System Identification, Lecture 8

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Lecture 8

- Identification of Closed Loop Systems (Ch.10)

Feedback

Consider the system:

$$\begin{cases} y(t) = G(q^{-1})u(t) + H(q^{-1})e(t) \\ u(t) = -F(q^{-1})y(t) + L(q^{-1})v(t) \end{cases}$$

where

- The input $u(t)$ is determined through feedback.
- F and L are called regulators.
- The signal $v(t)$ can be the reference signal or noise entering the regulator.

Why?

- Many realworld systems have feedback.
- The open-loop system is unstable.
- Feedback is required due to safety reasons.

What Happens in a Closed Loop Experiment?

- The input $u(t)$ depends on past $y(t)$ (and hence on past $e(t)$).

- The aim of control is to apply a $u(t)$ which minimizes the deviation between $y(t)$ and a reference signal $v(t)$. Good control often requires a $u(t)$ of bounded energy.

- SI requires PE, hence substantial energy of $u(t)$.

- The frequency content of $u(t)$ is limited by the true system.

An example

System:

$$\begin{cases} y(t) + ay(t-1) = bu(t-1) + e(t), \;\; E[e^2(t)] = \lambda^2 \\ u(t) = -fy(t) \end{cases}$$

Model structure:

$$y(t) + \hat{a}y(t-1) = \hat{b}u(t-1) + \epsilon(t)$$

Estimate by PEM

$$\begin{cases} \hat{a} = a + f\gamma \\ \hat{b} = b - \gamma \end{cases}$$

where γ is any scalar. There is no unique solution, hence the parameters are not estimated consistently.

Closed-loop behavior

Open-loop system:

$$\begin{cases} y(t) = G(q^{-1})u(t) + H(q^{-1})e(t) \\ u(t) = -F(q^{-1})y(t) + L(q^{-1})v(t). \end{cases}$$

Closed loop system:

$$\begin{cases} y(t) = (I + GF)^{-1}GLv(t) + (I + GF)^{-1}He(t) \\ u(t) = \left(L - (I + GF)^{-1}GL\right) v(t) - F(I + GF)^{-1}He(t). \end{cases}$$

Some assumptions

- The open loop system is strictly proper: $y(t)$ depends only on past values of the input $u(s)$ or $s < t$.
- The closed loop system is asymptotically stable.
- The external signal $v(t)$ is stationary and PE of sufficiently high order.
- The external signal $v(t)$ and the disturbance $e(s)$ are independent $\forall s, t$.

Prediction Error Methods

- In most cases it is not necessary to assume that the external signal $v(t)$ is measurable.
- Gives statistically efficient estimates under mild conditions.
- Computationally demanding.

Notation $\hat{G}$ denotes $G(q^{-1}, \hat{\theta})$.

Different Approaches

- *Direct Identification*. Feedback is neglected during identification - the system is treated as an open loop system.
- *Indirect Identification*. It is assumed that $v(t)$ is measured and the feedback law is known. First the closed loop behavior is modeled, then the open-loop system is identified by 'subtracting' the effect of the regulators from this model.
- *Joint Identification*. The signals $u(t)$ and $y(t)$ are both considered as the outputs of a multivariate system driven by white noise.

Direct Identification

Model structure:

$$\begin{cases} y(t) = Gu(t) + He(t) \\ E[e^2(t)] = \lambda^2 \end{cases}$$

Use the signals $(u(t))_t$ and $(y(t))_t$

Goal: estimate (SISO)

$$\begin{cases} \hat{\theta} = \operatorname{argmin}_\theta V_N(\theta) \\ V_N(\theta) = \frac{1}{N}\sum_{t=1}^N \epsilon^2(t,\theta) \\ \epsilon(t,\hat{\theta}) = \hat{H}^{-1}\Big(y(t) - \hat{G}u(t)\Big) \end{cases}$$

Question: Identifiability? Desired solution $\hat{G} = G$ and $\hat{H} = H$.

Consistency: Analyze the asymptotic cost function:

$$V(\theta) = \lim_{N\to\infty} V_N(\theta) = E[\epsilon(t,\theta)]$$

- Will $\hat{G} = G$ and $\hat{H} = H$ be a global minimum to $V(\theta)$ (system identifiability)?
- Is the solution $\hat{G} = G$ and $\hat{H} = H$ unique (parameter identifiability)?

An Example

System:

$$y(t) + ay(t-1) = bu(t-1) + e(t),\ E[e^2(t)] = \lambda^2$$

Model structure:

$$y(t) + \hat{a}y(t-1) = \hat{b}u(t-1) + \epsilon(t)$$

Input

$$u(t) = \begin{cases} -f_1 y(t) & \text{for a fraction } \gamma_1 \text{ of the total time.} \\ -f_2 y(t) & \text{for a fraction } \gamma_2 \text{ of the total time.} \end{cases}$$

Then (for $i = 1, 2$) we get

$$\begin{cases} y_i(t) + (a + bf_i)y_i(t-1) = e(t) \\ y_i(t) + (\hat{a} + \hat{b}f_i)y_i(t-1) = \epsilon_i(t) \end{cases}$$

which gives

$$
\begin{aligned}
V(\hat{a}, \hat{b}) &= \gamma_1 E[\epsilon_1^2(t)] + \gamma_2 E[\epsilon_2^2(t)] \\
&= \lambda^2 + \gamma_1 \lambda^2 \frac{(\hat{a} + \hat{b} f_1 - a - b f_1)^2}{1 - (a + b f_1)^2} \\
&\quad + \gamma_2 \lambda^2 \frac{(\hat{a} + \hat{b} f_2 - a - b f_2)^2}{1 - (a + b f_2)^2}
\end{aligned}
$$

Consequently

$$V(\hat{a}, \hat{b}) \geq \lambda^2 = V(a, b)$$

we get

- A global minimum is obtained if $\hat{a} = a$ and $\hat{b} = b$
- Unique minimum?
- Solve $V(\hat{a}, \hat{b}) = \lambda^2$

$$
\begin{bmatrix} 1 & f_1 \\ 1 & f_2 \end{bmatrix} \begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = \begin{bmatrix} a + b f_1 \\ a + b f_2 \end{bmatrix}
$$

- Unique solution if and only if $f_1 \neq f_2$ (Compare to our previous example).

The General Case

- The desired solution $\hat{G} = G$ and $\hat{H} = H$ will be a global minimum to $V(\theta)$
- Unique global minimum is necessary for parameter identifiability (consistency). Consistency is assured by
  - Using an external input signal $v(t)$
  - Using a regulator $F(q^{-1})$ that shifts between different settings during the experiment.

Indirect Identification

- Two step approach
  1. **Step 1** Identify the closed loop system using $v(t)$ as input and $y(t)$ as output.
  2. **Step 2** Determine the open loop system parameters from the closed loop parameters, *using knowledge of the feedback F and L*.

- Closed-loop system:

$$y(t) = \bar{G}v(t) + \bar{H}e(t)$$

where

$$\begin{cases} \bar{G} = (I + GF)^{-1}GL \\ \bar{H} = (I + GF)^{-1}H \end{cases}$$

- Estimate $\bar{G}$ and $\bar{H}$ from $v(t)$ and $y(t)$ with a PEM.

- From the estimated $\bar{G}$ and $\bar{H}$, form the $\hat{G}$ and $\hat{H}$

- Identifiability conditions are the same as for the direct approach.
- Same identifiability conditions do not imply that both direct as indirect approach give the same result.
- Drawback of indirect approach: one needs to know $v(t)$ and the regulators.

Joint input-output identification.

- Regard $u(t)$ and $y(t)$ as outputs from a multivariable system, driven by white noise and the reference input $v(t)$.

$$\begin{cases} y(t) = H_{11}(q^{-1,\theta})e(t) + H_{12}(q^{-1,\theta})v(t) \\ u(t) = H_{21}(q^{-1,\theta})e(t) + H_{22}(q^{-1,\theta})v(t) \end{cases}$$

- Innovations model: let $z(t) = \Big(y(t), u(t)\Big)^T$, then

$$z(t) = \mathbf{H}(q^{-1}, \theta)\bar{e}(t)$$

with $E[\bar{e}(s)\bar{e}^T(t)] = \Lambda_{\bar{e}}(\theta)\delta_{t,s}$.

- Use PEM to identify θ in $\mathbf{H}$ and $\Lambda_{\bar{e}}$.

Properties

- Same identifiability conditions as for the direct method.
- Both system and the regulator can be identified.
- The spectral characterization of $v(t)$ can be identified;
- the drawback is the computational demand.

Conclusions

- Feedback makes identification more difficult.
- Three (parametric) strategies based on PEM:
  - Direct.
  - Indirect.
  - Joint input-output.
- Identifiability under weak conditions.
- From a computational point-of-view, the direct approach is the simplest one.