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Global stabilization and restricted tracking for multiple integrators with bounded controls *

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Abstract: Necessary and sufficient conditions for globally stabilizing linear systems with bounded controls are known. However, it has been shown in [5] that, for single-input systems, no saturation of a linear feedback can globally stabilize a chain of integrators of order n, with $n \geq 3$. In this paper, we propose a *nonlinear* combination of saturation functions of *linear* feedbacks that globally stabilizes a chain of integrators of arbitrary order. The appealing feature of the proposed control is that it is fairly easy to construct. It is linear near the origin and can also be used to achieve trajectory tracking for a class of trajectories restricted by the absolute on the input.

Keywords: Chain of integrators, bounded controls, saturation, global stabilization, tracking.

1. Introduction

The problem of stabilizing linear systems with bounded controls has been studied extensively (see [1,2,3]). Recently, in [4] from the point of view of nonlinear control, a (smooth) nonlinear (bounded) feedback was constructed to globally stabilize all asymptotically null-controllable linear systems. A system is null-controllable if every state of the system can be driven to zero asymptotically using a bounded measurable control. In some cases, a chain of integrators for example, the construction involved a complicated recursive argument which requires one to solve for a certain submanifold of the state space. In [5] it is shown that a certain simple strategy, namely any bounded function of a linear feedback, cannot possibly globally stabilize a chain of integrators of order n, for $n \geq 3$.

In this paper we propose two bounded feedback strategies for a chain of integrators of arbitrary order. The algorithm for constructing the bounded control law is fairly simple. The control law is linear near the origin and can easily be used to achieve trajectory tracking for a class of trajectories restricted by the absolute bounded on the input.

Our solutions to this linear problem were motivated by a more general nonlinear stabilizability problem (see [6]). In the nonlinear setting, the previous solutions to the linear problem were unable to assist us. However, the solutions presented here are crucial to solving the nonlinear problem solved in [6].

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2. Main result

2.1. Global stabilization

We start with the following definition:

Definition 1. Given two positive constants L, M with $L \le M$, a function $\sigma : \mathbb{R} \to \mathbb{R}$ is said to be a *linear saturation* for (L, M) if it is a continuous, nondecreasing function satisfying

(a) $s\sigma(s) > 0$ for all $s \neq 0$;
(b) $\sigma(s) = s$ when $|s| \le L$;
(c) $|\sigma(s)| \le M$ for all $s \in \mathbb{R}$.

In the subsequent control design, one can choose arbitrarily smooth functions out of this class.

Now consider the linear system consisting of multiple integrators

$$\dot{x}_1 = x_2, \quad \dots, \quad \dot{x}_n = u. \tag{1}$$

We are searching for a bounded control that will globally asymptotically stabilize (1). Our main result is:

Theorem 2.1. *There exist linear functions $h_i : \mathbb{R}^n \to \mathbb{R}$ such that, for any set of positive constants $\{(L_i, M_i)\}$ where $L_i \le M_i$ for $i = 1, \dots, n$ and $M_i < \frac{1}{2}L_{i+1}$ for $i = 1, \dots, n-1$, and for any set of functions $\{\sigma_i\}$ that are* linear saturations for $\{(L_i, M_i)\}$, *the bounded control*

$$u = -\sigma_n(h_n(x) + \sigma_{n-1}(h_{n-1}(x) + \cdots + \sigma_1(h_1(x))) \cdots)$$

results in global asymptotic stability for the system (1).

Proof. Consider the linear coordinate transformation $y = Tx$ which transforms (1) into $\dot{y} = Ay + Bu$ where A and B are given by

$$A = \begin{bmatrix} 0 & 1 & \cdots & 1 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 1 \\ 0 & \cdots & \cdots & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}.$$

The recursive nature involved yields a transformation characterized by

$$y_{n-i} = \sum_{j=0}^{i} \binom{i}{j} x_{n-i} \quad \text{where} \quad \binom{i}{j} = \frac{i!}{j!(i-j)!}.$$

The inverse of the transformation is characterized by

$$x_{n-i} = \sum_{j=0}^{i} (-1)^{i+j} \binom{i}{j} y_{n-i}.$$

A suitable control law is

$$u = -\sigma_n(y_n + \sigma_{n-1}(y_{n-1} + \cdots + \sigma_1(y_1)) \cdots) \tag{2}$$

which yields the closed loop system

$$\begin{aligned} \dot{y}_1 &= y_2 + \cdots + y_n - \sigma_n(y_n + \sigma_{n-1}(y_{n-1} + \cdots + \sigma_1(y_1)) \cdots), \\ \dot{y}_2 &= y_3 + \cdots + y_n - \sigma_n(y_n + \sigma_{n-1}(y_{n-1} + \cdots + \sigma_1(y_1)) \cdots), \\ &\vdots \\ \dot{y}_{n-1} &= y_n - \sigma_n(y_n + \sigma_{n-1}(y_{n-1} + \cdots + \sigma_1(y_1)) \cdots), \\ \dot{y}_n &= -\sigma_n(y_n + \sigma_{n-1}(y_{n-1} + \cdots + \sigma_1(y_1)) \cdots). \end{aligned} \tag{3}$$

We begin by considering the evolution of the state y_n. Consider the Lyapunov function $V_n = y_n^2$. The derivative of V_n is given by

$$\dot{V}_n = -2y_n[\sigma_n(y_n + \sigma_{n-1}(y_{n-1} + \cdots + \sigma_1(y_1)) \cdots)].$$

From Definition 1, condition (a) applied to σ_n and condition (c) applied to σ_{n-1} coupled with the fact that $M_{n-1} < \frac{1}{2}L_n$, we see that $\dot{V}_n < 0$ for all $y_n \notin Q_n = \{y_n: |y_n| \leq \frac{1}{2}L_n\}$. Consequently, y_n enters Q_n in finite time and remains in Q_n thereafter. Further, because the right-hand side of (3) is globally Lipschitz, the remaining states $y_1, \ldots, y_{n-1}$ remain bounded for any finite time.

Now consider the evolution of the state y_{n-1}. First observe that after y_n has entered Q_n, the argument of σ_n is bounded as

$$|y_n + \sigma_{n-1}(y_{n-1} + \cdots + \sigma_1(y_1)) \cdots)| \leq \tfrac{1}{2}L_n + M_{n-1} \leq L_n.$$

Consequently, after y_n enters Q_n, σ_n operates in its linear region from condition (b) of Definition 1. Then the evolution of y_{n-1} is given by

$$\dot{y}_{n-1} = -\sigma_{n-1}(y_{n-1} + \cdots + \sigma_1(y_1)) \cdots).$$

Using the same argument as for y_n we can show that y_{n-1} enters an analogous set Q_{n-1} in finite time and remains in Q_{n-1} thereafter. Again, all of the remaining states stay bounded. This procedure can be continued to show that after some finite time the argument of every function σ_i has entered the region where the function is linear. After this finite time, the closed loop equations have the form

$$\begin{aligned}
\dot{y}_1 &= -y_1,\\
\dot{y}_2 &= -y_1 - y_2,\\
&\vdots\\
\dot{y}_n &= -y_1 - y_2 - \cdots - y_n.
\end{aligned}$$

Clearly, the dynamics, after the prescribed finite time, are exponentially stable. □

The number of saturation functions required can be decreased by stabilizing the states in pairs rather than one at a time. We employ a slightly more restrictive class of linear saturation functions.

Definition 2. Given two positive constants L, M with $L \leq M$, a function $\sigma: \mathbb{R} \to \mathbb{R}$ is said to be a *simple linear saturation* for (L, M) if it is a continuous, nondecreasing function satisfying
(a) $s\sigma(s) > 0$ for all $s \neq 0$;
(b) $\sigma(s) = s$ when $|s| \leq L$;
(c) $|\sigma(s)| = M$ when $|s| \geq M$.

Where before we needed n saturation functions, now we need one function for each pair of states. If the dimension of the state space is odd, we will need one additional saturation function for the additional state. Accordingly, define $\tilde{n} = \frac{1}{2}n$ if n is even and $\tilde{n} = \frac{1}{2}(n+1)$ if n is odd.

Theorem 2.2. *There exist linear functions $h_i: \mathbb{R}^n \to \mathbb{R}$ such that for any set of positive constants $\{(L_i, M_i)\}$ where $L_i \leq M_i$ for $i = 1, \ldots, \tilde{n}$ and $M_i < L_{i+1}/(1+\sqrt{2})$ for $i = 1, \ldots, \tilde{n}-1$, and for any set of functions $\{\sigma_i\}$ that are* simple linear saturations for $\{(L_i, M_i)\}$, *the bounded control*

$$u = -\sigma_{\tilde{n}}(x) + \sigma_{\tilde{n}-1}(h_{\tilde{n}-1}(x) + \cdots + \sigma_1(h_1(x))) \cdots)$$

results in global asymptotic stability for the system (1).

Proof. Consider the same coordinate change as in the proof of the previous theorem. We will proceed in a similar manner as before, this time showing that the states y_{n-1}, y_n enter within finite time and thereafter remain in a region where the function $\sigma_{\tilde{n}}$ is linear. Since the differential equation is globally Lipschitz, the remaining states $y_1, \dots, y_{n-2}$ remain bounded. With $\sigma_{\tilde{n}}$ operating in its linear region we can iterate to show that y_{n-3}, y_{n-2} enter and remain in a region where $\sigma_{\tilde{n}-1}$ is linear. Eventually, this leads to the conclusion that after some finite time, the closed loop equations have the form

$$\dot{y}_1 = -y_1,$$
$$\dot{y}_2 = -y_1 - y_2,$$
$$\vdots$$
$$\dot{y}_n = -y_1 - y_2 - \cdots - y_n,$$

which is an exponentially stable linear system.

Consider the dynamics of y_{n-1}, y_n:

$$\dot{y}_{n-1} = y_n - \sigma_{\tilde{n}}(y_{n-1} + y_n + \sigma_{\tilde{n}-1}(y)), \qquad \dot{y}_n = -\sigma_{\tilde{n}}(y_{n-1} + y_n + \sigma_{\tilde{n}-1}(y)). \tag{4}$$

To show that y_{n-1}, y_n enters a sufficiently small neighborhood of the origin we use the following Lyapunov-like function:

$$W(y_{n-1}, y_n) = \tfrac{1}{2}y_{n-1}^2 + \tfrac{1}{2}y_n^2. \tag{5}$$

This positive definite function is only a Lyapunov-*like* function because there will be points in the state space where $\dot{W} > 0$. However, we will show that the integral of $\dot{W}$ is negative over known closed form solutions of (4) in the region where $\sigma_{\tilde{n}}$ is saturated. Further, when it is possible that $\sigma_{\tilde{n}}$ is not saturated, W is strictly decreasing (outside a neighborhood of the origin).

Consider the following regions of the state space:

region I: $y_{n-1} + y_n > M_{\tilde{n}} + M_{\tilde{n}-1}$,
region II: $y_{n-1} + y_n < -M_{\tilde{n}} - M_{\tilde{n}-1}$,
region III: $|y_{n-1} + y_n| \le M_{\tilde{n}} + M_{\tilde{n}-1}$.

We begin by showing that any bounded initial condition in region I yields a trajectory that enters region III in finite time. Observe that in region I, (4) is given by

$$\dot{y}_{n-1} = y_n - M_{\tilde{n}}, \quad \dot{y}_n = -M_{\tilde{n}}.$$

Consequently, the closed form solution of the trajectories in region I are given by

$$y_{n-1}(t) = y_{n-1}(t_o) + y_n(t_o)t - \tfrac{1}{2}M_{\tilde{n}}t^2 - M_{\tilde{n}}t, \qquad y_n(t) = y_n(t_o) - M_{\tilde{n}}t.$$

Combining, we have

$$y_{n-1}(t) + y_n(t) = y_{n-1}(t_o) + y_n(t_o) - 2M_{\tilde{n}}t + y_n(t_o)t - \tfrac{1}{2}M_{\tilde{n}}t^2.$$

We assume that $y_{n-1}(t_o) + y_n(t_o) > M_{\tilde{n}} + M_{\tilde{n}-1}$ and we solve for a t_b such that

$$y_{n-1}(t_b) + y_n(t_b) = M_{\tilde{n}} + M_{\tilde{n}-1}.$$

Using the quadratic formula it is straightforward to show that such a t_b exists and is finite and positive. The same argument holds for region II by symmetry.

Now consider an initial condition such that

$$y_{n-1}(t_o) + y_n(t_o) = M_{\tilde{n}} + M_{\tilde{n}-1}.$$

To enter region I, we must have $\dot{y}_{n-1}(t_o) > -\dot{y}_n(t_o)$ since the boundary of region I is a line of slope -1. This implies

$$y_n(t_o) > 2M_{\tilde{n}}, \qquad y_{n-1}(t_o) < -M_{\tilde{n}} + M_{\tilde{n}-1}.$$

Assume we enter region I. We show that we return to region III in finite time $t_b > 0$ and that $W(t_b) - W(t_o) < 0$. From the discussion above for trajectories in region I and since $y_{n-1}(t_o) + y_n(t_o) = y_{n-1}(t_b) + y_n(t_b)$, it follows that

$$\tfrac{1}{2}M_{\bar{n}}t_b^2 + [2M_{\bar{n}} - y_n(t_o)]t_b = 0.$$

This implies

$$t_b = \frac{2}{M_{\bar{n}}}(y_n(t_o) - 2M_{\bar{n}})$$

which is positive because $y_n(t_o) > 2M_{\bar{n}}$. Now consider

$$W(t_b) - W(t_o) = \tfrac{1}{2}(y_{n-1}^2(t_b) + y_n^2(t_b) - y_{n-1}^2(t_o) - y_n^2(t_o)).$$

First consider $\frac{1}{2}(y_{n-1}^2(t_b) - y_{n-1}^2(t_o))$. Observe that in terms of $y_{n-1}(t_o)$,

$$t_b = \frac{2}{M_{\bar{n}}}(-y_{n-1}(t_o) + M_{\bar{n}-1} - M_{\bar{n}}).$$

Evaluating the closed form solution for y_{n-1} at t_b yields

$$y_{n-1}(t_b) = -y_{n-1}(t_o) + 2(M_{\bar{n}-1} - M_{\bar{n}}).$$

A straightforward calculation then shows that

$$y_{n-1}^2(t_b) - y_{n-1}^2(t_o) = 4y_{n-1}(t_o)(M_{\bar{n}} - M_{\bar{n}-1}) + 4(M_{\bar{n}} - M_{\bar{n}-1})^2.$$

Since $y_{n-1}(t_o) < -M_{\bar{n}} + M_{\bar{n}-1}$ and $M_{\bar{n}} > M_{\bar{n}-1}$, it follows that $y_{n-1}^2(t_b) - y_{n-1}^2(t_o) < 0$.

Now consider $\frac{1}{2}(y_n^2(t_b) - y_n^2(t_o))$. Evaluating the closed form solution for y_n at t_b yields

$$y_n(t_b) = -y_n(t_o) + 4M_{\bar{n}}.$$

A straightforward calculation then shows that

$$y_n^2(t_b) - y_n^2(t_o) = -8y_n(t_o)M_{\bar{n}} + 16M_{\bar{n}}^2.$$

Since $y_n(t_o) > 2M_{\bar{n}}$, it follows that $y_n^2(t_o) - y_n^2(t_o) < 0$.

By symmetry, the same analysis holds for trajectories originating on the boundary of region II and entering region II.

Now consider trajectories in region III. We have

$$\begin{aligned}\dot{W} &= y_{n-1}[y_n - \sigma_{\bar{n}}(y_{n-1} + y_n + \sigma_{\bar{n}-1}(y))] + y_n[-\sigma_{\bar{n}}(y_{n-1} + y_n + \sigma_{\bar{n}-1}(y))] \\ &= (y_{n-1} + y_n)[-\sigma_{\bar{n}}(y_{n-1} + y_n + \sigma_{\bar{n}-1}(y))] + y_{n-1}y_n \\ &= (y_{n-1} + y_n)[y_{n-1} + y_n - \sigma_{\bar{n}}(y_{n-1} + y_n + \sigma_{\bar{n}-1}(y))] - (y_{n-1} + y_n)^2 + y_{n-1}y_n \\ &= (y_{n-1} + y_n)[y_{n-1} + y_n - \sigma_{\bar{n}}(y_{n-1} + y_n + \sigma_{\bar{n}-1}(y))] - \tfrac{1}{2}(y_{n-1} + y_n)^2 - \tfrac{1}{2}y_{n-1}^2 - \tfrac{1}{2}y_n^2 \\ &\le |(y_{n-1} + y_n)| M_{\bar{n}-1} - \tfrac{1}{2}y_{n-1}^2 - \tfrac{1}{2}y_n^2 - \tfrac{1}{2}(y_{n-1} + y_n)^2.\end{aligned}$$

Consider the level set $W = \frac{1}{2}M_{\bar{n}-1}^2$. On this level set, a circle of radius $M_{\bar{n}-1}$ in the y_{n-1}, y_n plane, we have

$$M_{\bar{n}-1} \le |y_{n-1} + y_n| \le \sqrt{2}M_{\bar{n}-1}.$$

Consider $|y_{n-1}+y_n| = kM_{\bar{n}-1}$ where $k \in [1, \sqrt{2}]$. Then

$$\dot{W} \leq -\tfrac{1}{2}M_{\bar{n}-1}^2 - \tfrac{1}{2}(kM_{\bar{n}-1})^2 + kM_{\bar{n}-1}^2 = -(\tfrac{1}{2} - k + \tfrac{1}{2}k^2)M_{\bar{n}-1}^2.$$

Since $k \in [1, \sqrt{2}]$, $\dot{W} \leq 0$. Since $\dot{W}$ is bounded by a quadratic negative definite function plus a linear perturbation in region III, $\dot{W} < 0$ outside of the level set $W = \frac{1}{2}M_{\bar{n}-1}^2$ and inside region III. Further, if the trajectory leaves region III, it returns in finite time and at a lower energy level W. Consequently, for any $\varepsilon > 0$, the trajectories of y_{n-1}, y_n enter a circle of radius $M_{\bar{n}+1} + \varepsilon$ in finite time and remain in that circle thereafter. If $M_{\bar{n}-1}$ is chosen so that

$$L_{\bar{n}} = \sqrt{2}(M_{\bar{n}-1} + \varepsilon) + M_{\bar{n}-1}$$

(i.e. $L_{\bar{n}} > M_{\bar{n}-1}(\sqrt{2}+1)$), then $\sigma_{\bar{n}}$ operates in its linear region after some finite time. Once $\sigma_{\bar{n}}$ becomes a strictly linear function we have

$$\dot{y}_{n-3} = y_{n-2} - \sigma_{\bar{n}-1}(y_{n-3} + y_{n-2} + \sigma_{\bar{n}-2}(y)), \qquad \dot{y}_{n-2} = -\sigma_{\bar{n}-1}(y_{n-3} + y_{n-2} + \sigma_{\bar{n}-2}(y))$$

and the same analysis applies to show that y_{n-3}, y_{n-2} eventually enters a sufficiently small neighbourhood of the origin. The iterative process continues until it can be shown that, after some finite time, every saturation function is operating in its linear region. After this time, the dynamics of (1) are those of an exponentially stable linear system. □

Remark. The results of [5] indicate that it is not possible to further reduce the number of saturation functions by trying to stabilize three states at a time.

2.2. Restricted tracking

Consider the nonlinear system

$$\dot{x}_1 = x_2, \quad \ldots, \quad \dot{x}_n = \sigma_{n+1}(u), \quad y = x_1. \tag{6}$$

Here σ_{n+1} is a linear saturation for (L_{n+1}, M_{n+1}). The task is to cause y to track a desired reference trajectory y_d given by $y_d, \dot{y}_d, \ldots, y_d^{(n)}$.

Corollary 2.1. *If $|y_d^{(n)}(t)| \leq L_{n+1} - \varepsilon$ for all $t \geq t_0$ and for some $\varepsilon > 0$ then there exist linear functions $h_i : \mathbb{R}^n \to \mathbb{R}$ such that for any set of positive constants $\{(L_i, M_i)\}$ where $M_n \leq \varepsilon$, $L_i \leq M_i$ for $i = 1, \ldots, n$ and $M_i \geqslant \frac{1}{2}L_{i+1}$ for $i = 1, \ldots, n-1$ and for any set of functions $\{\sigma_i\}$ that are* linear saturations for $\{(L_i, M_i)\}$, *the feedback*

$$u = y_d^{(n)} - \sigma_n(h_n(\tilde{x}) + \sigma_{n-1}(h_{n-1}(\tilde{x}) + \cdots + \sigma_1(h_1(\tilde{x}))) \cdots)$$

where $\tilde{x}$ is defined as $\tilde{x}_i = x_i - y_d^{(i-1)}$ for $i = 1, \ldots, n$, results in asymptotic tracking for the system (6).

Proof. In terms of $\tilde{x}$, (6) becomes

$$\dot{\tilde{x}}_1 = \tilde{x}_2, \quad \ldots, \quad \dot{\tilde{x}}_n = -y_d^{(n)} + \sigma_{n+1}(u).$$

Observe that, with the specified control law, if we choose $M_n \leq \varepsilon$, then $\sigma_{n+1}(\cdot)$ is always operating in its linear region so the closed loop system becomes

$$\dot{\tilde{x}}_1 = \tilde{x}_2, \quad \ldots, \quad \dot{\tilde{x}}_n = -\sigma_n(h_n(\tilde{x}) + \sigma_{n-1}(h_{n-1}(\tilde{x}) + \cdots + \sigma_1(h_1(\tilde{x}))) \cdots).$$

Now if $\{(L_i, M_i)\}$ satisfy $M_i < \frac{1}{2}L_{i+1}$ for $i = 1, \ldots, n-1$ and $\sigma_i(\cdot)$ satisfies Definition 1, then we have the conditions of the stabilization theorem of Section 2.1. Consequently, $\tilde{x}$ asymptotically approaches zero. In turn, this implies that $y(t)$ asymptotically approaches $y_d(t)$. □

For a result with fewer saturation functions we assume, for (6), that σ_{n+1} is a linear saturation for $(L_{\tilde{n}+1}, M_{\tilde{n}+1})$.

Corollary 2.2. *If* $|y_d^{(n)}(t)| \le L_{\tilde{n}+1} - \varepsilon$ *for all* $t \ge t_0$ *and for some* $\varepsilon > 0$ *then there exist linear functions* $h_i : \mathbb{R}^n \to \mathbb{R}$ *such that for any set of positive constants* $\{(L_i, M_i)\}$ *where* $M_{\tilde{n}} \le \varepsilon$, $L_i \le M_i$ *for* $i = 1, \ldots, \tilde{n}$ *and* $M_i < L_{i+1}/(1+\sqrt{2})$ *for* $i = 1, \ldots, \tilde{n}-1$ *and for any set of functions* $\{\sigma_i\}$ *which are* simple linear saturations for $\{(L_i, M_i)\}$, *the feedback*

$$u = y_d^{(n)} - \sigma_{\tilde{n}}\big(h_{\tilde{n}}(\tilde{x}) + \sigma_{\tilde{n}-1}\big(h_{\tilde{n}-1}(\tilde{x}) + \cdots + \sigma_1\big(h_1(\tilde{x})\big)\cdots\big)$$

where $\tilde{x}$ *is defined by* $\tilde{x}_i = x_i - y_d^{(i-1)}$ *for* $i = 1, \ldots, n$, *results in asymptotic tracking for the system* (6).

3. Conclusion

Two simple bounded control algorithms have been presented to globally stabilize a chain of integrators. The algorithms are implemented with saturation functions that are linear near the origin and can be chosen arbitrarily smooth. The control laws naturally extend to the task of trajectory tracking using bounded controls.

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