A note on models for spatial logic based on transition systems with spatial structure*

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Abstract

In this note we describe models for a simple spatial logic $\mathcal{L}$ whose only spatial connectives are inaction and parallel composition. The models are transition systems with an additional structure on the states to interpret the spatial operators. Both the transition and the spatial structures are described in coalgebraic terms, since interpreting the logic requires observing the structure of states rather than building new ones. The corresponding notion of bisimulation takes into account both structures. We show that bisimilarity is equivalent to logical equivalence, thus extending Hennessy-Milner's result to the present framework. We discuss the possible role of $\mathcal{L}$ in characterizing classes of models and show that the spatial operators are derived operators in a more primitive coalgebraic modal logic.

1 Introduction

There has been recently a growing interest in logics that support in a integrated way both behavioural and so-called spatial properties of concurrent systems [1, 3, 4, 6, 15, 17]. The models for spatial logic that have been considered so far have been mostly for concrete domains like the ambient calculus [6], the asynchronous [3] or synchronous [2] π-calculus, semistructured data [5] and mutable data structures [17], among others. In this note we propose a general family of models consisting of transition systems whose states has been endowed with a structure intended to capture their spatiality in a broad sense. The traditional approach has been to consider the set of states as an algebra for an appropriate set of operators (e.g. [11, 7]). Instead, we propose to treat space in coalgebraic terms, the main reason being that for the applications we have in mind we need to observe the structure of given states rather then using them to construct new states (for coalgebras see [10, 19, 9]). Furthermore, this way we have a uniform treatment of space and time, since the dynamics of transition systems is naturally described in coalgebraic terms. This decision has important consequences. For example, the notion of bisimulation associated with the models

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takes into account both the spatial and the transition structures, and so is finer than the bisimulation associated with just the underlying transition structure. This provides an explanation for the observation made by several researchers that spatial logic has an intensional character, in that it discriminates between states that are behaviourally equivalent. A somewhat unexpected outcome of this approach is that such transition systems with spatial structure can be seen as a noninterleaving model of concurrency [14].

In this note we consider a very simple spatial logic, which is basically Hennessy-Milner logic [13] with the spatial operators 0 (*void*, nullary) and | (*composition*, binary): 0 is satisfied by any inactive process and $A|B$ is satisfied by any process which is a parallel composition of a process satisfying A with a process satisfying B. It turns out that to interpret these operators we only need to describe the spatial structure on the set S of states by a "spatial function" $\mathrm{sp} : S \to \mathcal{P}_f(S \times S)$, where $\mathcal{P}_f$ is the finite powerset functor; a pair (s', s'') in $\mathrm{sp}(s)$ means that s can be observed as a parallel composition of s' with s''; if $\mathrm{sp}(s)$ is empty, s is inactive. We then say that s satisfies 0 if s is inactive, and s satisfies $A|B$ if s can be observed as a parallel composition of a state satisfying A with a state satisfying B.

In this note we show for the logic and models briefly outlined above that bisimilarity is equivalent to logical equivalence, thus extending Hennessy-Milner's result to the present framework. We also discuss the possible role of the logic in characterizing particular classes of models and show that the spatial operators are derived operators in a more primitive coalgebraic modal logic. We end with some suggestions for future work.

2 A simple spatial logic

We consider in this note a very modest spatial logic comprising only the spatial operators of inaction and parallel composition. Our logic $\mathcal{L}$ is the set of formulas described by the following syntax:

$$A, B ::= \top \mid \neg A \mid A \wedge B \mid \Diamond A \mid 0 \mid A|B$$

Thus, we have the propositional constants $\top$ for true, $\neg$ for negation and $\wedge$ for conjunction. The modality $\Diamond$ is associated with an internal transition (for simplicity we disregard labelled transitions). The remaining two formulas are the spatial formulas. The formula 0 expresses inaction, and $A|B$ a parallel combination of a process satisfying A with a process satisfying B. This small logic may be understood a mild extension of a Hennessy-Milner-like logic (apart from the absence of labels in the transition modality) with two basic spatial operators. We write $\mathcal{L}_{\mathrm{tr}}$ for the logic without the spatial operators and $\mathcal{L}_{\mathrm{sp}}$ for the logic without the temporal modality.

Labelled transition systems with spatial structure

The models of $\mathcal{L}$ we consider here are *labelled transition systems with spatial structure*, which consist of a set S together with functions

$$\mathrm{tr} : S \to \mathcal{P}_f(S),$$
$$\mathrm{sp} : S \to \mathcal{P}_f(S \times S),$$

where $\mathcal{P}_f$ is the finite powerset functor. These functions are intended to describe the transition and the spatial structures of the system, respectively. The pair $\langle S, \text{tr} \rangle$ is an ordinary (unlabelled, for simplicity) transition system, to which a spatial structure has been added; we put as usual $s \to t$ iff $t \in \text{tr}(s)$. The pairs in $\text{sp}(s) \subseteq S \times S$ are the decompositions of s into parallel components. In the particular case where $sp(s) = \emptyset$, the state s is considered to be inactive.

It is reasonable to expect that these data obey some conditions, for example to capture the idea that parallel composition is commutative or to relate the transition and the spatial structures. Such conditions will be discussed later.

Satisfaction

Given the logic $\mathcal{L}$ and a lts with space $\langle S, tr, sp \rangle$, we define a *satisfaction* relation $\models \subseteq \mathcal{L} \times S$ by the following clauses:

- $s \models \top$ always.
- $s \models \neg A$ iff $s \not\models A$.
- $s \models A \wedge B$ iff $s \models A$ and $s \models B$.
- $s \models \Diamond A$ iff $\exists t : s \to t$ and $t \models A$.
- $s \models 0$ iff $sp(s) = \emptyset$.
- $s \models A|B$ iff $\exists (s', s'') \in sp(s) : s' \models A$ and $s'' \models B$.

As an example, the formula $A \triangleq \neg 0 \wedge \neg(\neg 0 | \neg 0)$ describes a state that is neither inactive nor a parallel composition of non-inactive states. We have $s \models A$ iff $\text{sp}(s) \neq \emptyset$ and whenever $(s', s'') \in \text{sp}(s)$, then $\text{sp}(s') = \emptyset$ or $\text{sp}(s') = \emptyset$.

3 Bisimilarity and logical equivalence

Bisimulation

In a lts with space $\langle S, tr, sp \rangle$, a *bisimulation* is a symmetric relation $R \subseteq S \times S$ such that sRt implies:

- For all $s \to s'$, there is $t \to t'$ such that $s'Rt'$.
- For all $(s', s'') \in sp(s)$, there is $(t', t'') \in sp(t)$ such that $s'Rt'$ and $s''Rt''$.

Bisimilarity $\sim$ is the greatest bisimulation.

If we take into account only the transition condition, we obtain the standard notions of bisimulation and bisimilarity in lts's, which in this context we call *transition bisimulation* and *transition bisimilarity.* Likewise, by considering only the spatial condition we obtain *spatial bisimulation* and *spatial bisimilarity.* We denote the two particular bisimilarities by $\sim_{tr}$ and $\sim_{sp}$.

Logical equivalence

Given a lts with space $\langle S, tr, sp\rangle$ and $s, t \in S$, we write

$$s =_{\mathcal{L},S} t$$

or just $s =_{\mathcal{L}} t$ iff $\{A \in \mathcal{L} : s \models A\} = \{A \in \mathcal{L} : t \models A\}$. Similar equivalences are defined by considering the logics $\mathcal{L}_{\mathrm{tr}}$ and $\mathcal{L}_{\mathrm{sp}}$. In the transition case, it is known that $\sim_{tr}$ coincides with $=_{\mathcal{L}_{\mathrm{tr}}}$, in the sense that $s \sim_{tr} t$ iff $s =_{\mathcal{L}_{\mathrm{tr}}} t$ for all $s, t \in S$. We next proceed to prove that the same is true for the purely spatial case. It is interesting to note that the proof technique in the spatial case is similar to the standard proof for the transition case, which is one of the advantages of treating behaviour and structure in the same framework.

Spatial bisimilarity as logical equivalence

Lemma 3.1 *$=_{\mathcal{L}_{sp}}$ is a spatial bisimulation, hence is contained in $\sim_{sp}$.*

Proof. The relation $=_{\mathcal{L}_{sp}}$ is clearly symmetric. Now suppose $s =_{\mathcal{L}_{sp}} t$ for some $s, t \in S$. Given $(s', s'') \in sp(s)$, we must find $(t', t'') \in sp(t)$ such that $s' =_{\mathcal{L}_{sp}} t'$ and $s'' =_{\mathcal{L}_{sp}} t''$, that is,

$$\{A \in \mathcal{L}_{sp} : s' \models A\} = \{A \in \mathcal{L}_{sp} : t' \models A\},$$
$$\{A \in \mathcal{L}_{sp} : s'' \models A\} = \{A \in \mathcal{L}_{sp} : t'' \models A\}.$$

If $s' \models A'$ and $s'' \models A''$, then $s \models A'|A''$. As $s =_{\mathcal{L}_{sp}} t$, we have $t \models A'|A''$, hence there exists $(t', t'') \in sp(t)$ such that $t' \models A'$ and $t'' \models A''$. In principle, the pair (t', t'') depends on the formulas A' and A''. Assume we have shown that there exists a pair (t', t'') that satisfies the required condition for all choices of A' and A''. With this hypothesis, we can prove that $s' =_{\mathcal{L}_{sp}} t'$ and $s'' =_{\mathcal{L}_{sp}} t''$. Let us suppose that $s' \models A'$. As $s'' \models \top$, we have $t' \models A'$ and $t'' \models \top$. Conversely, if $t' \models A'$ but $s' \not\models A'$, then $s' \models \neg A'$ hence, by what has just been proved, $t' \models \neg A'$, a contradiction. Thus, s' and t' satisfy the same formulas, therefore, $s' =_{\mathcal{L}_{sp}} t'$. Similarly, $s'' =_{\mathcal{L}_{sp}} t''$.

To finish the proof we only need show that there exists a pair (t', t'') such that for all A' and A'', if $s' \models A'$ and $s'' \models A''$, then $t' \models A'$ and $t'' \models A''$. As $sp(t)$ is finite, there exists a finite set $\{(t'_1, t''_1), \ldots (t'_n, t''_n)\} \subseteq sp(t)$ such that if $s' \models A'$ and $s'' \models A''$, then $t'_i \models A'$ and $t''_i \models A''$ for some i. Reasoning by contradiction, let us suppose that for every i it was possible to find formulae A'_i and A''_i such that $s' \models A'_i$ and $s'' \models A''_i$ but $t'_i \not\models A'_i$ or $t''_i \not\models A''_i$. Forming the conjunctions $A' \triangleq A'_1 \wedge \cdots \wedge A'_n$ and $A'' \triangleq A''_1 \wedge \cdots \wedge A''_n$, we have $s' \models A'$ and $s'' \models A''$. By hypothesis, there exists i such that $t'_i \models A'$ and $t''_i \models A''$. But this implies $t'_i \models A'_i$ and $t''_i \models A''_i$, contradicting our assumption. The assumption must then be false, which means that there exists i such that for all formulas A' and A'' such that $s' \models A'$ and $s'' \models A''$, we have $t'_i \models A'$ and $t''_i \models A''$. □

To prove the converse we introduce the approximations relations $\sim_{sp,n}$, defined inductively for all $n \geq 0$ as follows:

- $s \sim_{sp,0} t$ always.
- $s \sim_{sp,n+1} t$ iff for all $(s', s'') \in sp(s)$, there exists $(t', t'') \in sp(t)$ such that $s' \sim_{sp,n} t'$ and $s'' \sim_{sp,n} t''$, and for all $(t', t'') \in sp(t)$, there exists $(s', s'') \in sp(s)$ such that $s' \sim_{sp,n} t'$ and $s'' \sim_{sp,n} t''$.

Note that $m \geq n$ implies $\sim_{sp,m} \subseteq \sim_{sp,n}$.

Lemma 3.2 *$s \sim_{sp} t$ iff $s \sim_{sp,n} t$ for all n.*

Proof. Let $\approx$ be the intersection of the $\sim_{sp,n}$. It is easy to see that $\sim_{sp}$ is contained in $\approx$. To prove the converse, we show that $\approx$ is a spatial bisimulation. Clearly, $\approx$ is symmetric. Assume $s \approx t$ and let $(s', s'') \in sp(s)$. For every n, there exists $(t'_n, t''_n) \in sp(t)$ such that $s' \sim_{sp,n} t'_n$ and $s'' \sim_{sp,n} t''_n$. Since $sp(t)$ is finite, one such pair at least, let us call it (t', t''), is used infinitely many times, which means that in fact it can be used for all n since the relations $\sim_{sp,n}$ are decreasing. But this implies $s' \approx t'$ and $s'' \approx t''$, as required. □

Lemma 3.3 *$\sim_{sp}$ is contained in $=_{\mathcal{L}_{sp}}$.*

Proof. The "depth" $d(A)$ of a formula A is defined inductively by the following cases:

- $d(\top) = 0$.
- $d(\neg A) = d(A)$.
- $d(A \wedge B) = \max\{d(A), d(B)\}$.
- $d(0) = 1$.
- $d(A|B) = 1 + \max\{d(A), d(B)\}$.

Let $\mathcal{L}_{sp,n}$ be the set of formulas $A \in \mathcal{L}_{sp}$ such that $d(A) \leq n$. Put $s =_{\mathcal{L}_{sp,n}} t$ iff $\{A \in \mathcal{L}_{sp,n} : s \models A\} = \{A \in \mathcal{L}_{sp,n} : t \models A\}$. Clearly, $s =_{\mathcal{L}_{sp}} t$ iff $s =_{\mathcal{L}_{sp,n}} t$ for all $n \geq 0$. To prove that $\sim_{sp} \subseteq =_{\mathcal{L}_{sp}}$ we only need to show that $\sim_{sp,n} \subseteq =_{\mathcal{L}_{sp,n}}$ for all n.

For $n = 0$, we have $s \sim_{sp,0} t$ for all s and t. As $\{A \in \mathcal{L}_{sp,0} : s \models A\}$ is the set of propositional tautologies for all s, we conclude that $s =_{\mathcal{L}_{sp,0}} t$. Assume that the result has been established for n and suppose that $s \sim_{sp,n+1} t$. As we also have $s \sim_{sp,n} t$, the inductive hypothesis implies that s and t satisfy the same formulas of depth $\leq n$, so we only have to check that they satisfy the same formulas of depth $n + 1$. If $d(A) = n + 1$ and $s \models A$, we will show that $t \models A$; the converse is similar. In the case $n = 0$ we may have $A = 0$, which implies $sp(s) = \emptyset$. In that case also $sp(t) = \emptyset$, since $s \sim_{sp,1} t$, hence $t \models 0$. If A has the form $A'|A''$, there exists $(s', s'') \in sp(s)$ such that $s' \models A'$ and $s'' \models A''$. By the hypothesis $s \sim_{sp,n+1} t$, there exists $(t', t'') \in sp(t)$ such that $s' \sim_{sp,n} t'$ and $s'' \sim_{sp,n} t''$. By inductive hypothesis, $t' \models A'$ and $t'' \models A''$, since $d(A') \leq n$ and $d(A'') \leq n$. It follows that $t \models A'|A''$. In the case in which A is a propositional combination of the two previous cases, the proof is by induction on the number of propositional connectives in A, taking the cases just considered as the base cases. □

Theorem 3.4 *$\sim_{sp}$ coincides with $=_{\mathcal{L}_{sp}}$.*

The general case

Combining the transition and the spatial cases we obtain:

Theorem 3.5 *$\sim$ coincides with $=_{\mathcal{L}}$.*

We omit the details.

4 Classes of models described by axioms

As noted above, concrete instances of lts's with space satisfy certain properties that we might like to introduce as axioms to characterize classes of models. For example, we might require that whenever $(s', s'') \in \mathrm{sp}(s)$ it is also the case that $(s'', s') \in \mathrm{sp}(s)$, so as to capture observationally the commutativity of parallel composition. Now in a coalgebraic framework we are not supposed to observe states directly, so we should only mention states up to bisimilarity. The commutativity conditions should then be rephrased as follows: If $(s', s'') \in \mathrm{sp}(s)$, there exist $t'' \sim s''$ and $t' \sim s'$ such that $(t'', t') \in \mathrm{sp}(s)$. The interesting thing is that this formulation allows us to express commutativity with the logic $\mathcal{L}$.

Lemma 4.1 *Let $\langle S, \mathrm{tr}, \mathrm{sp}\rangle$ be a lts with space and $s \in S$. The following statements are equivalent:*

1. *If $(s', s'') \in \mathrm{sp}(s)$, there exist $t'' \sim s''$ and $t' \sim s'$ such that $(t'', t') \in \mathrm{sp}(s)$.*
2. *$s \models (A|B) \Rightarrow (B|A)$ for all formulas A and B, where $\Rightarrow$ is the implication sign.*

Proof. (*1* $\Longrightarrow$ *2*) This is immediate, because bisimilar states satisfy the same formulas.

(*2* $\Longrightarrow$ *1*) Suppose $(s', s'') \in \mathrm{sp}(s)$. If A' and A'' are two formulas such that $s' \models A'$ and $s'' \models A''$, then $s \models A'|A''$, hence $s \models A''|A'$, so there is $(t'', t') \in \mathrm{sp}(s)$ such that $t'' \models A''$ and $t' \models A'$. In principle, the states t', t'' depend on the formulas A', A'', but as in the proof of Lemma 3.1, we can show that it is possible to choose t', t'' so that $s' \models A'$ and $s'' \models A''$ imply $t' \models A'$ and $t'' \models A''$ for all A', A''. As in the same proof, we conclude that $s' \sim t'$ and $s'' \sim t''$. This ends the proof. □

We state additional examples of common properties without further comment:

- $A|0 \Leftrightarrow A$ (inactive states are neutral with respect to parallel composition).
- $A|B \Rightarrow B|A$ (parallel composition is commutative).
- $(A|B)|C \Leftrightarrow A|(B|C)$ (parallel composition is associative).
- $0 \Rightarrow \neg\Diamond\top$ (inactive states have no transitions).
- $(\Diamond A)|B \Rightarrow \Diamond(A|B)$ (local transitions cause global transitions).

5 Relation with modal logics for coalgebras

A benefit of the coalgebraic models of spatial logic is that there are very general ways to associate modal logics with coalgebras. The question arises: What is the relationship of spatial logic to those modal logics? We next show that the fragment of spatial logic that we have been considering can be obtained by introducing appropriate "spatial modalities" in terms of which the spatial connectives can be defined as derived operators. For convenience, we follow here the approach proposed by Rößiger (see e.g. [18]; an alternative more general approach can be found in [16]; for a general introduction to the field of modal logics for coalgebras see [12]).

Again, we concentrate on the purely spatial case for illustrative purposes. So we are going to consider the endofunctor $FX = \mathcal{P}_f(X \times X)$ on the category of sets and "purely spatial" systems $\langle S, \mathrm{sp}\rangle$ with $\mathrm{sp} : S \to FS$.

The coalgebraic modal logic of $\langle S, \mathrm{sp}\rangle$

Following [18], we associate with each subterm S_i of $\mathcal{P}_f(S \times S)$ a description language $\mathcal{L}_i$ whose basic statements have a modal nature that relate the languages among themselves. The basic statements are then combined by the propositional (non-spatial) connectives to give the full languages $\mathcal{L}_i$.

Term	Language	Basic statement
$S_1 = S$	$\mathcal{L}_1$	$\langle \mathrm{sp}\rangle A,\quad A \in \mathcal{L}_2,$
$S_2 = \mathcal{P}_f(S \times S)$	$\mathcal{L}_2$	$\langle \mathcal{P}_f\rangle A,\quad A \in \mathcal{L}_3,$
$S_3 = S \times S$	$\mathcal{L}_3$	$\langle \pi_i\rangle A,\quad A \in \mathcal{L}_1 \quad (i = 1, 2).$

Here, $\pi_i : S \times S \to S$ $(i = 1, 2)$ are the canonical projections.

Satisfaction relation

We define a satisfaction relation $\models_i$ between elements of S_i and statements of $\mathcal{L}_i$ by simultaneous induction for all languages. We present the definition for the basic modal statements only:

$$\begin{aligned}
\mathcal{L}_1 :&\quad s \models_1 \langle \mathrm{sp}\rangle A \iff \mathrm{sp}(s) \models_2 A\\
\mathcal{L}_2 :&\quad P \models_2 \langle \mathcal{P}_f\rangle A \iff \exists p \in P, p \models_3 A\\
\mathcal{L}_3 :&\quad p \models_3 \langle \pi_i\rangle A \iff \pi_i(p) \models_1 A \ (i = 1, 2).
\end{aligned}$$

The spatial connectives are derived connectives in $\mathcal{L}_1$

The spatial connectives 0 and | are easily defined:

$$\begin{aligned}
0 &\triangleq \langle \mathrm{sp}\rangle \neg \langle \mathcal{P}_f\rangle \top,\\
A|B &\triangleq \langle \mathrm{sp}\rangle \langle \mathcal{P}_f\rangle (\langle \pi_1\rangle A \wedge \langle \pi_2\rangle B).
\end{aligned}$$

Indeed, it is easy to see that $s \models_1 \langle \mathrm{sp}\rangle \neg \langle \mathcal{P}_f\rangle \top$ iff $\mathrm{sp}(s) = \emptyset$, iff s satisfies 0. On the other hand, $s \models_1 \langle \mathrm{sp}\rangle \langle \mathcal{P}_f\rangle (\langle \pi_1\rangle A \wedge \langle \pi_2\rangle B)$ iff there exists $(s_1, s_2) \in \mathrm{sp}(s)$ such that $s_1 \models_1 A$ and $s_2 \models_1 B$, which is the same as saying that s satisfies $A|B$.

6 Discussion and extensions

This note proposed a general approach to the study of models for spatial logic where both the behavioural and the spatial properties are interpreted in coalgebraic terms. For simplicity we considered here a logic with only the two most basic spatial operators, namely, inaction and parallel composition. To interpret these operators it was enough to consider a spatial function of the form $\mathrm{sp} : S \to \mathcal{P}_f(S \times S)$, but other forms are possible, and in fact in [14] two other types of spatial function were considered. What is required is a systematic study of these basic models, establishing the appropriate correspondences between the respective categories.

Of course the logic considered in this note can be extended in several ways, requiring more sophisticated notions of model. Noteworthy in this respect are spatial operators that deal with names, like the fresh name quantifier and others [3]. In this case it is not enough to consider coalgebras over sets, we need the richer structure of sets on which act permutations of names [8].

We end with two research directions already pointed out in this note: the characterization of classes of models by sentences of the logic, and the relationship between spatial logics and the coalgebraic modal logics associated with the types of models under consideration.

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