Exam in Programming Theory

Department of Information Technology
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Location: PB
Time: 08 – –13
No books or calculator allowed

Directions:

1. Each proof step should be motivated formally (no credit for informal proofs)
2. You have to state the **simple** arithmetic rules you use in the proofs
3. All the axioms and theorems in the appendix may be used in the proofs
4. Answer only one problem on each sheet of paper
5. Do not write on the back of the paper
6. Write your anonymous exam code (if you do not have one, your name) on each sheet of paper

Good Luck!

Problem 1 (12 p)

Let **S: while** $(x \neq 0)$ **do x:=x+1 od** be a statement. In the following multiple choice question, state, without proof, the correct answer:

1. $wp(S, x > -2) = (x \geq 0)$
2. $wp(S, x > -2) = (x \leq 0)$
3. $wp(S, x > -2) = (x = 0)$

Problem 2 (15 p)
Prove the following theorem:

$$(p \implies \neg q) \implies ((p \wedge (q \implies r)) = p)$$

Problem 3 (15 p)
Prove the following:

$$(\forall i \ : \ m \leq i < n \ : \ (p_i \ \vee \ q_i) \ \Rightarrow \ r_i) \quad \implies \quad \begin{pmatrix} \forall i \ : \ m \leq i < n \ : \ (p_i \ \Rightarrow \ r_i) \\ \vee \\ \forall i \ : \ m \leq i < n \ : \ (q_i \ \Rightarrow \ r_i) \end{pmatrix}$$

Problem 4 (18 p)
Using the alternative command theorem, prove this program correct:

$$\begin{array}{l} \{(x \geq y) \wedge (x \geq z)\} \\ \ \textbf{if} \ \ y \leq z \ \ \rightarrow \ \ w := \ y \\ \ \square \ \ \ z \leq y \ \ \rightarrow \ \ w := \ z \\ \ \textbf{fi} \\ \{w := min(x, y, z)\} \end{array}$$

where *min* is defined as follows:

$$\begin{array}{c} (w := min(x, y, z)) \\ = \\ \Big(\big((w = x) \vee (w = y) \vee (w = z)\big) \wedge \big((w \leq x) \wedge (w \leq y) \wedge (w \leq z)\big)\Big) \end{array}$$

Problem 5 (40 p)
Consider the following program:

$$\{Q: \ n > 0\}$$

$$i := \ 1;$$

$$\{\text{inv } P: \ (0 < i) \land (i \leq n) \ \land \ (\exists p \ : \ i = 2^p)\}$$
$$\{\text{bound } t: \ n - i\}$$

$$\textbf{do} \quad 2 * i \leq n \quad \rightarrow \quad i := \ 2 * i \quad \textbf{od}$$

$$\{R: \ (0 < i) \land (i \leq n) \land (n < 2 * i) \ \land \ (\exists p \ : \ i = 2^p)\}$$

1. Prove partial correctness
2. Prove termination

Hint: you may use these simple arithmetic rules (or similar ones):

$$\begin{array}{lcl}
(\exists p \ : \ 1 = 2^p) & = & T \\
(0 < 1) & = & T \\
(x \leq y) & = & (x - 1 < y) \\
\neg(x \leq y) & = & (y < x) \\
(x > y) & = & (x - y > 0) \\
(0 < 2 * i) & = & (0 < i) \\
\big((\exists p \ : \ i = 2^p) \ \Rightarrow \ (\exists p \ : \ 2 * i = 2^p)\big) & = & T \\
\big((0 < i) \ \land \ (2 * i \leq n) \ \Rightarrow \ (n > i)\big) & = & T \\
(0 < i) & = & (i < 2 * i) \\
\big((a < b) \ \Rightarrow \ (c - b < c - a)\big) & = & T
\end{array}$$