What is this course about?

- **The control problem**: Minimize $e$, including the effect of $v$, and keep $u$ small ($u_{min} \leq u \leq u_{max}$).
- **Ultimate course goal**: Find the optimal solution!
Example 1: Mission to Mars
Bring’em back alive!

- Predefined optimal trajectory minimizes required amount of fuel.
- Steer the rocket so that the deviation from the optimal trajectory is minimized, with minimal fuel consumption (we want to get back!).
Example 2: Wastewater treatment

A wastewater treatment plant removes contaminants from sewage, e.g. nutrients like ammonia and phosphorus.

- One step (of many) is nitrogen removal in a bioreactor:
  - **Output:** Concentrations of ammonia and nitrate in effluent water
  - **Input:** Air flow, external carbon, recirculation...
  - **Disturbance:** Variations of flow rates and concentrations of nutrients in the incoming water...

- **Objective:** Remove as much contaminants as possible, to as low cost as possible.

- **Characteristics:** Multivariable system, disturbances to a large extent periodical (on daily, weekly and yearly bases),...
How do we solve the problem?

Course goal in parts

Optimal controllers

Minimize a cost function

\[ V = ||y||^2 + \rho ||u||^2, \]

\( \rho > 0 \) design parameter.

Two strategies:

- **LQG** gives a linear, observer based state feedback controller.
  Truly optimal under idealized conditions. *(Chap. 9)*

- **MPC** exploits numerical optimization. Constraints on \( u \) and \( y \) are easily handled. Gives a nonlinear controller. *(Chap. 16)*
With a good characterization of the disturbances, a better performance can be achieved. *(Chap. 5)*

- Regard disturbances as random signals = stochastic processes.
- Characterize them by their frequency content = their spectrum.
- Model them as output from a linear system with a totally random input (= white noise).
- Their behaviour can be predicted by an optimal observer = the Kalman filter.
Controllers are generally implemented in computers. A computer cannot handle continuous-time signals, only numbers ⇒ use uniform sampling. \((Chap. \ 2, \ 3, \ 4, \ 5, \ 9, \ 16)\)

- All signals are discrete in time, i.e. sequences.
- Discrete-time models are described by difference equations \(=\) recurrence relations (rather than differential equations).
- Use the Z-transform (instead of the Laplace transform).
- A continuous-time system can be represented by a discrete-time model
  - exactly if the inputs are piecewise constant,
  - approximately using a discrete-time approximation of the time derivative.
Typically a system has more than one input and more than one output. MIMO = Multi-Input Multi-Output (and SISO = Single-Input Single-Output). *(Chap. 3)*

- In the model $Y(s) = G(s)U(s)$ of a MIMO system $Y(s)$ and $U(s)$ are vectors, and $G(s)$ is a matrix.
- Convenient to use state space models for MIMO systems, looks and works the same way as for SISO systems.
- The bioreactor in example 2 is a MIMO system.
Course outline

- **F1–F2, L1**: Intro, repetition
- **F3–F5, L2–L3, BL1**: Discrete-time system, sampling, MIMO
- **F6–F8, L4–L5, BL2**: Disturbance models
- **F9–F11, L6, BL3, Lab (proc.+demo)**: Optimal control (LQG, MPC)
- **F12, L7**: Summary and repetition
- **Compulsory parts**: Process labs, final exam