Lecture 8

Identification of Closed Loop Systems – (Ch. 10)

System Model

System:

$$y(t) = G_s(q^{-1})u(t) + H_s(q^{-1})e(t)$$
$$u(t) = -F(q^{-1})y(t) + L(q^{-1})v(t)$$

- The input $u(t)$ is determined through feedback.
- $F(q^{-1})$ and $L(q^{-1})$ are called regulators.
- The signal $v(t)$ can be a reference signal, or noise entering the regulator.

Why?

Why is closed loop identification of interest?

- Many systems have feedback.
- The open loop system is unstable.
- Feedback is required due to safety reasons.

What Happens in a Closed-loop Experiment?

- The input $u(t)$ depends on the output $y(t)$ (dependence between $u(t)$ and $e(t)$).
- The aim with control (feedback) is to minimize the deviation between $y(t)$ and the reference value $v(t)$. Good control implies a small value of $u(t)$.
- System identification requires good excitation, implying in some sense large variations in $u(t)$. Hence, there is a conflict with the previous aspect.
- The frequency contents of the input is limited by the true system.

An Example

System:

$$y(t) + ay(t-1) = bu(t-1) + e(t), \quad Ee^2(t) = \lambda^2$$
$$u(t) = -fy(t)$$

Model structure:

$$y(t) + \hat{a}y(t-1) = \hat{b}u(t-1) + \varepsilon(t)$$

Estimate (PEM or LS):

$$\hat{a} = a + f\gamma$$
$$\hat{b} = b - \gamma$$

where γ is any scalar. There is no unique solution. Consequently the parameters are not consistently estimated.

The Closed-loop Behavior

Open-loop system:

$$y(t) = G_s(q^{-1})u(t) + H_s(q^{-1})e(t)$$
$$u(t) = -F(q^{-1})y(t) + L(q^{-1})v(t)$$

Closed-loop system (omitting q^{-1}):

$$y(t) = (I + G_sF)^{-1}\Big(G_sLv(t) + H_se(t)\Big)$$
$$u(t) = \Big[I - F\big(1 + G_sF\big)^{-1}G_s\Big]Lv(t) - F\big(I + G_sF\big)^{-1}H_se(t)$$

Some Assumptions

1. The open loop system is strictly proper. $y(t)$ depends only on *past* input values $u(s)$, $s < t$.
2. The closed loop system is asymptotically stable.
3. The external signal $v(t)$ is stationary and persistently exciting of sufficient order.
4. The external signal $v(t)$ and the disturbance $e(s)$ are independent for all t and s.

Use of Prediction Error Methods

- In most cases it is not necessary to assume that the external input $v(t)$ is measurable.
- Gives statistically efficient estimates under mild conditions.
- Computationally demanding.

Notation: $\hat{G}$ means $G(q^{-1}, \hat{\boldsymbol{\theta}})$.

Different Approaches

- *Direct identification.* The existence of possible feedback is neglected. The system is treated as an open loop system.
- *Indirect identification.* It is assumed that $v(t)$ is measurable and the feedback law is known. First the closed loop is identified. Then the open loop is determined from the known regulators and the identified closed loop.
- *Joint identification.* The data $u(t)$ and $y(t)$ are treated as the outputs of a multivariable system driven by white noise. The multivariable system is identified.

Direct Identification

Model used for prediction:

$$y(t) = \hat{G}u(t) + \hat{H}e(t)$$
$$Ee^2(t) = \hat{\lambda}^2$$

Data Used: $\{y(t), u(t)\}_{t=1}^{N}$

Goal: Estimate (SISO-case)

$$\hat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} V_N(\boldsymbol{\theta})$$

$$V_N(\boldsymbol{\theta}) = \frac{1}{N}\sum_{t=1}^{N} \varepsilon^2(t, \boldsymbol{\theta}), \quad \varepsilon(t, \boldsymbol{\theta}) = \hat{H}^{-1}[y(t) - \hat{G}u(t)]$$

Question: Identifiability? Desired solution:

$$\hat{G} \equiv G_s \quad \hat{H} \equiv H_s$$

Consistency?

Analyze the asymptotic cost function:

$$V(\boldsymbol{\theta}) = \lim_{N\to\infty} V_N(\boldsymbol{\theta}) = E\varepsilon^2(t)$$

- Will $\hat{G} \equiv G_s$ and $\hat{H} \equiv H_s$ be a global minimum to $V(\boldsymbol{\theta})$ (system identifiable)?
- Is the solution $\hat{G} \equiv G_s$ and $\hat{H} \equiv H_s$ unique (consistency) ?

An Example

System:

$$y(t) + ay(t-1) = bu(t-1) + e(t)$$
$$Ee^2(t) = \lambda^2$$

Model structure:

$$y(t) + \hat{a}y(t-1) = \hat{b}u(t-1) + \varepsilon(t)$$

Input:

$$u(t) = \begin{cases} -f_1 y(t) & \text{for a fraction } \gamma_1 \text{ of the total time} \\ -f_2 y(t) & \text{for a fraction } \gamma_2 \text{ of the total time} \end{cases}$$

Then (for $i = 1, 2$),

$$y_i(t) + (a + bf_i)y_i(t-1) = e(t)$$
$$\varepsilon_i(t) = y_i(t) + (\hat{a} + \hat{b}f_i)y_i(t-1)$$

which gives:

$$\begin{aligned} V(\hat{a}, \hat{b}) &= \gamma_1 E\varepsilon_1^2(t) + \gamma_2 E\varepsilon_2^2(t) \\ &= \lambda^2 + \gamma_1\lambda^2 \frac{(\hat{a} + \hat{b}f_1 - a - bf_1)^2}{1 - (a + bf_1)^2} \\ &+ \gamma_2\lambda^2 \frac{(\hat{a} + \hat{b}f_2 - a - bf_2)^2}{1 - (a + bf_2)^2} \end{aligned}$$

Consequently,

$$V(\hat{a}, \hat{b}) \geq \lambda^2 = V(a, b)$$

We get:

- $\hat{a} = a, \ \hat{b} = b$ is a global minimum.
- *Unique* minimum ?
- Solve $V(\hat{a}, \hat{b}) = \lambda^2$.

$$\begin{pmatrix} 1 & f_1 \\ 1 & f_2 \end{pmatrix} \begin{pmatrix} \hat{a} \\ \hat{b} \end{pmatrix} = \begin{pmatrix} a + bf_1 \\ a + bf_2 \end{pmatrix}$$

- Unique solution if and only if $f_1 \neq f_2$. (Compare with our previous example)

The General Case

- The desired solutions $\hat{H} = H_s$ and $\hat{G} = G_s$ will be a global minimum of $V(\boldsymbol{\theta})$.
- Unique global minimum is necessary for parameter identifiability (consistency). Consistency is assured by:
  - Using an external input $v(t)$.
  - Using a regulator $F(q^{-1})$ that shifts between different settings during the experiment.

Indirect Identification

- Two step approach:

  Step 1 Identify the closed loop system using $v(t)$ as input and $y(t)$ as the output.

  Step 2 Determine the open loop system parameters from the closed loop model obtained in step 1, *using the knowledge of the feedback* ($F(q^{-1})$ and $L(q^{-1})$).

- Closed loop system:

$$y(t) = \bar{G}v(t) + \bar{H}e(t)$$

$$\begin{aligned} \bar{G} &\triangleq (I + G_sF)^{-1}G_sL \\ \bar{H} &\triangleq (I + G_sF)^{-1}H_s \end{aligned}$$

- Estimate $\bar{G}$ and $\bar{H}$ from $y(t)$ and $v(t)$ using a Prediction Error Method.
- From the estimated $\hat{\bar{G}}$ and $\hat{\bar{H}}$ form the estimates $\hat{H}$ and $\hat{G}$.
- Identifiability conditions: Same as that for direct method.
- Same identifiability properties do not mean that direct and indirect methods give same results.
- Drawbacks of indirect method: Need to know $v(t)$ and the regulators.

Joint Input-Output Identification

- Regard $y(t)$ and $u(t)$ as outputs from a multivariable system driven by white noise

$$\begin{bmatrix} y(t) \\ u(t) \end{bmatrix} = \begin{bmatrix} \mathcal{H}_{11}(q^{-1};\boldsymbol{\theta}) & \mathcal{H}_{12}(q^{-1};\boldsymbol{\theta}) \\ \mathcal{H}_{21}(q^{-1};\boldsymbol{\theta}) & \mathcal{H}_{22}(q^{-1};\boldsymbol{\theta}) \end{bmatrix} \begin{bmatrix} e(t) \\ v(t) \end{bmatrix}$$

- Innovations model: $z(t) \triangleq [y(t)\, u(t)]^T$

$$z(t) = \mathcal{H}(q^{-1};\boldsymbol{\theta})\bar{e}(t)$$

$$E\bar{e}(t)\bar{e}^T(s) = \Lambda_{\bar{e}}(\boldsymbol{\theta})\delta_{t,s}$$

- Use PEM to identify $\mathcal{H}$ and $\Lambda_{\bar{e}}$.

Properties:

- Same identifiability conditions as for the direct method.
- The system and the regulator can be identified.
- The spectral characteristics of $v(t)$ can also be identified.
- The drawback is an increased computational complexity.

Conclusions

- Feedback makes the identification procedure more difficult.
- Three parametric approaches (based on PEM) to identify systems operating in a closed loop:
  1. Direct approach.
  2. Indirect approach.
  3. Joint input-output approach.
- Identifiability under weak conditions.
- From a computational point of view, the direct approach is the simplest.