Loop gain: \( G_o(s) = F_y(s)G(s) \)

Closed loop system: \( Y(s) = G_c(s)Y_{ref}(s) + S(s)V(s) - T(s)N(s) \)

\[
G_c(s) = \frac{F_r(s)G(s)}{1 + G_o(s)}, \quad S(s) \triangleq \frac{1}{1 + G_o(s)}, \quad T(s) \triangleq \frac{G_o(s)}{1 + G_o(s)}
\]
The step response

The step response is a graphical representation of how a system responds to a sudden change in input. It is typically shown as a function of time, with the input step being applied at time $t=0$. The response is measured at various points to define key characteristics:

- **Risetime ($T_r$)**: The time it takes for the response to go from 10% to 90% of the final value ($y_f$). It is often used to measure the system's ability to track changes in the input.

- **Settling time ($T_s$)**: The time it takes for the response to settle to within a certain percentage of the final value ($y_f$). It is typically defined as when the error $e$ is less than a specified percentage of $y_f$.

- **Overshoot ($M$)**: The maximum deviation of the response from the final value. It is often measured as a percentage of $y_f$.

- **Steady state error ($e_\infty$)**: The difference between the final value of the response and the desired value. It is calculated as $e_\infty = r - y_f$.

In the diagram:

- $M y_f$ represents the maximum deviation from the final value.
- $y_f$, $0.9 y_f$, and $0.1 y_f$ mark the final value and its 90% and 10% points, respectively.
- $T_r$ is the risetime, the time it takes for the response to go from 10% to 90% of $y_f$.
- $T_s$ is the settling time, the time it takes for the response to settle to within 10% of $y_f$.

The diagram illustrates how these characteristics are measured and defined in the context of a step response.
The Nyquist criterion

Let $\gamma$ be a semicircle with radius $R \to \infty$. $G_o(s)$ maps $\gamma$ onto $\gamma'$. 

The Nyquist criterion: If $\gamma'$ encircles $-1$ $k$ times counter clockwise, the closed loop system has $k$ more poles in the RHP than $G_o(s)$ has. If $\gamma'$ encircles $-1$ $k$ times clockwise, the closed loop system has $k$ fewer poles in the RHP than $G_o(s)$ has.
Bode plots

\[ G(s) = \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2}, \]  
poles in \(-\omega_0 \left( \zeta \pm i \sqrt{1 - \zeta^2} \right)\)

\(\omega_0 = \) distance from the origin, \(\zeta = \) damping ratio.

Small damping ratio, \(\zeta \ll 1 \implies\) resonance peak \(M_p \geq |G(i\omega_0)| = \frac{1}{2\zeta}\)
Specifications on the Nyquist curve $G_o(i\omega)$

\[ \omega = \omega_p \]

\[ \frac{1}{A_m} \]

\[ \varphi_m \]

\[ \omega_c \]

\[ \arg G_o \]

\[ \omega_p \]
The system with the transfer function

\[ G(s) = \frac{b_1 s^{n-1} + \cdots + b_{n-1} s + b_n}{s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n} \]

can be represented by the controller canonical form:

\[
\begin{bmatrix}
-a_1 & -a_2 & \cdots & -a_{n-1} & -a_n \\
1 & 0 & \cdots & 0 & 0 \\
0 & 1 & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & 1 & 0
\end{bmatrix}
\begin{bmatrix}
x \\
x \\
x \\
x \\
x
\end{bmatrix} + 
\begin{bmatrix}
1 \\
0 \\
0 \\
\vdots \\
0
\end{bmatrix} u
\]

\[ y = \begin{bmatrix} b_1 & b_2 & \cdots & b_n \end{bmatrix} x \]
The system with the transfer function

\[ G(s) = \frac{b_1 s^{n-1} + \cdots + b_{n-1} s + b_n}{s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n} \]

can be represented by the observer canonical form:

\[
\begin{align*}
\dot{x} &= \begin{bmatrix}
-a_1 & 1 & 0 & \cdots & 0 \\
-a_2 & 0 & 1 & \cdots & 0 \\
& \vdots & \vdots & \ddots & \vdots \\
-a_{n-1} & 0 & 0 & \cdots & 1 \\
-a_n & 0 & 0 & \cdots & 0
\end{bmatrix} x + \begin{bmatrix}
b_1 \\
b_2 \\
\vdots \\
b_{n-1} \\
b_n
\end{bmatrix} u \\
y &= \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \end{bmatrix} x
\end{align*}
\]
Solution of the state equation

The initial value problem

\[ \dot{x}(t) = Ax(t) + Bu(t), \quad x(t_0) = x_0, \]

has the solution

\[ x(t) = e^{A(t-t_0)}x_0 + \int_{t_0}^{t} e^{A(t-\tau)}Bu(\tau)d\tau. \]

The matrix exponential function:

\[ e^{At} \triangleq \sum_{k=0}^{\infty} \frac{1}{k!}(At)^k, \quad \mathcal{L}[e^{At}] = (sI - A)^{-1} \]