Final Exam in Programming Theory

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Location: PB
Time: 08 − −13
No books or calculator allowed

Directions:

1. Each proof step should be motivated formally (no credit for informal proofs)
2. **Simple** arithmetic rules and all the axioms and theorems in the appendix may be used in the proofs
3. Answer only one problem on each sheet of paper
4. Do not write on the back of the paper
5. Write your name on each sheet of paper

Good Luck!

Problem 1 (15 p)

Consider the integers x and y. Prove the following claim.

$$(0 < x \wedge x < y) \implies wp\,(\text{"}x, y := \ 0, y - 1\text{"},\ x + 1 \leq y)$$

Problem 2 (25 p)

Consider the program S below:

$$\textbf{do} \quad (y > 0) \quad \rightarrow \quad \big(x := \ y; \big(\ \textbf{do} \quad (x \neq 2) \quad \rightarrow \quad x := \ x - 3 \quad \textbf{od} \ \big)\, ; y := \ y - 9 \big) \quad \textbf{od}$$

Give, without proof, $wp\,(S,\ R)$ for the following values of R.

1. T
2. $y = -1$
3. $y = 0$
4. $x = 2$
5. $x = 0$

Problem 3 (20 p)
Assume three integers x, y and z.

- Write a predicate $P(x, y, z)$ such that P evaluates to true if and only if z equals the smallest of x and y.
- Find a (sequence of) command(s) S such that: $\{T\}S\{P\}$
- Prove $\{T\}S\{P\}$

Problem 5 (40 p)
Consider an array $b[0:n]$ (n inclusive) of integers, and the following skeleton where The (sequences of) assignments S_1, S_2 do not modify the array b:

$$\begin{array}{l} \{Q : 0 \leq n\} \\ i, r :=\ 0, 0 \\ \{inv\, P : ?\} \\ \{bound\, t : ?\} \\ \ \textbf{do} \quad P_1 \ \rightarrow \ S_1 \\ \quad\ \ \square \ \ P_2 \ \rightarrow \ S_2 \\ \quad \textbf{od} \\ \{R : \quad \} \end{array}$$

1. First program. Assume $R : r = \Sigma j \ : \ 0 \leq j < n+1 \ : \ (b[j] * j\%2)$[1]
   - Choose suitable guards P_1, P_2 and (sequences of) assignments S_1, S_2,
   - Choose an invariant P and use it to prove partial correctness.
2. Second program. Assume $R : (\forall j \ : \ 0 \leq j < n+1 \ : \ b[j] \neq n) \vee (b[r] = n)$
   - Choose suitable guards P_1, P_2 and (sequences of) assignments S_1, S_2,
   - Propose an invariant P for proving partial correctness (**you do not need to prove partial correctness!**).
3. Choose one bound function t for both programs, and prove termination for the second program.

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[1] You can use the following simple arithmetic definitions:

(a) $(j\%2 = 0) \vee (j\%2 = 1)$
(b) $(j\%2 = 0) = ((j+1)\,\%2 = 1)$
(c) $(j\%2 = 1) = ((j+1)\,\%2 = 0)$