System identification and sensor fusion in dynamical systems

UPPSALA
UNIVERSITET

Thomas Schön
Division of Systems and Control,
Uppsala University,
Sweden.

The system identification and sensor fusion problem

- Inertial sensors
- Camera
- Barometer

- Inertial sensors
- Radar
- Barometer
- Map

- Inertial sensors
- Cameras
- Radars
- Wheel speed sensors
- Steering wheel sensor

- Inertial sensors
- Ultra-wideband

How do we combine the information from the different sensors?

Might all seem to be very different problems at first sight. However, the same strategies can be used in dealing with all of these applications (and many more).

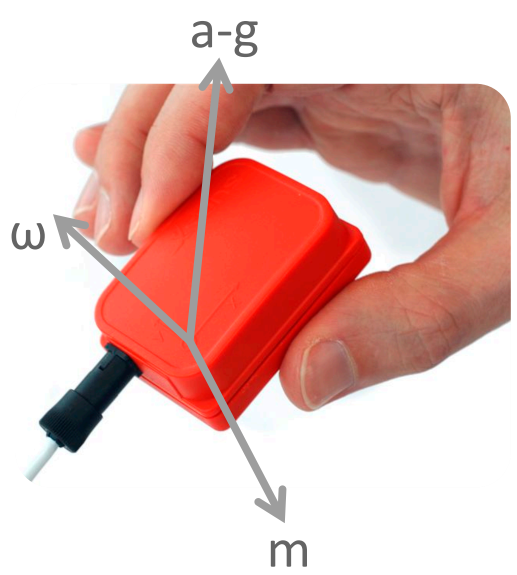
Introductory example (I/III)

Aim: Motion capture, find the motion (position, orientation, velocity and acceleration) of a person (or object) over time.

Industrial partner: Xsens Technologies.

Sensors used:

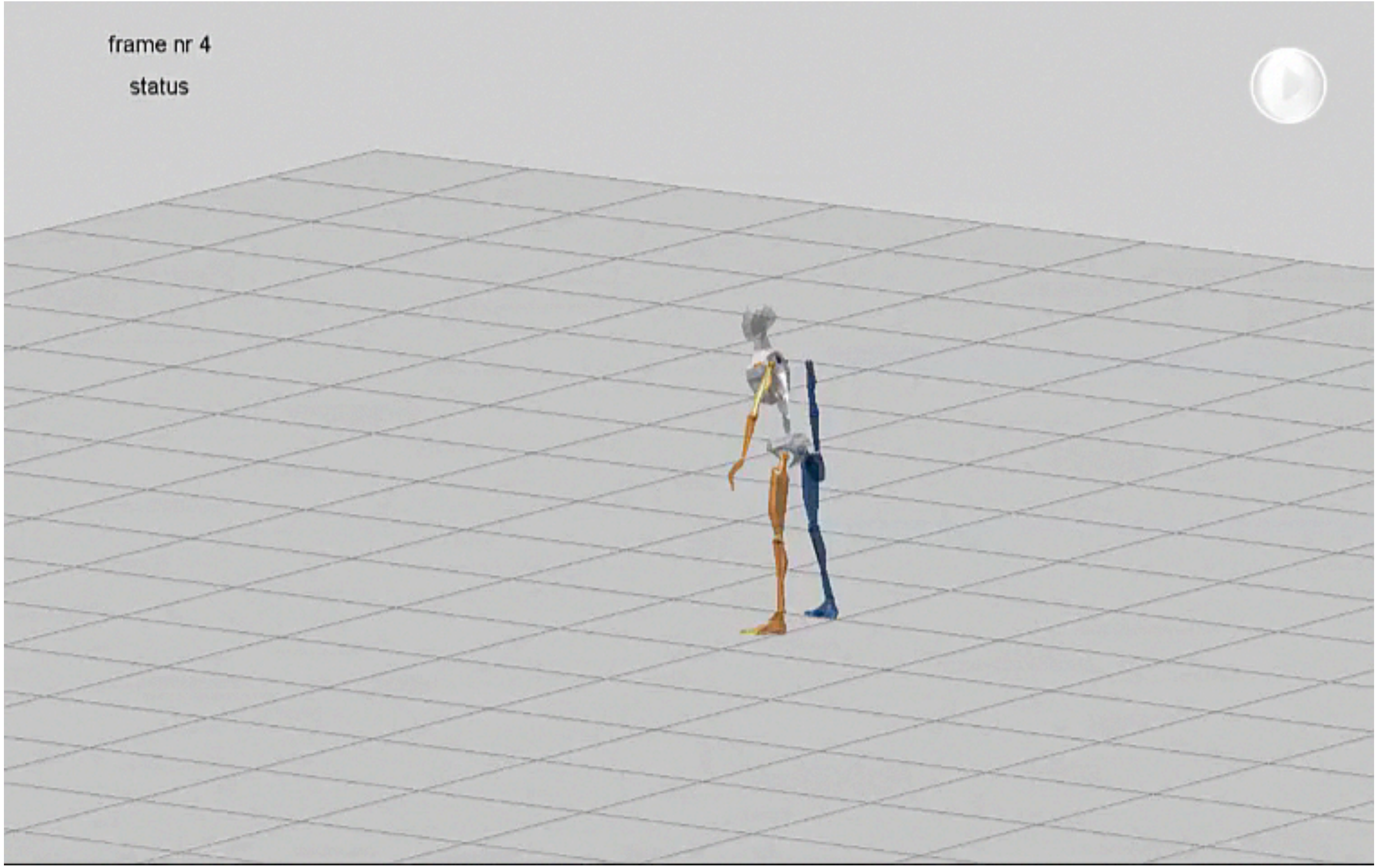
- 3D accelerometer (acceleration)
- 3D gyroscope (angular velocity)
- 3D magnetometer (magnetic field)



17 sensor units are mounted onto the body of the person.

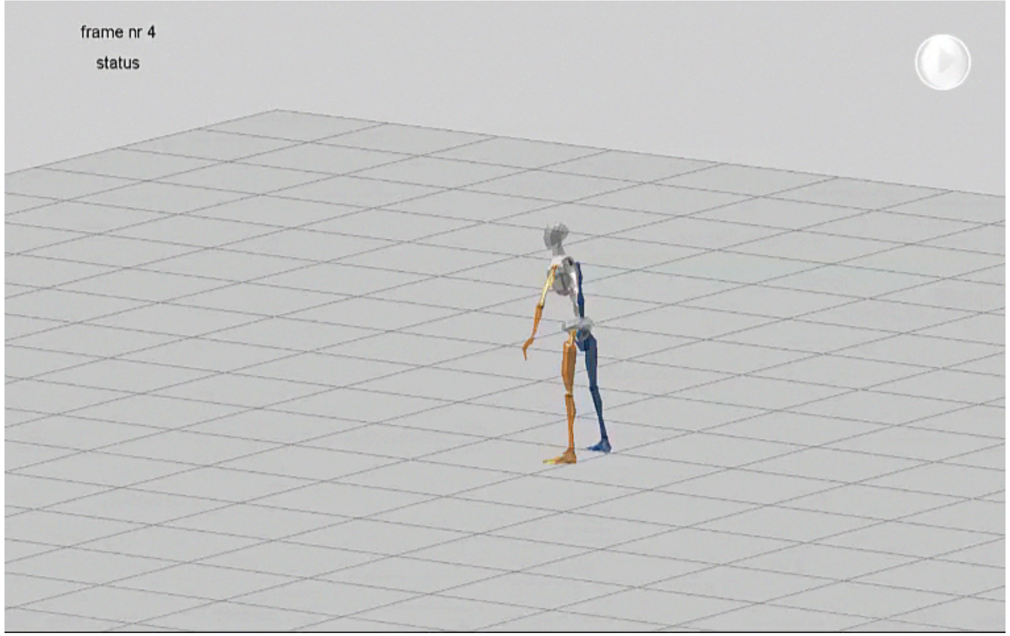
Introductory example (II/III)

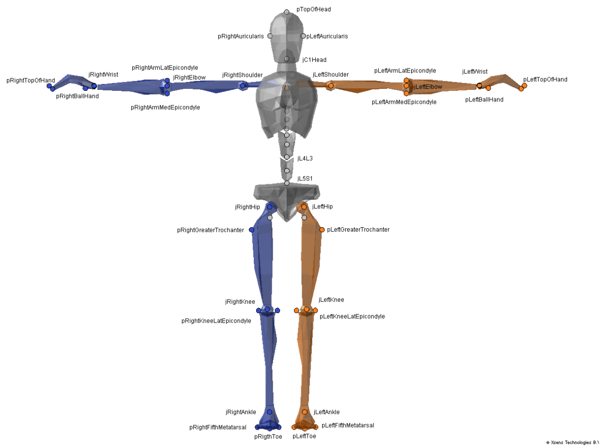
1. Only making use of the inertial information.



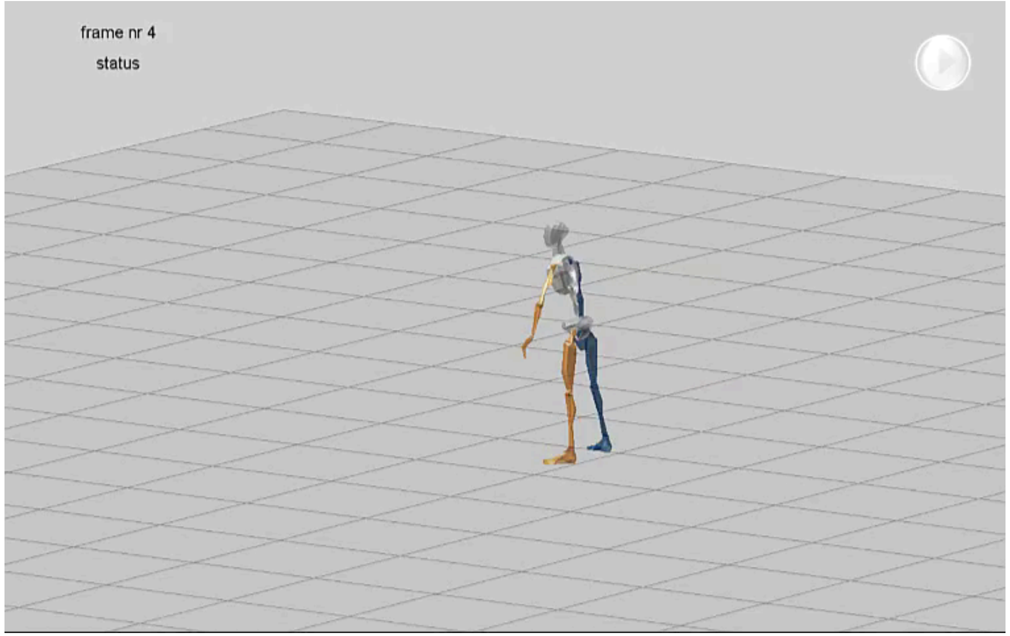
Introductory example (III/III)

2. Inertial + biomechanical model





3. Inertial + biomechanical model + world model



Outlook

These introductory examples leads to several questions, e.g.,

- Can we incorporate more sensors?
- Can we make use of more informative world models?
- How do we solve the inherent inference problem?
- Perhaps most importantly, can this be solved systematically?

There are many interesting problems that can be solved systematically, by addressing the following problem areas

Sensor fusion

1. Probabilistic models of dynamical systems (sys. id.)
2. Probabilistic models of sensors and the world
3. Formulate and solve the state inference problem
4. Surrounding infrastructure

1. Probabilistic models of dynamical systems

Basic representation: Two discrete-time stochastic processes,

- $\{x_t\}_{t \geq 1}$ representing the state of the system
- $\{y_t\}_{t \geq 1}$ representing the measurements from the sensors

The probabilistic model is described using two (f and g) probability density functions (PDFs):

State ↓ x_{t+1}; Known input ↓ u_t

$$x_{t+1} \mid x_t \sim f_\theta(x_{t+1} \mid x_t, u_t), \quad \text{Dynamics}$$

Measurements → y_t

$$y_t \mid x_t \sim g_\theta(y_t \mid x_t, u_t). \quad \text{Measurements}$$

Static parameters ↑ θ

Model = PDF

This type of model is referred to as a **state space model (SSM)** or a **hidden Markov model (HMM)**.

The dynamical systems exist in a context.

This requires a **world model**.

Valuable (indeed often necessary) source of information in computing situational awareness. There are more and more complex world models being built all the time.

An example is our new models of the magnetic contents in various objects, which opens up for interesting new possibilities....

(a) Estimated shape of table (b) Real shape of table

Fig. 1: Estimated magnetic content in a table turned upside down.

Some early results:

Niklas Wahlström, Manon Kok, Thomas B. Schön and Fredrik Gustafsson. **Modeling magnetic fields using Gaussian processes**. *Proceedings of the 38th International Conference on Acoustics, Speech, and Signal Processing (ICASSP)*, Vancouver, Canada, May 2013.

Manon Kok, Niklas Wahlström, Thomas B. Schön and Fredrik Gustafsson. **MEMS-based inertial navigation based on a magnetic field map**. *Proceedings of the the 38th International Conference on Acoustics, Speech, and Signal Processing (ICASSP)*, Vancouver, Canada, May 2013.

3. Formulate and solve the inference problem

The inference problem amounts to **combining** the knowledge we have from the models (dynamic, world, sensor) and from the measurements.

The **aim** is to compute

$$p(x_{1:t}, \theta \mid y_{1:t})$$

and/or some of its marginal densities,

$$p(x_t \mid y_{1:t})$$
$$p(\theta \mid y_{1:t})$$

These densities are then commonly used to form point estimates, **maximum likelihood** or **Bayesian**.

- Everything we do rests on a firm foundation of probability theory and mathematical statistics.
- If we have the wrong model, there is no estimation/learning algorithm that can help us.

3. Inference - the filtering problem

$$p(x_t \mid y_{1:t}) = \frac{\overbrace{p(y_t \mid x_t)}^{\text{sensor model}} \overbrace{p(x_t \mid y_{1:t-1})}^{\text{prediction density}}}{p(y_t \mid y_{1:t-1})}$$

$$p(x_{t+1} \mid y_{1:t}) = \int \underbrace{p(x_{t+1} \mid x_t)}_{\text{dynamical model}} \underbrace{p(x_t \mid y_{1:t})}_{\text{filtering density}} dx_t$$

In the application examples these equations are solved using particle filters (PF), Rao-Blackwellized particle filters (RBPF), extended Kalman filters (EKF) and various optimization based approaches.