Final Exam in Programming Theory

Department of Information Technology
Uppsala University
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Lecturers: Parosh A. A., J. Cederberg
Location: Gimogatan
Time: $15 - 20$
No books or calculator allowed

Directions:

1. Each proof step should be motivated formally (no credit for informal proofs)
2. **Simple** arithmetic rules and all the axioms and theorems in the appendix may be used in the proofs
3. Answer only one problem on each sheet of paper
4. Do not write on the back of the paper
5. Write your name on each sheet of paper

Good Luck!

Problem 1 (25 p)

Prove the following claim.

$$\begin{pmatrix} (w \leq y) \\ \wedge \\ (w \leq z) \\ \wedge \\ (x \leq y) \vee (x \leq z) \end{pmatrix} \implies (x + w \leq y + z)$$

Problem 2 (20 p)

Consider the program S below.

$$\textbf{do} \quad 0 < x \quad \rightarrow \quad x := \; x - 2 \quad \textbf{od}$$

Give $wp(S, R)$ for the following values of R.

- $R = odd(x)$.
- $x = -1$.

- $x = -3$.
- $x = 0$.
- $x \geq 2$.

It is sufficient that you only give the definition of $wp(S, R)$. Do **not** explain how you found $wp(S, R)$. Do **not** prove that your solution is correct. Points will be deducted if you attempt that !!

Problem 3 (25 p)
Using the alternative command theorem, show that the following program is correct.

$$
\begin{array}{l}
\{(x = max(x, y, z)) \ \vee \ (y = max(x, y, z))\} \\
\ \textbf{if} \ \ (x \leq z) \wedge (y \leq z) \ \ \rightarrow \ \ w := \ z \\
\ \square \qquad\qquad\qquad\qquad\quad T \ \ \rightarrow \ \ w := \ x + y \\
\ \textbf{fi} \\
\{w \geq max(x, y, z)\}
\end{array}
$$

where x, y, and z are natural numbers.
Hint: Formulate the two predicates $b = max(a_1, a_2, a_3)$ and $b \geq max(a_1, a_2, a_3)$.

Problem 4 (30 p)
Consider the following program:

$$
\begin{array}{l}
\{Q: \ (s = 1) \wedge (i = 0) \wedge (n > 0)\} \\
\{\text{inv } P: \ ??\} \\
\{\text{bound } t: \ ??\} \\
\ \textbf{do} \ \ i < n \ \ \rightarrow \ \ s, i := \ s + 2i + 3, i + 1 \ \ \textbf{od} \\
\{R: \ s = (n + 1)^2\}
\end{array}
$$

Define an invariant P and a bound function. Show the correctness of the program through the iterative command theorem, using your definitions of P and t.