Numerical methods for studying the vortex patch problem

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Summary

Our work has consisted of implementing two discrete methods, one relying on interpolation with cubic splines and one pseudo-spectral method, and then comparing the performance of these when solving the vortex patch problem.

Why the vortex patch problem?

The vortex patch problem addresses a situation where a bounded region exhibits constant vorticity, whereas its surroundings does not.

In this project we have studied an interpolation between the 2D Euler equation (2DEE) and the Surface Quasi Geostrophic Equation (SQGE), where the vortex patch problem seeks a non-smooth solution to this interpolation.

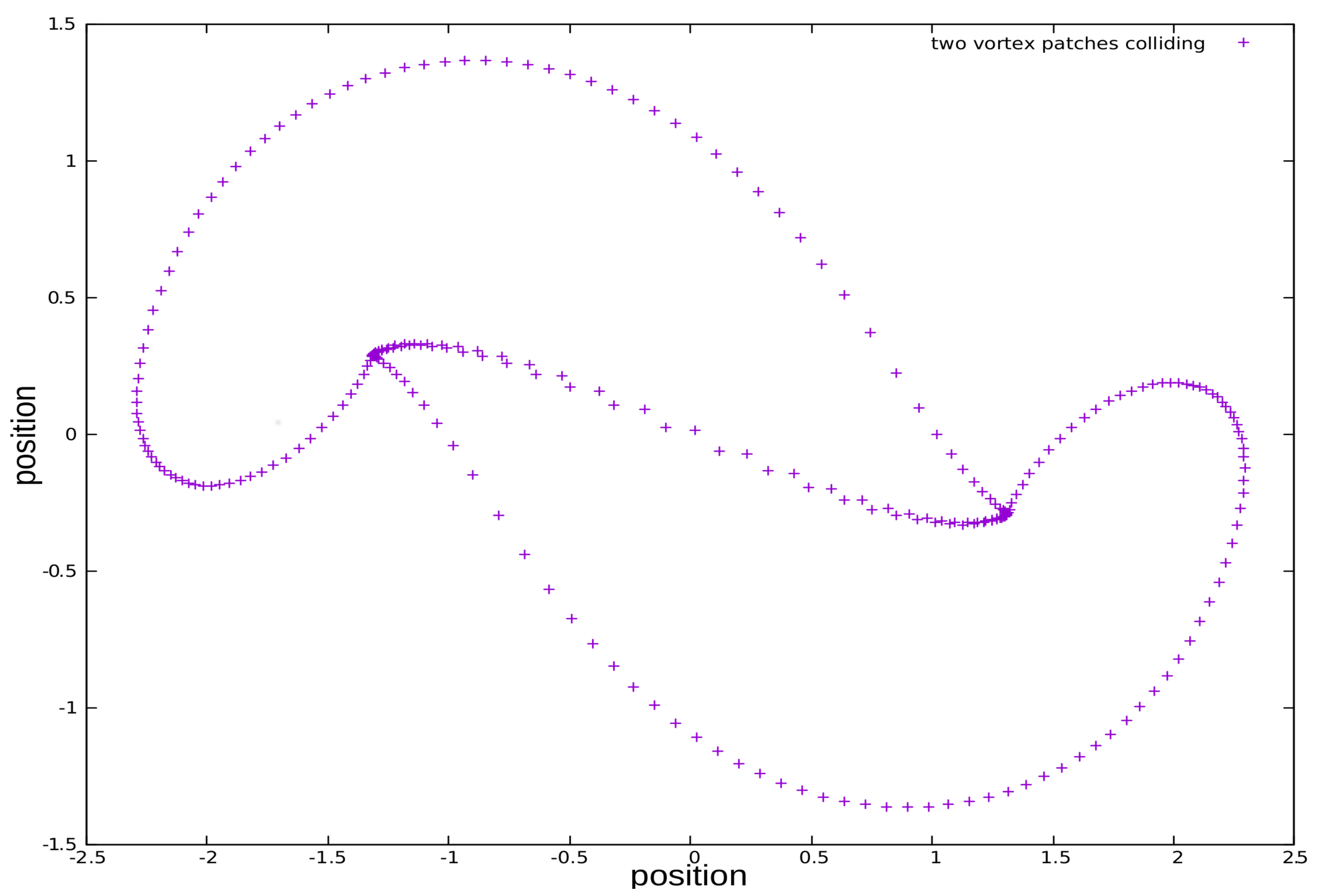

Figure 1: The evolution of two vortex patches using the interpolation method, framed after 0.5 seconds.

The vortex patch problem is a scenario where singularities may appear for some equations. Since the SQGE has strong connections to the 3DEE, which in turn has strong connections to the Navier-Stokes equations, whose existence and smoothness is still an open problem in mathematics, it is of interest to study whether singularities do appear for the vortex patch problem.

Methods

Two different methods were implemented in C, and compared. In each time step, the velocity field of the vortex patch's contour is calculated by an integral evaluation.

Interpolation method

Uses cubic splines to represent the contour curve. The velocity integral is calculated in four different ways, but mostly with an adaptive Runge-Kutta method. An example of two patches colliding when simulated by this method may be seen in fig. 1.

The pseudo-spectral method

Approximates the velocity as a Riemann sum over coefficients retrieved with the Fast Fourier Transform.

Conclusions

- The pseudo-spectral method is significantly faster than the interpolation method, see fig. 2.
- Both methods converge towards the solution.
- The pseudo-spectral method has an exponential accuracy, while the interpolation method is third order accurate.

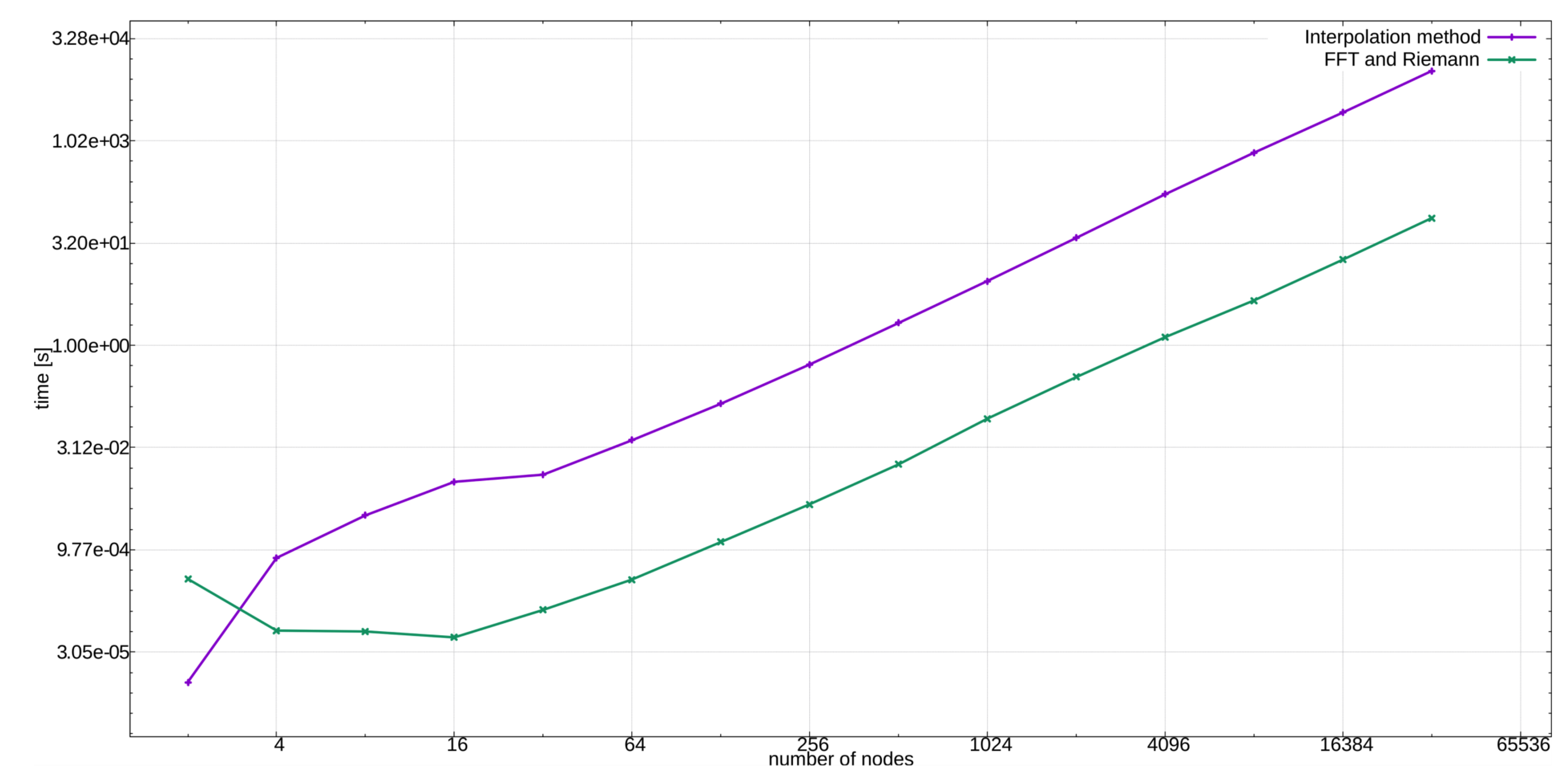

Figure 2: Computational time as a function of number of nodes representing the contour curve on a log-log scale.

Discussion

- Pseudo-spectral method is better than interpolation method in regard to both computational speed and precision.
- Interpolation method has an adaptive node handling. Can add nodes when sharp corners, i.e. singularities, appear. Not the case for pseudo-spectral, which also has aliasing issues for high modes.
- Combining the two might be a good option.

References

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