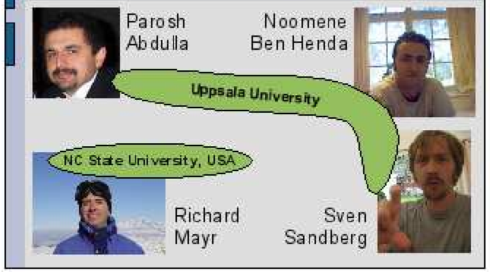
The Good, the Bad, and the Random

Stochastic Games with Lossy Channels



Overview

- Background
- Model
- Algorithm
- Conclusion

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Context

- **Reactive System**
  - Never terminates
  - Input & output during execution
- **Program Verification**
  - Communication protocols
  - Hardware

One approach to verification:

- **Model checking**
  - Input: model + formula
  - Output: "yes, formula true" / "no, formula false"

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Stochastic Infinite-State Games

- **Stochastic**
  - Unreliable communication
  - Fault-Tolerant systems
  - Randomized algorithms
- **Infinite-State**
  - Unbounded queues, stacks, counters, ...
  - Time
  - Unbounded parallelism
- **Games**
  - Model interacts with environment
  - Alternation in logics
  - Abstraction

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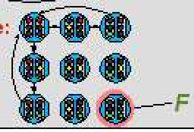
Reachability

- *F* – set of "target states"
- *F* reached?

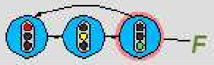
- **Example:**



Repeated Reachability

- *F* – set of "target states"
- *F* reached infinitely often?

- **Example:**
  - Will it always return to the green state?



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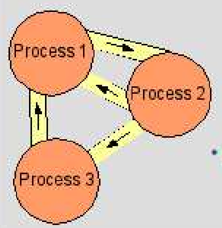
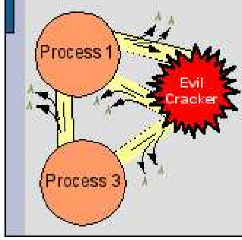
Lossy Channel Systems (LCS)



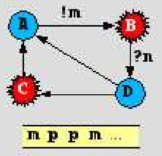
- **Model:**
  - Finite state processes
  - Unbounded lossy channels
  - Send & receive operations
- **Motivation:**
  - Models of communication protocols

Game Probabilistic Lossy Channel Systems (GPLCS)



- **Game:**
  Interaction with evil cracker
- **Probabilistic:**
  Messages lost randomly (probability λ)

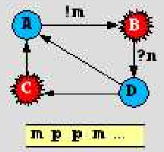
Game Probabilistic Lossy Channel Systems (GPLCS)



Sufficient:
- One channel
- One process
- Each state controlled by a player

Properties:
- Infinite state space
- Perfect channel = Turing machine

Game Probabilistic Lossy Channel Systems (GPLCS)



State: s = (A, mpnn)

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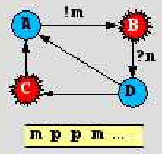
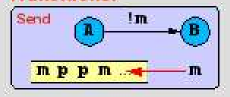


Transitions:



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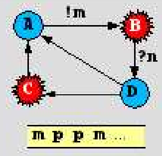

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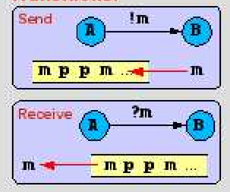

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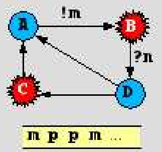

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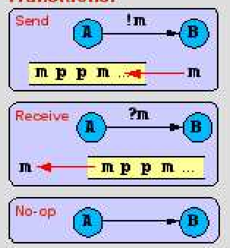
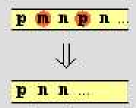


Game Probabilistic Lossy Channel Systems (GPLCS)

Probabilistic message loss:



- **Each transition:**

  each message lost with prob $\lambda > 0$, independently

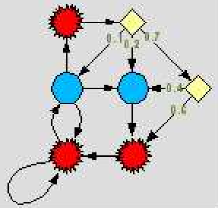
Stochastic Games

- **Every GPLCS induces a stochastic game**
- **3 types of states:**

Player Good

Player Bad

Player Random



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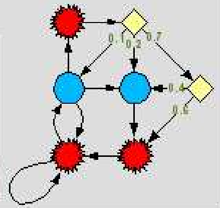


Stochastic Games

- **Strategy**

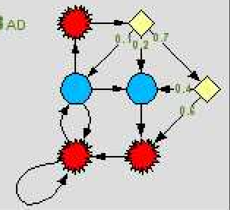
  = selection of outgoing transitions
- **Strategies for Good & Bad**

  ⇓

  Only probabilistic choices remain

  ⇓

  "**Prob(event)**" well-defined



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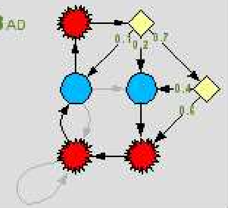

Overview

- **Background**
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- **Conclusion**

Repeated Reachability for GPLCS

- **Input:**
  - GPLCS
  - Set F of final states
- **Output: Partition of states:**

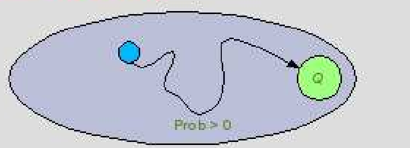
Winning for Bad

Winning for Good

Background Model Algorithm Conclusion

Subroutine: Force-set

- Force-set ≈ “Reachability for games”
  - Given target set Q
  - Compute the set of states where GOOD can force Prob(reach Q) > 0
- Backward search



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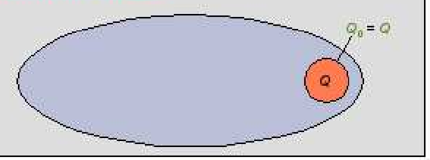

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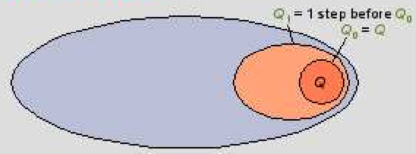

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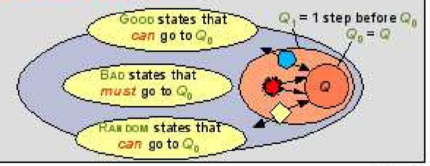

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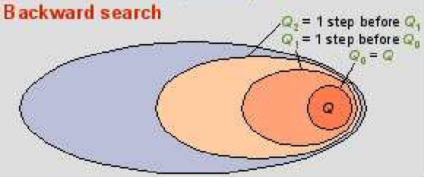

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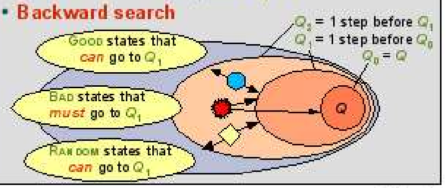

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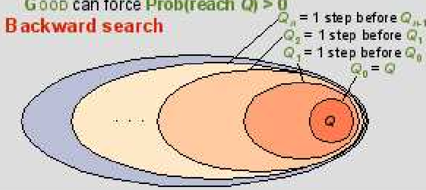
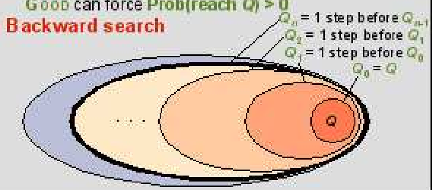


Background Model Algorithm Conclusion

Subroutine: Force-set Convergence

- Force-set … mes”
  - Given targ…
  - Compute the set of states where GOOD can force Prob(reach Q) > 0
- Backward search

Converges?



Background Model Algorithm Conclusion

Subroutine: Force-set Convergence

- Force-set … mes”
  - Given targ…
  - Compute the se… here
  GOOD can…
- Backward…

Converges?

YES! for GPLCS (well quasi orders)

Background Model Algorithm Conclusion

Subroutine: Force-set Correctness

- Force-set … mes”
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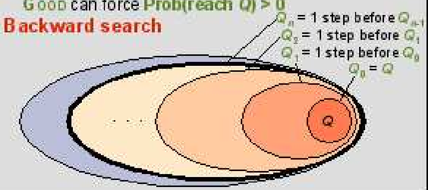
- Backward search

Correct?



Background Model Algorithm Conclusion

Subroutine: Force-set Correctness

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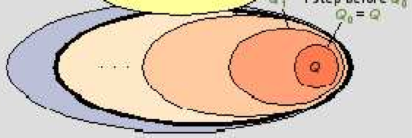
  - Compute the se... ...here GOOD can ...
- Backward...

Correct?

YES! (and I'll show why)



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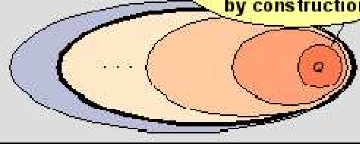
Correct?

In Q_n, GOOD can force Prob(reach Q) > 0 by construction



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Subroutine: Force-set Correctness

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Correct?

Outside Q_n, BAD can force Prob(reach Q) = 0

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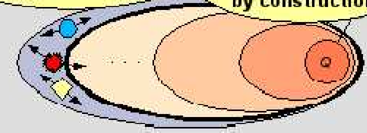
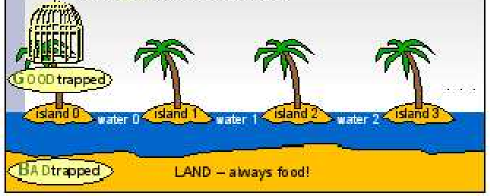


Background Model Algorithm Conclusion

Algorithm Idea

- A world of islands and water
- No food (= F) on islands
- In water, may float to island (possibly elsewhere, too)
- Find BAD islands and water!



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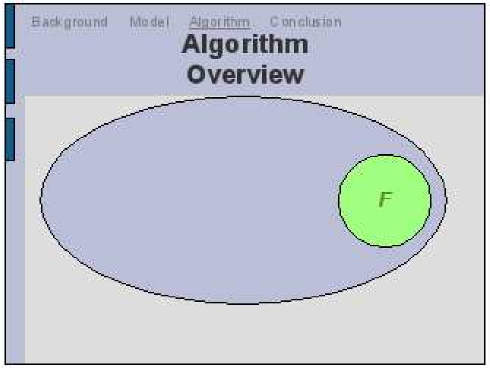

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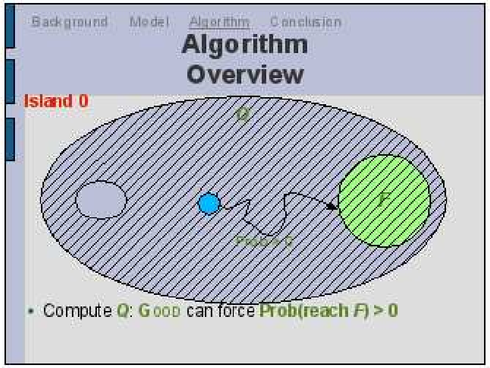

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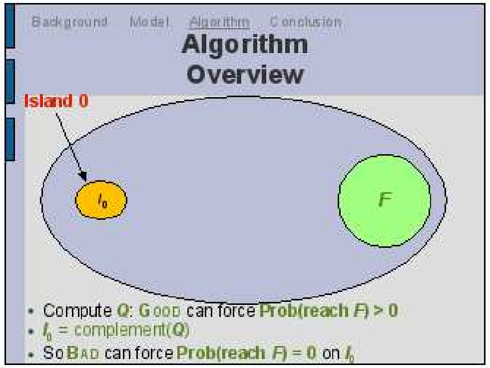

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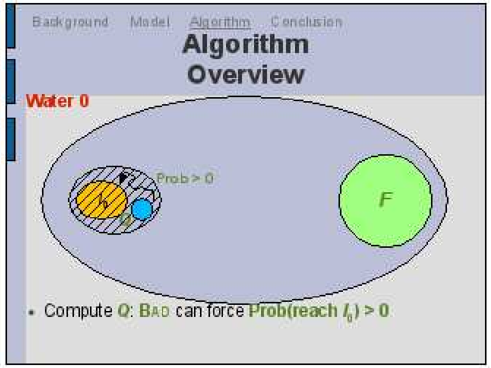

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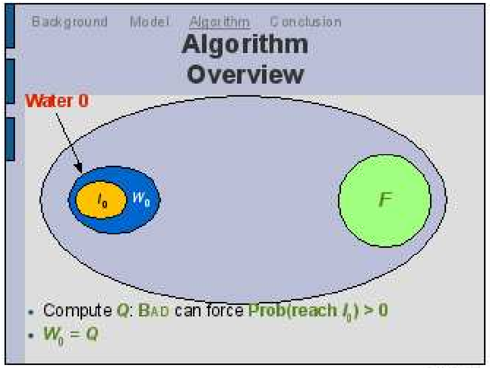

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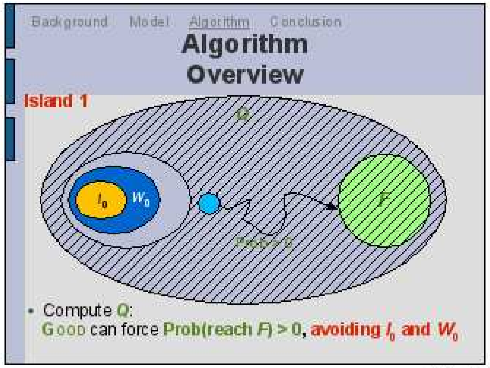

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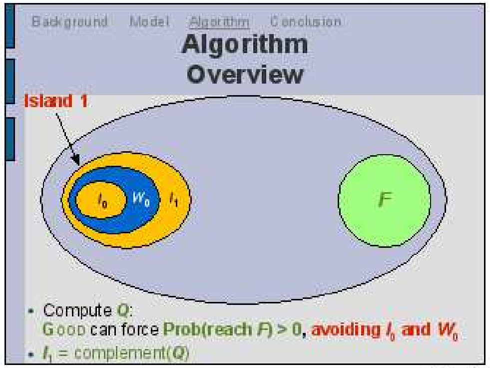

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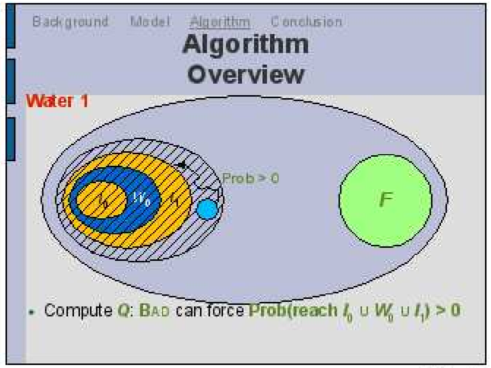

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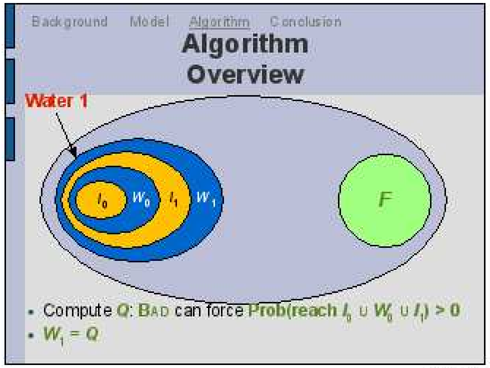

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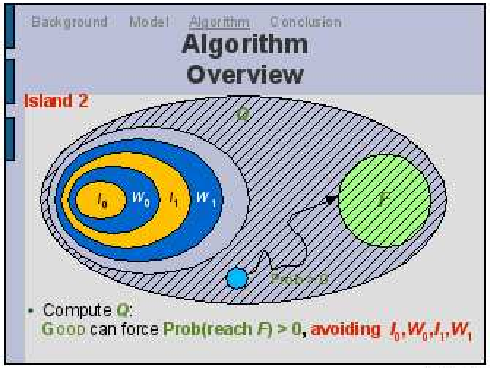

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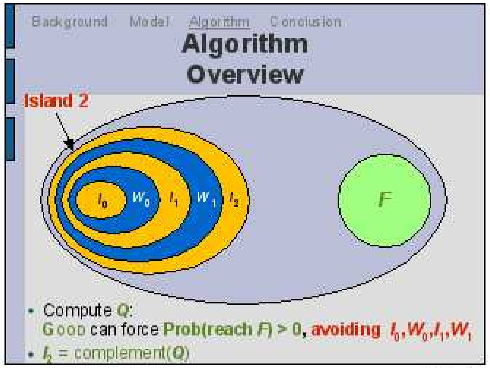

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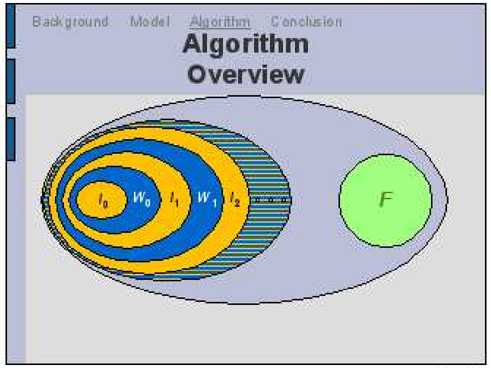

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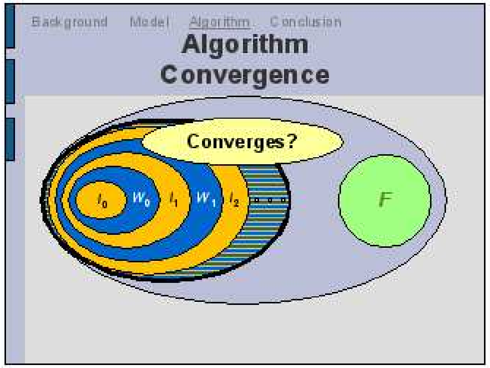

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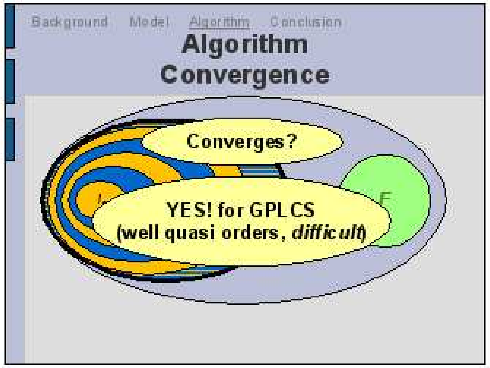

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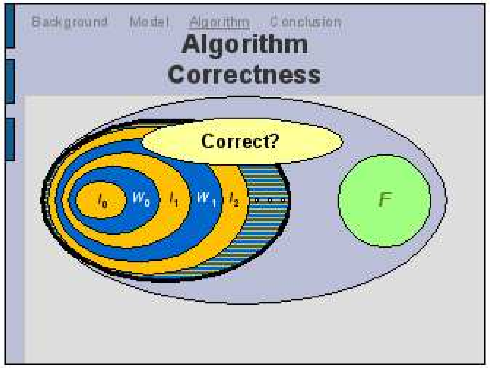

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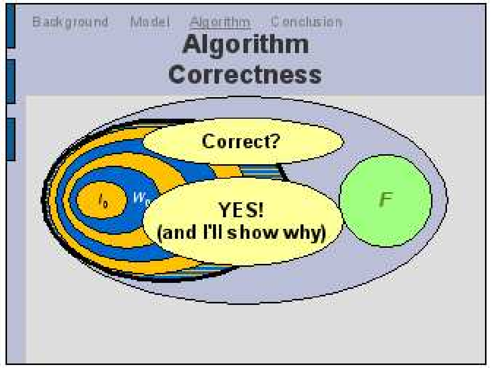

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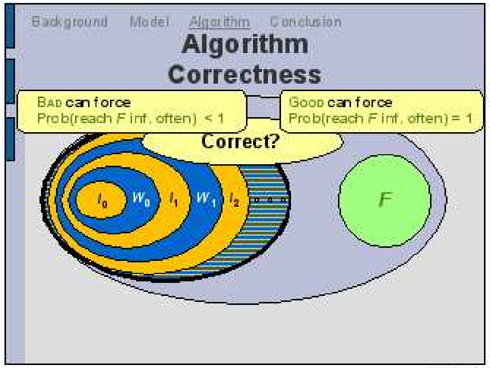

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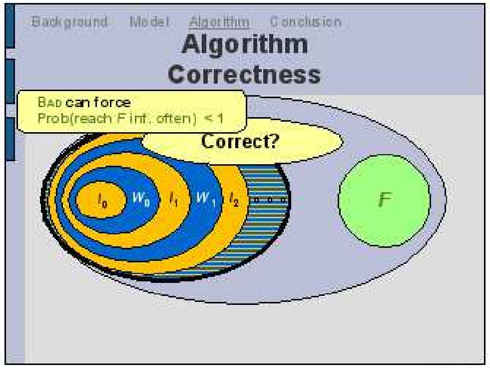

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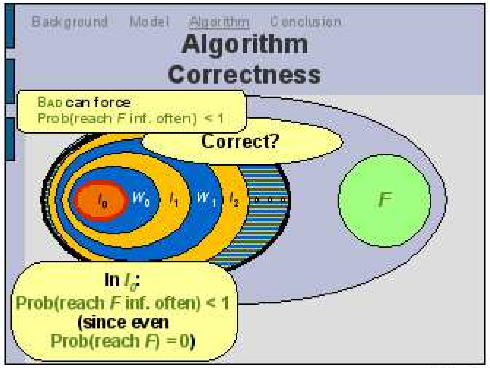

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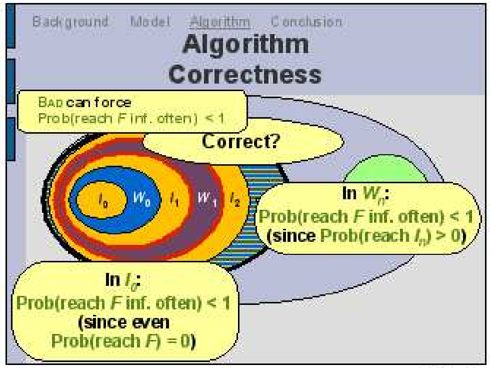

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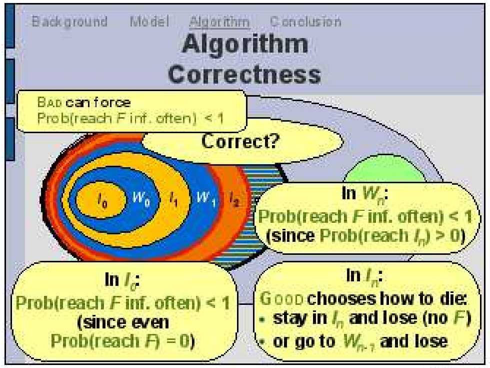
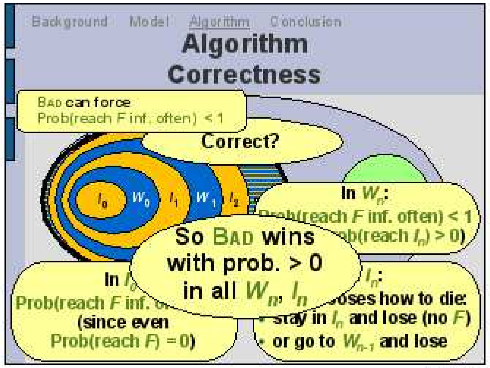
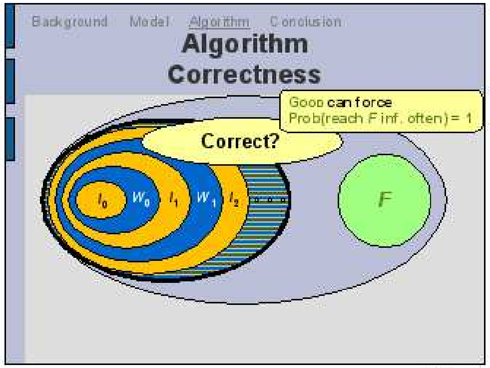


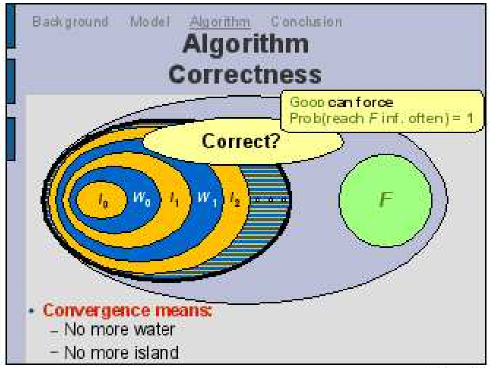
Slide 69



Slide 70

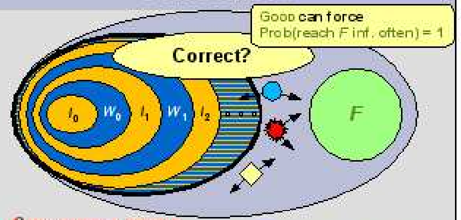


Slide 71



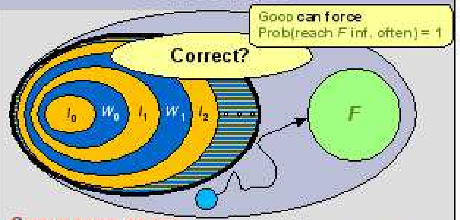
Slide 72

Algorithm Correctness



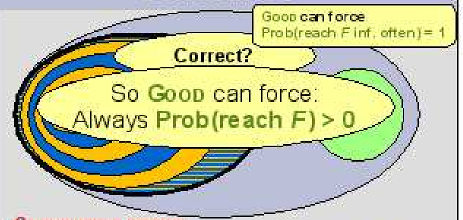
- Convergence means:
  - No more water $\Rightarrow$ Good can stay outside I_n, W_n
  - No more island

Algorithm Correctness



- Convergence means:
  - No more water $\Rightarrow$ Good can stay outside I_n, W_n
  - No more island $\Rightarrow$ Good can force Prob(reach F) > 0

Algorithm Correctness



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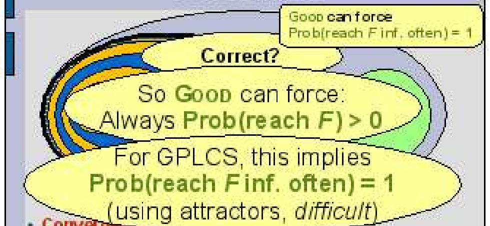
Algorithm Correctness



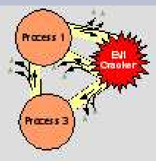
- Converg...
  - No more water $\Rightarrow$ Good can stay outside I_n, W_n
  - No more island $\Rightarrow$ Good can force Prob(reach F) > 0

Overview

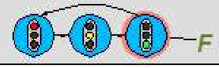
- Background
- Models
- Algorithm
- Conclusion

The Problem We Studied

- GPLCS:
  Protocol vs. Cracker

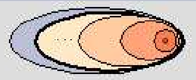


- Repeated reachability:
  reach "good" state infinitely many times



Algorithm

- Force-set:
  - Prob(reach Q) > 0
  - Backward search



- Winning sets for Bad:
  - Prob(reach F inf. often) < 1
  - Search for islands and water

Extensions

- Construct winning strategies
- Concurrent games
  Simultaneous moves
- Reachability games
  Can Good force Prob(reach F) = 1 ?

Can do

- Parity games
  Generalization of repeated reachability

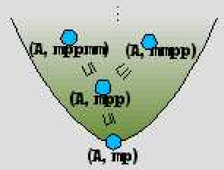
Unclear

The End

Questions?

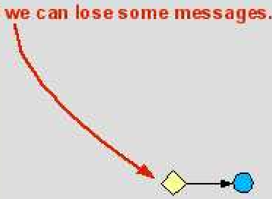
Convergence: Well Quasi Orders

- Recall: State: s = (A, mppp)
- Subword ordering:
  pmp $\sqsubseteq$ mpmmp
- Upward closure:
  all "bigger" states
- Message loss
  $\Rightarrow$ subword



Convergence: Well Quasi Orders

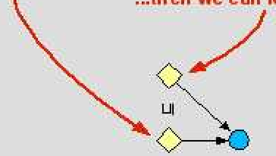
If we can lose some messages...



Convergence: Well Quasi Orders

If we can lose some messages...

...then we can lose more!



Convergence: Well Quasi Orders

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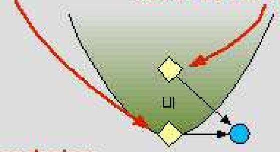
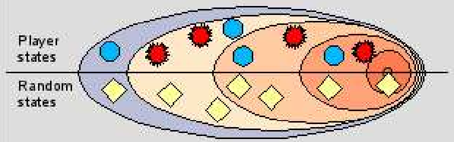


Conclusion:
RANDOM states one step before are upward closed

Convergence of Force-sets: Well Quasi Orders

- **Theorem (Higman 1952):** Any sequence of increasing, upward closed sets converges.
- **We apply it:**



Convergence of Force-sets: Well Quasi Orders

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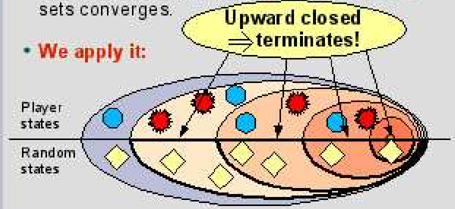

Convergence of Force-sets: Well Quasi Orders

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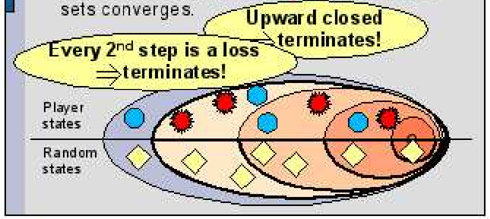
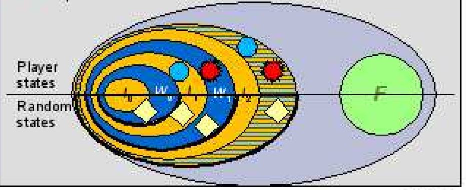


Convergence of Algorithm: Well Quasi Orders

- Argument is much more difficult!
- RANDOM states in water upward closed
- Sequence of waters terminates
- Sequence of islands terminates



The Fine Print: Finite-Memory Strategies

- **We assume BAD is restricted** to *finite-memory strategies*
- **BAD cannot remember** unbounded amount of history
- **Intuition:** Evil cracker has a finite hard disk
- Needed to prove correctness

Leftover

Repeated Reachability

- ***F* – set of "target states"**
- **Is *F* reached infinitely often?**
- **Example:**
  - Will it always return to the green state?