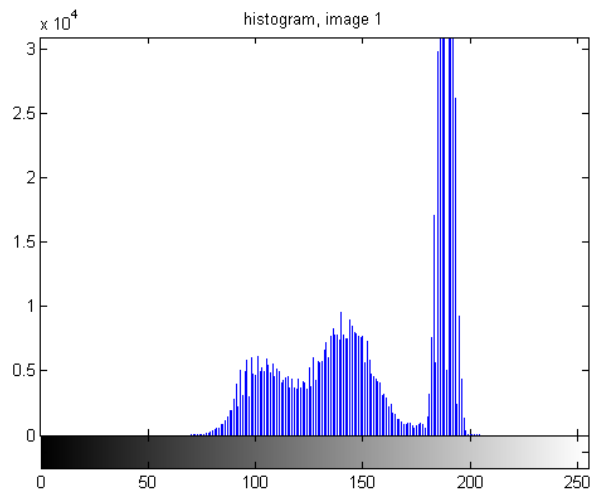
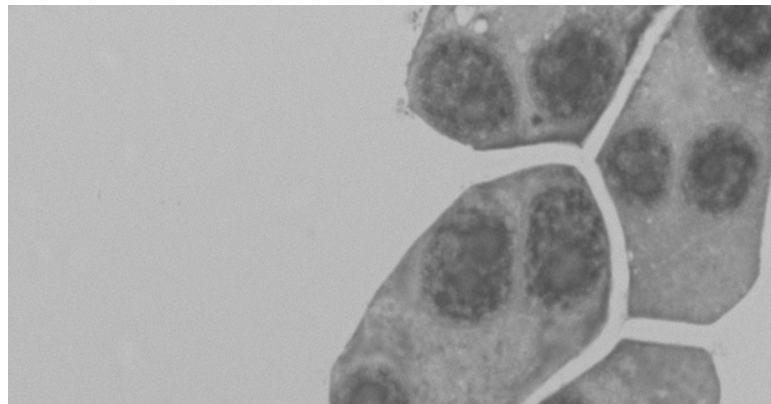
*For an image,  $I$ , find the transformation,  $T$ , that gives the histogram some ideal shape,  $s$ .*

# Image 1 histogram matched to image 2



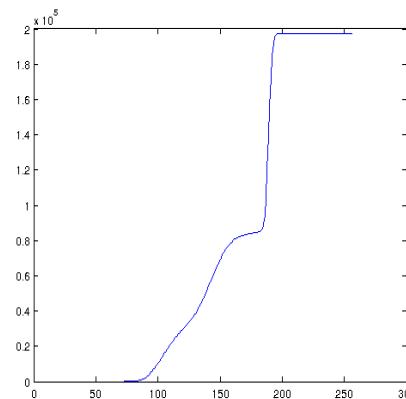
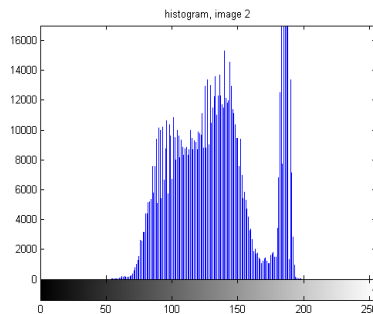
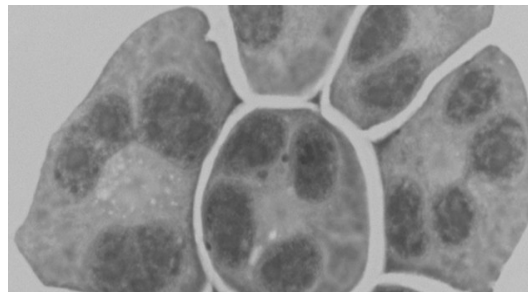
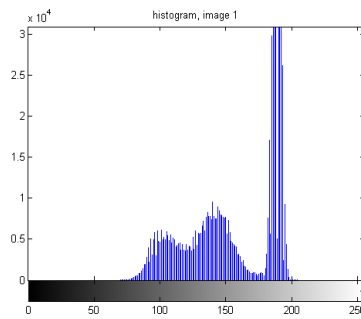
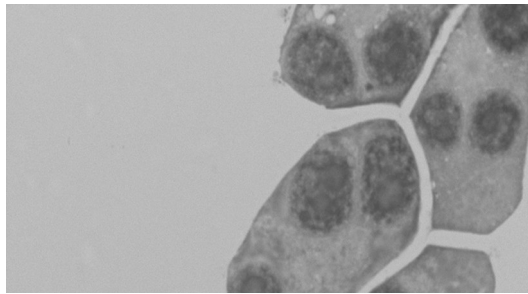


# Image 2 histogram matched to image 1

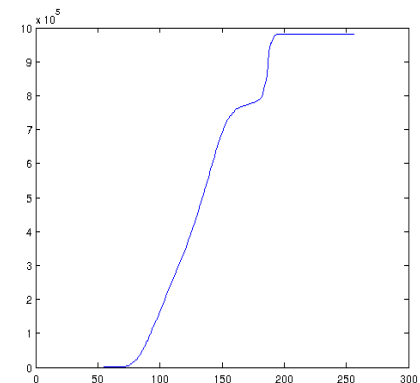


# Histogram matching

- Compute the histograms for image  $I_1$ ,  $I_2$
- Calculate the cumulative distribution function  $F_1()$ ,  $F_2()$
- For each gray level  $G_1$   $[0,255]$  find gray level  $G_2$  for which  $F_1(G_1)=F_2(G_2)$
- Histogram matching function  $M(G_1) = G_2$



img1



img2

# Summary

- Many common tasks can be described by image arithmetics.
- Histogram equalizations can be useful for visualization.
- Watch out for information leaks!

Try at home!

A few things to think about....

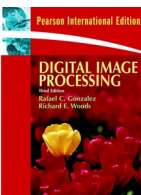
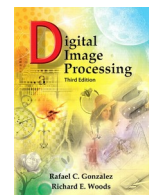
- What is the relation between image arithmetics and linear transfer functions?
- What can you know about an image from the histogram?
- If you have an 8-bit image, A; how will the 8-bit image  $B=255*(A+1)$  look like (exactly!)?
- What conclusions can you draw from the histogram if the first/last column is really high?
- Can you get better resolution by combining multiple images of the same sample?

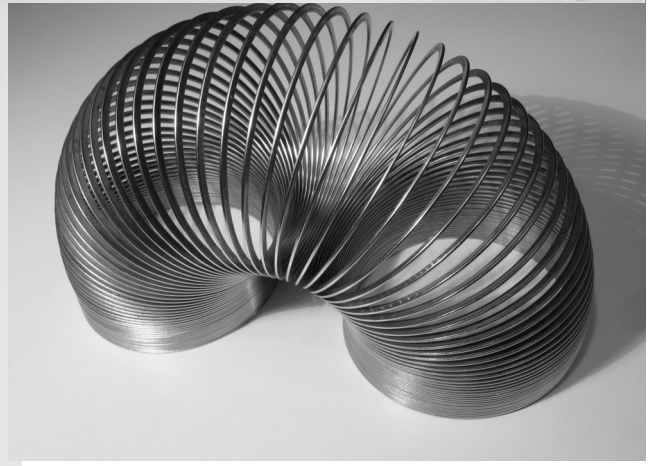
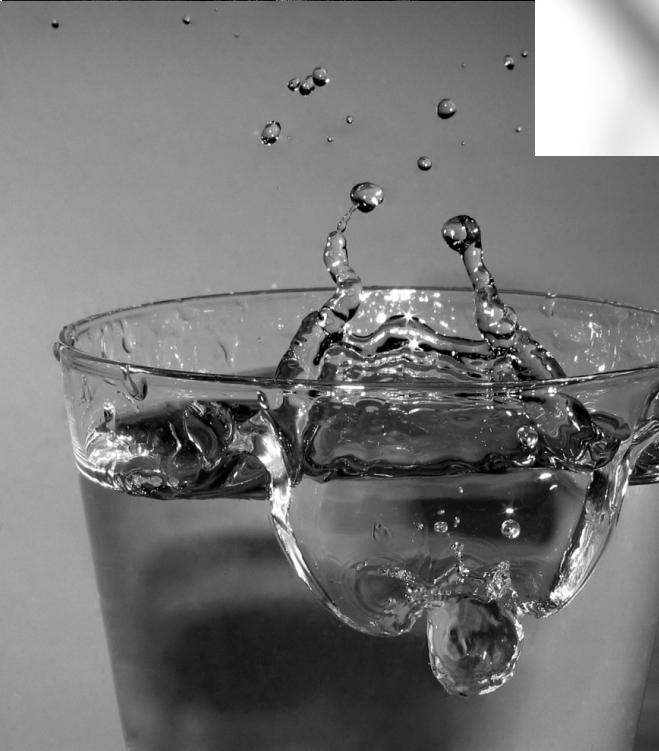
Suggested problems:

2.22, 2.18, 2.9, 3.1, 3.6

Next lecture:

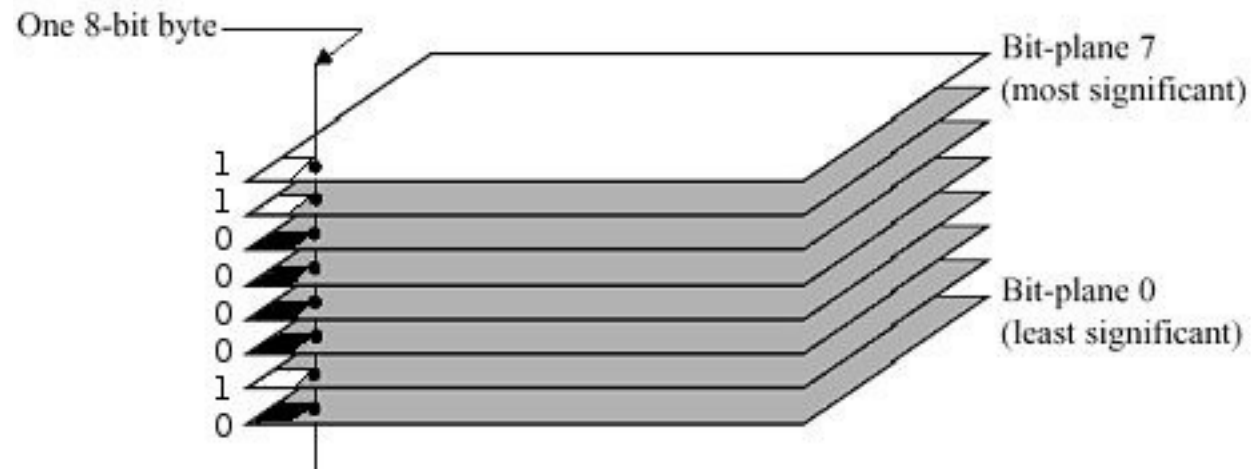
Spatial filtering (Ch. 3.4-3.8)





# Bit plane slicing

- Pixels → digital numbers composed of bits
- Computer → Binary number system
- Basic unit, bit



194 = 11000010

- 8- and 16- bits are common for file formats.

But how many bits are  
necessary?

Next slides:

- The eight bit planes for an image.
- The same image using 7,...,0 bit planes.



