ODEnum 2020, problem set 3

A. Dq notes: problem 2, p114
B. Order reduction: Prove that the stage order of an SDIRK method is as most 1, and of a DIRK method at most 2. (See Hairer, Wanner IV.15)

C. Consider a system of ODEs *y'=L(y),* where y is a vector with *s* components v_i, *i=1,… s*. Assume the Euler forward discretization of the system $u^{n+1}= u^n+hL(u^n)$ has the property that for all $h \le h_E$ the solution satisfies $TV(u^{n+1}) \le TV(u^n)$, where $TV(v) = \sum_{1 \le j \le s-1} |v_{j+1} - v_j|$ denotes the total variation of the vector *v.* A one-step method is called strong stability preserving (SSP) if there exists a *C*>0 such that the solution at consecutive time levels satisfies $TV(u^{n+1}) \le TV(u^n)$ for all $h \le C h_E$. Consider the following 2 second order Runge-Kutta methods, sometimes denoted Heun's and Ralston's methods, respectively.

$$y^{n+1} = y^n + \frac{h}{2}\left(L(y^n) + L\left(y^n + hL(y^n)\right)\right)$$

$$y^{n+1} = y^n + \frac{h}{4}\left(L(y^n) + 3L\left(y^n + \frac{2h}{3}L(y^n)\right)\right)$$

Are these methods SSP under some restriction $h \le C h_E$?
Read about SSP methods in *Gottlieb, Shu, Tadmore*, Strong Stability-Preserving High-Order Time Discretization Methods, SIAM Rev., 43(1), 89–112