Course PM
1TD253: Finite element methods, 5.0 hp

Stefan Engblom

[Exit Diomedes] Cressida: –The error of our eye directs our mind;
What error leads must err; ...
William Shakespeare: *Troilus and Cressida*, Act 5, scene 2.

What is NOT in this document

- Exact dates and times

- Chapters to read, suitable exercises

- Pointers to the literature

- Actual laborations and assignments

For these matters, please refer to the continuously updated course web-page at

http://www.it.uu.se/edu/course/homepage/fem/ht12/

Course contents

Key concepts: elliptic boundary value problems, hyperbolic and parabolic time dependent problems, well-posedness, energy method, natural and essential boundary conditions, A-stability, discrete function spaces in one and two spatial dimensions, piecewise polynomial approximation: interpolation and projection, quadratures, *a priori/posteriori* error bounds.

Finite element methods in one and two spatial dimensions, variational formulation, Galerkin orthogonality, stability/error bounds for the finite element approximation of elliptic problems, adaptive mesh refinement, FEM for time dependent problems (finite elements in space, finite differences in time).

Software: use and qualitative knowledge of existing FEM-software, such as Comsol Multiphysics.

Learning objectives

This course will teach you how to solve problems formulated as partial differential equations (PDEs) with the finite element method. This method is important thanks to its *generality*: it applies in a wide variety of situations and has many important applications. For example, the FEM is used in computational methods in Biophysics, Chemical engineering, Electromagnetics, Fluid mechanics, General relativity, Geophysics, Structural mechanics...

To pass the course you should be able to

P1 derive the variational formulation for an elliptic PDE in 1 and 2 dimensions with Dirichlet-, Neumann-, or, Robin boundary conditions,

P2 discretize the variational formulation with suitable basis functions and hence formulate a finite element method,

P3 implement the finite element method on a computer,

P4 for *time dependent parabolic and hyperbolic* PDEs, see P1–P3 above,

P5 predict the convergence behavior of finite element methods and -codes,

P6 discuss how finite element software works and use such software to solve more complicated problems.

For higher grades you are also required to know how to

H1 derive *a priori* and *a posteriori* error bounds for elliptic equations in one and two spatial dimensions,

H2 construct adaptive algorithms for local mesh refinement using these error estimates,

H3 for time dependent problems, evaluate different time discretization strategies with respect to computational efficiency and stability.

Teaching

The course consists of 12 lectures, 6 exercise classes and 3 computer labs. *Prepare each lecture and each class by reading through the material beforehand and by working through some of the exercises. This is much more efficient and saves a lot of time! You make the class less effective to yourself and to your fellow students by being unprepared! Instead, be prepared!*

Assignments   There are three mandatory assignments A1–A3.

	Details	Goals targeted
A1	A Posteriori error estimates and adaptivity	P5, H1, H2 (P6)
A2	The wave equation	P3–P5, H3
A3	Adaptivity in 2D	P3, P5, H1, H2

Laborations   The laborations are designed to be useful when doing the assignments.

	Details	Goals targeted
Lab 1	FEM in 1D	P1–P3 (P5, P6)
Lab 2	FEM in 2D	P1–P3, P5 (P6)
Lab 3	Comsol multiphysics	P5, P6

Final exam The final exam consists of 5 exam questions Q1–Q5. Each of these will test a few of the goals P1–P6 and/or H1–H3. Rather than stating a precise minimum number of points to pass, each question will be graded separately with respect to the goals it targets. Hence in order to *pass* the course you must demonstrate acceptable knowledge (that is, grade 3) on all basic goals P1–P6. For *higher* grades you should demonstrate *good/excellent* solutions for the basic goals, but *also* for H1–H3.

Since this method of grading might be a bit of a novelty, it will be discussed and motivated at the introductory lecture. We will also work through a couple of examples to see how it actually works. Should doubts still persist, never hesitate to bring it up in class or otherwise feel free to contact me through e-mail or visit me in my office.

Goals	Question	Grade
P1	Q1(a),Q2(a),Q3(a)	5
P2	Q1(b),Q2(b),Q3(a)	5
P3	Q5(a)	4
P4	Q2(c),Q3(c)	3
P5	Q3(d),Q4(a)	5
P6	Q5(b)	5
H1	Q3(e),Q4(b)	5
H2	Q5(c)	4
H3	Q2(d),Q4(c),Q5(d)	F
	Total grade:	???

Table 1: The grading matrix for an imaginary exam with 5 exercises. *Example:* Question Q4 tests P5 and H1+3. -What grade should the student have?

Advice from previous students

1. *"läs häftet/boken, häng med från början", "...do more reference readings before each lecture", "prepare ahead"*

2. *"Read the book yourself before the lecture, it makes it easy to understand the difficult things during the lecture."*

3. *"Don't be afraid to participate actively in the lectures and prepare yourself for each lecture by following the recommendations given at the end of the previous one. I regret not doing so."*

4. *"get familiarised with the inequalities used in the course..."*

5. *"Read compendium and when you do exercise try to link theory. And obviously read other sources not only depend on compendium"*

6. *"Do alot of experimenting on your own! There is not that much time for examples of actually setting up a FEM-problem so do some on your own with the Heat equation or whatever."*

7. *"The labs are really good so do your best to understand them and you will be well prepared for the exam."*