Course PM
1TD253: Finite element methods, 5.0 hp

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Polonius: *[Aside]* –Though this be madness, yet there is method in't.
William Shakespeare: *Hamlet*, Act 2, scene 2.

What is NOT in this document

- Exact dates and times
- Chapters to read, suitable exercises
- Pointers to the literature
- Actual laborations and assignments

For these matters, please refer to the continuously updated course web-page at

`http://www.it.uu.se/edu/course/homepage/fem/ht10/`

Course contents

Key concepts: elliptic boundary value problems, hyperbolic and parabolic time dependent problems, variational formulation, natural and essential boundary conditions, discrete function spaces in one and two spatial dimensions, piecewise polynomial approximation: interpolation and projection, quadratures, *a priori/posteriori* error bounds.

Finite element methods in one and two spatial dimensions, error bounds for the finite element approximation of elliptic problems, adaptive mesh refinement, FEM for time dependent problems (finite elements in space, finite differences in time).

Software: use and qualitative knowledge of existing FEM-software, such as Comsol Multiphysics.

Learning objectives

This course will teach you how to solve problems formulated as partial differential equations (PDEs) with the finite element method. This method is important thanks to its *generality*: it applies in a wide variety of situations and has many important applications. For example, the FEM is used in computational methods in Biophysics, Chemical engineering, Electromagnetics, Fluid mechanics, General relativity, Geophysics, Structural mechanics...

To pass the course you should be able to

- P1 derive the variational formulation for an elliptic PDE in 1 and 2 dimensions with Dirichlet-, Neumann-, or, Robin boundary conditions,
- P2 discretize the variational formulation with suitable basis functions and hence formulate a finite element method,
- P3 implement the finite element method on a computer,
- P4 predict the convergence behavior of finite element methods and -codes,
- P5 for *time dependent parabolic and hyperbolic* PDEs, see P1–P3 above,
- P6 discuss how finite element software works and use such software to solve more complicated problems.

For higher grades you are also required to know how to

- H1 derive *a priori* and *a posteriori* error bounds for elliptic equations in one and two spatial dimensions,
- H2 construct adaptive algorithms for local mesh refinement using these error estimates,
- H3 for time dependent problems, evaluate different time discretization strategies with respect to computational efficiency and stability.

Teaching

The course consists of 12 lectures, 6 exercise classes and 3 computer labs. *Prepare each lecture and each class by reading through the material beforehand and by working through some of the exercises. This is much more efficient and saves a lot of time!*

Lectures The guest lectures L7 and L11 are obligatory and are strongly recommended! By listening to these lectures you partially demonstrate P6.

	Goals targeted		Goals targeted
L1	P2	**L2**	P2, P3
L3	P1, P2, H1	**L4**	P3, H1, H2
2D:			
L5	P2, P3	**L6**	P3
L7(*)	P1, **P6**	**L8**	P3, H1, H2
Time-dependent:			
L9	P5, H1, H3	**L10**	P5, H1, H3
L11(*)	H3, **P6**	**L12**	Repetition

Exercises Prepare yourself by working through the stated exercises. Amongst other things, we will work in small groups and present different solutions and discuss different strategies. *You make the class less effective to yourself and to your fellow students by being unprepared! Instead, be prepared!*

	Material from		Material from
E1	L1, L2	**E2**	L3 (L4)
E3	L5 (L6)	**E4**	L7 (L8)
E5	L9 (L10)	**E6**	L1–L10

There are three *voluntarily* assignments A1–A3. If you submit them within deadline I will correct them and give feedback.

	Details	Goals targeted
A1	A Posteriori error estimates and adaptivity	P4, H1, H2 (P6)
A2	The wave equation	P3–P5, H3
A3	Adaptivity in 2D	P3, P4, H1, H2

Laborations The laborations are mandatory and count as 2.0 hp. Each report should contain a section with theory, a section with implementation details, and a concluding section with results and discussions. The report should be preceded by a short summary and the code should be attached. Since the code is an essential part of the report, it should be as readable as the report.

	Details	Goals targeted
Lab 1	FEM in 1D	P1–P3 (P4, P6)
Lab 2	FEM in 2D	P1–P4 (P6)
Lab 3	Comsol multiphysics	P4, P6

Final exam The final exam consists of 5 exam questions Q1–Q5. Each of these will test a few of the goals P1–P6 and/or H1–H3. Rather than stating a precise minimum number of points to pass, each question will be graded separately with respect to the goals it targets. Hence in order to *pass* the course you must present acceptable solutions (that is, grade 3) on all basic goals P1–P6. For *higher* grades you should demonstrate *good/excellent* solutions for the basic goals, but *also* for H1–H3.

Since this method of grading might be a bit of a novelty, it will be discussed and motivated at the introductory lecture. We will also work through a couple of examples to see how it actually works. Should doubts still persist, never hesitate to bring it up in class or otherwise feel free to contact me through e-mail or visit me in my office.

Goals	Grade(Question No.)	Total grade (Comment)
P1	5(Q1),5(Q2),5(Q3)	5
P2	5(Q1),5(Q2),4(E3)	5
P3	3(Q5)	4 (Lab 1+2)
P4	4(Q3),4(Q4)	5 (Lab 2+3)
P5	3(Q2),3(Q3)	3
P6	4(Q5)	5 (Lab 3, mandatory lectures)
H1	4(Q3),5(Q4)	5
H2	4(Q5)	4
H3	4(Q2),F(Q4),F(Q5)	3
	Total grade:	???

Table 1: The grading matrix for an imaginary exam with 5 exercises. *Example:* Question Q4 tests P4 and H1+3. -What grade should the student have?