Uppsala University
Dept. of Systems and Control

Final Exam in System Identification for F4

Answers and brief solutions

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1.

$$\begin{bmatrix} \sum i^2(t-1) & -\sum i(t-1)u(t-1) \\ -\sum i(t-1)u(t-1) & \sum u^2(t-1) \end{bmatrix} \begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = \begin{bmatrix} -\sum i(t)i(t-1) \\ \sum i(t)u(t-1) \end{bmatrix}$$

2. a)

$$\begin{aligned} r_{yu}(\tau) &= \mathrm{E}\{y(t+\tau)u(t)\} \\ &= \mathrm{E}\left\{\left(\sum_{k=0}^{\infty} g(k)u(t+\tau-k) + v(t)\right) u(t)\right\} \\ &= \mathrm{E}\left\{\sum_{k=0}^{\infty} g(k)u(t+\tau-k)u(t)\right\} \\ &= \sum_{k=0}^{\infty} g(k) r_u(\tau-k) \\ &= g(k)\delta_{\tau,k}\lambda^2 \\ &= \lambda^2 g(\tau) \end{aligned}$$

b)

$$\hat{r}_{yu}(\tau) = \frac{1}{N} \sum_{t=1-\min(\tau,0)}^{N-\max(\tau,0)} y(t+\tau)u(t), \quad \tau = 0, \pm 1, \pm 2, \ldots$$

which gives

$$\hat{g}(\tau) = \frac{1}{\hat{\lambda}^2} \hat{r}_{yu}(\tau)$$

where $\hat{\lambda}^2$ is an estimate of λ^2.

3. a)

$$\begin{aligned}\hat{a} &= \left(\frac{1}{N}\sum_{t=1}^{N}\varphi^2(t)\right)^{-1}\left(\frac{1}{N}\sum_{t=1}^{N}\varphi(t)y(t)\right)\\ &= \left(\frac{1}{N}\sum_{t=1}^{N}y^2(t-1)\right)^{-1}\left(-\frac{1}{N}\sum_{t=1}^{N}y(t-1)y(t)\right)\\ &\underset{N\to\infty}{\longrightarrow} -\frac{r_y(1)}{r_y(0)} = a_0\end{aligned}$$

b)

$$\operatorname{var}(\hat{a}) \approx \frac{\lambda^2}{N}\bar{R}^{-1}$$

where

$$\bar{R} = \mathrm{E}\{\varphi^2(t)\} = r_y(0) = \frac{\lambda^2}{1-a_0^2}$$

so

$$\operatorname{var}(\hat{a}) \approx \frac{1-a_0^2}{N}$$

4.

$$\begin{aligned}\hat{c}(t) &= \left(\sum_{k=1}^{t}0.9^{t-k}\right)^{-1}\sum_{k=1}^{t}0.9^{t-k}y(k)\\ \hat{c}(t) &= \hat{c}(t-1) + \frac{0.1}{1-0.9^t}\left(y(t)-\hat{c}(t-1)\right)\end{aligned}$$

5. A discussion can be made based on the material in the course book.

6. a)

$$\mathrm{E}\{\hat{c}\} = \frac{1}{2}c$$

b)

$$\operatorname{var}(\hat{c}) = \frac{\sigma^2}{4N}$$

7. a)

$$\mathrm{E}\{z(t)\varphi^T(t)\} = \begin{bmatrix} b_0\lambda^2 & kb_0\lambda^2 \\ 0 & \lambda^2 \end{bmatrix}$$

b) Choose $u(t)$ so that it is sufficiently variational, persistently exciting of sufficient order, and so that $\frac{1}{N}\sum_{t=1}^{N}\varphi(t)\varphi^T(t)$ is invertible.