Memoryless Determinacy of Parity and Mean Payoff Games: A Simple Proof

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Abstract

We give a simple, direct, and constructive proof of memoryless determinacy for Parity and Mean Payoff Games. First, we prove by induction that the finite-duration versions of these games, played until some vertex is repeated, are determined and both players have memoryless winning strategies. In contrast to (Ehrenfeucht & Mycielski 1979), our proof does not refer to the infinite-duration versions. Second, we show that the results straightforwardly generalize to infinite duration games.

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Contents

1 Introduction

Parity Games are infinite duration games played by two adversaries on finite graphs with colored vertices. The problem of deciding the winner in Parity Games is polynomial time equivalent to the Rabin Chain Tree Automata (or Parity Tree Automata) nonemptyness and the μ-calculus[1] model checking (Emerson, Jutla & Sistla 1993). For these reasons, Parity Games are of considerable importance and have been extensively studied by the complexity-theoretic, automata-theoretic, and verification communities.

One of the fundamental properties of Parity Games, used by almost all decision algorithms, is the so-called *memoryless determinacy*. Vertices of every game can be partitioned into winning sets of both players, and both players possess positional winning strategies from all vertices in their winning sets. This means that for every vertex where a player has a choice, he can make up his mind in advance on what to do if the play reaches it, independently of the history of the play, and revealing this strategy is not a disadvantage.

Emerson (1985) sketched the first memoryless determinacy proof for Parity Games[2] as early as in 1985. His proof is based on a (fairly complicated) simplification by Hossley & Rackoff (1972) (relying on König's lemma) of Rabin's original proof (Rabin 1969) of the nonemptiness problem for Rabin automata. A later, more self-contained, determinacy proof by Emerson & Jutla (1991) relies heavily on the μ-calculus, and is non-constructive (e.g., the definition of the strategy uses properties of all paths in a binary tree, a set of continuum cardinality). Later McNaughton (1993) and Zielonka (1998) gave alternative proofs.

Today it is interesting to note that memoryless determinacy of Parity Games is a one-line consequence (using a simple reduction due to Puri (1995) sometimes attributed to M. Jerrum) of the earlier more general result of (Ehrenfeucht & Mycielski 1973, Ehrenfeucht & Mycielski 1979) on memoryless determinacy of the so-called *Mean Payoff Games*.

Mean Payoff Games are infinite games played by two adversaries on finite graphs with weighted edges. Players try to maximize/minimize the mean value of weights of encountered edges. It was proved in (Ehrenfeucht & Mycielski 1973, Ehrenfeucht & Mycielski 1979) that every Mean Payoff Game has a unique value ν such that Player 0 can ensure that he wins at

[1] One of the most expressive temporal logics of programs.

[2] Actually for Rabin Pairs automata, but easily transfers to Parity Games/automata

least ν and Player 1 can ensure he loses at most ν, i.e., the games are *determined*. Furthermore, each player can ensure this value by using a *positional (memoryless)* strategy.

The proof for Mean Payoff Games given in (Ehrenfeucht & Mycielski 1979) utilizes a cyclic interplay between infinite games and their finite counterparts, and is nonconstructive. The authors asked as to whether it is possible to give a direct, rather than roundabout proof, a question we answer affirmatively in this paper. Memoryless determinacy for Mean Payoff Games was later shown constructively in (Gurvich, Karzanov & Khachiyan 1988), but the proof is rather involved, using estimates of norms of solutions to systems of linear equations, convergence to the limit, and precision arguments.

The purpose of this paper is to give a simple and direct proof that works uniformly for both Parity Games and Mean Payoff Games. Our proof does not involve any auxiliary constructions or machinery (like μ-calculus or linear algebra), proceeds by induction, and constructs positional strategies for more complicated games from strategies for simpler ones. Similar to (Ehrenfeucht & Mycielski 1979), we rely on finite versions of the games, played until the first vertex repetition, but in contrast to (Ehrenfeucht & Mycielski 1979) we completely avoid roundabout proofs of the properties of finite games using infinite ones and vice versa. Our proof is constructive, although the algorithm it provides is not very efficient. Due to the importance of Parity Games and Mean Payoff Games, we feel that a straightforward and constructive proof of memoryless determinacy, common to both games, and without involving external and powerful methods should be interesting and useful.

The proof of memoryless determinacy of McNaughton (1993) holds for a broad abstract class of games. It even provides a necessary and sufficient condition for a game to have memoryless determinacy. This proof is constructive but apparently does not apply directly to Mean Payoff Games. Zielonka (1998) gives two simple and elegant proofs that work for Parity Games with a possibly infinite number of vertices, but not for Mean Payoff Games. The first version is constructive and uses induction on the number of vertices (transfinite induction if there are infinitely many vertices). The second version is shorter but nonconstructive.

The interest in Parity Games and Mean Payoff Games is to a large extent motivated by their complexity-theoretic importance. For both games, the corresponding decision problems belong to NP $\cap$ coNP, but their PTIME membership status remains open. Much of the research in the complexity of solving the games relies on memoryless determinacy, e.g., (Zwick &

Paterson 1996, Gurvich et al. 1988, Jurdzinski 2000, Vöge & Jurdzinski 2000, Björklund, Sandberg & Vorobyov 2002). Some papers rely essentially on memoryless determinacy, while others prove it independently, explicitly or implicitly (sometimes difficult to understand).

It would be interesting to know whether our proofs can be extended to more general classes of *discounted* and *simple stochastic games* (Zwick & Paterson 1996).

Outline of the Paper. Section 2 gives some basic definitions for Parity Games. Theorem 3.1 of Section 3 shows determinacy of Finite Parity Games, with strategies requiring memory. The proof is by elementary induction on the number of vertices in a finite tree. Section 4 provides some intermediate results needed in later sections. In Section 5, we prove, also by induction, but on the number of edges in the games, that Finite Parity Games are solvable in positional strategies. This is the main theorem in the paper. We extend the proof to infinite duration Parity Games in Section 6. In Section 7 we show how the proof for Parity Games generalizes to prove memoryless determinacy for Mean Payoff Games.

2 Parity Games

We assume the standard definition of *Parity Games* (PGs). These are *infinite* duration games played on *finite* directed graphs $G = (V, E)$ with vertices colored by natural numbers. The vertices of the graph are partitioned into sets V_0 and V_1, and every vertex has at least one outgoing edge (i.e., there are no sinks). The game starts in some vertex, and Player i chooses a successor when a play comes to a vertex in V_i. In this way, the players construct an infinite sequence of vertices. The parity of the *largest* color hit infinitely often in this sequence determines the winner: even/odd is winning for Player 0/Player 1, respectively.

A (general) *strategy* of Player i is a function that for every prefix of a play, ending in a vertex $v \in V_i$, selects a successor of v (move of the player). A *positional strategy* for Player i is a mapping selecting a unique successor of every vertex $v \in V_i$. When a play comes to $v \in V_i$, Player i unconditionally selects the unique successor determined by the positional strategy, independently of the history of the play. Thus positional strategies are *memoryless*.

When considering a positional strategy σ of Player i, it is often useful to examine the graph G_σ, which is obtained from G by removing all outgoing edges from vertices in V_i, except those used by σ.

3 Finite Parity Games

Together with the (infinite duration) Parity Games defined above, we also consider *finite duration* games played on the same graphs as PGs, but only until the first loop is constructed.

A *Finite Parity Game* (FPG) G_a starts in vertex a, and players alternate moves until some vertex v_l is visited for the second time. At this point the loop from v_l to v_l constructed during the play is analyzed, and the maximal vertex color c occurring on this loop determines the winner. If c is even, then Player 0 wins; otherwise, Player 1 wins.

FPGs are finite perfect information zero-sum games (the loser pays \$1 to the winner). Therefore, each such game has a value. For any vertex a there is a value ν and strategies σ_i for both players such that in G_a Player i using σ_i secures for himself the payoff ν.

We give an explicit and simple (without reference to the general Brouwer's or Kakutani's fixpoint theorems) proof of this fact. Moreover, we will show that the strategies need not be mixed (probabilistic), but may require memory. Additionally, the possible values are just 1 and -1 (Player 0 or 1 wins, respectively). Based on this result (Theorem 3.1), we will construct positional optimal winning strategies and thus prove memoryless determinacy of FPGs in the following sections.

Theorem 3.1 *The vertices of any FPG can be partitioned into sets* $W_0(G)$ *and* $W_1(G)$ *such that Player* i *can win* G_v *when* $v \in W_i(G)$*.*

(The set $W_i(G)$ in Theorem 3.1 is called the *winning set* for Player i.)

Proof. Starting from every vertex v construct an `AND-OR` tree of all possible developments of an FPG starting at v. This tree is finite, and leaves correspond to first vertex repetitions on paths from the root. Mark those leaves with 0 or 1 corresponding to which player wins in the leaf. Evaluate the root of the tree bottom up by repeatedly using the rules: 1) if a vertex u of Player i has a successor marked i, then mark u with i; 2) if all successors of a vertex u of Player i are marked $1-i$, then mark u with $1-i$. This evaluation uniquely determines the mark of the root, 0 or 1. □

Note that implementing a winning strategy according to the construction in the above proof requires memory to keep the history of the play in order to decide where to move at each step. In the following sections we show how to construct *positional* strategies.

4 Positional Strategies in Finite Parity Games

This section summarizes useful properties of positional strategies in FPGs needed later.

Lemma 4.1 *If Player i has a positional strategy σ winning from some successor u of $v \in V_i$, then v is winning for Player i.*

Proof. Consider the graph G_σ obtained from G by deleting the outgoing edges from vertices in V_i not used in σ. If $\sigma(v) = u$, then the sets of loops reachable (constructible by Player $1 - i$) from v and u in G_σ are the same. Therefore, since σ is winning from u, it is also winning from v. Otherwise ($\sigma(v) \neq u$), if v is reachable from u in G_σ, then all loops reachable from v in G_σ must be winning for Player i. If, on the other hand, v is not reachable from u, we can change σ only at v, creating σ' such that $\sigma'(v) = u$. This new strategy will still be winning from u, and the sets of loops reachable from u and v in $G_{\sigma'}$ will again be the same. □

Lemma 4.2 *Suppose σ_0, σ_1 are positional strategies for Player 0 and 1 such that Player i by using σ_i wins every game that starts in $W_i(G)$ (for $i \in \{0, 1\}$). Then all plays starting in $W_i(G)$ and played according to σ_i stay in $W_i(G)$.*

Proof. Consider a game on the graph G_{σ_i} starting in some vertex from $W_i(G)$. In this game, we can assume that all vertices belong to Player $1 - i$ because Player i has no choice in his vertices. By Lemma 4.1 any edge (u, v) with $v \in W_{1-i}(G)$ must also have $u \in W_{1-i}(G)$, so there are no edges from $W_i(G)$ to $W_{1-i}(G)$ in G_{σ_i}.

□

5 Memoryless Determinacy of Finite Parity Games

In this section we prove the main theorem of this paper, which implies memoryless determinacy of the infinite duration Parity Games (Section 6). The proof itself does not use any reference to the infinite games. In Section 7 we extend this theorem to finite and infinite duration Mean Payoff Games. Actually, the proof of Theorem 5.1 is not modified, we just show how to adjust winning conditions so as to reuse the theorem and its proof as they are.

Theorem 5.1 *In every FPG G, there are positional strategies σ_0 of Player 0 and σ_1 of Player 1 such that Player i wins by using σ_i whenever a play starts in $W_i(G)$, no matter which strategy the adversary Player $1-i$ applies.*

Proof. The proof is by induction on the number $|E|-|V|$ of choices of both players. The base case is trivial: if $|E|=|V|$ then there are no vertices with a choice, and all strategies must be positional.

For the inductive step we split into three cases, depending on whether there are vertices from which the player who owns the vertex can win. Let $V_0^* \subseteq V_0$ and $V_1^* \subseteq V_1$ be the sets of vertices of respective player with outdegree more than 1. Assume the theorem holds whenever $|E|-|V|<n$ and consider $|E|-|V|=n$.

Case 1: $V_0^* \cap W_0(G) \neq \emptyset$. Take $x \in V_0^* \cap W_0(G)$ and let e be an edge leaving x such that Player 0 can win G_x after selecting e in the first move. (The rest of Player 0's strategy does not need to be positional.) Consider the game G' in which we remove all edges leaving x, except e. By inductive assumption, there are positional strategies σ_0 of Player 0 and σ_1 of Player 1 in G' such that Player i applying σ_i wins any play that starts in $W_i(G')$. These strategies are also positional strategies in G. We will prove that σ_i wins for Player i in G if the play starts in a vertex in $W_i(G')$.

Take any $v \in V_1 \cup V_2$ and consider the game G_v. Assume $v \in W_0(G')$ and Player 0 uses σ_0. Any play in G_v following σ_0 was also possible in G'. Since it is winning for Player 0 in G', it is also winning for Player 0 in G. Therefore, $W_0(G') \subseteq W_0(G)$.

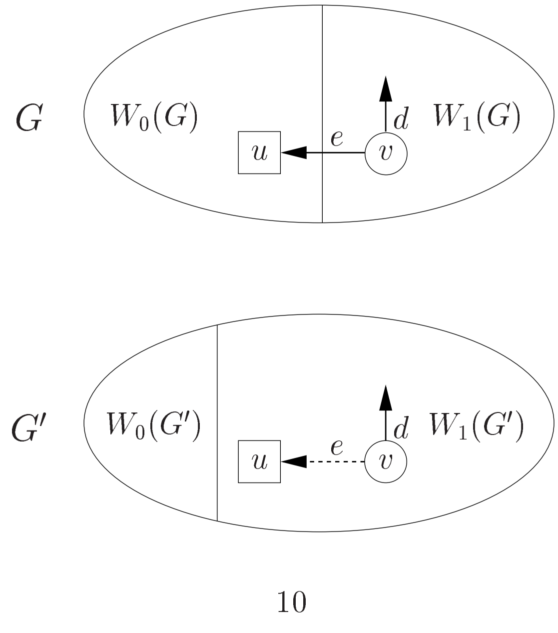
Now assume $v \in W_1(G')$ and Player 1 uses σ_1. Note that $x \in W_0(G')$, because Player 0 wins G_x after moving through e, and this situation is equivalent to making the same initial move in G'_x. Thus by Lemma 4.2 the play cannot reach x. But any play according to σ_1 not reaching x was possible also in G'_v, so Player 1 wins it. Hence, $W_1(G') \subseteq W_1(G)$.

We thus showed that $W_i(G) = W_i(G')$ and Player i wins G_v using σ_i, for any $v \in W_i(G)$.

Case 2: $V_1^* \cap W_1(G) \neq \emptyset$ and Case 1 does not apply. The proof is symmetrical to Case 1.

Case 3: $V_0^* \cap W_0(G) = V_1^* \cap W_1(G) = \emptyset$. Since only Player i may have choices in $W_{1-i}(G)$, we will assume that all vertices in $W_i(G)$ belong to V_{1-i} (for $i \in \{0, 1\}$). Assume one of the players has choices; otherwise the base case considered above applies.

We first prove that there are no edges between the winning sets. Towards a contradiction, assume that there is an edge e from $v \in W_1(G)$ to $u \in W_0(G)$ (the case of an edge back is symmetric). Although u is losing for Player 1, he must have a way of winning a play that starts in v and moves by e to u. This can only be done by forcing the play back to v. Player 1 cannot win from u, however, so if the game starts there, and is forced to v, Player 0 must be able to do something else. Thus there must be another edge d leaving v; see the figure below (round vertices belong to Player 0 and square vertices belong to Player 1).



Now create the game G' by removing the edge e. Since Player 0 has less choices, we must have $W_0(G') \subseteq W_0(G)$. By inductive assumption, there are positional strategies σ_0 and σ_1, winning from $W_0(G')$ and $W_1(G')$, respectively. Since all vertices in $W_0(G')$ can be considered as belonging to V_1, Lemma 4.1 implies that there are no edges leaving $W_0(G')$. This in turn means that u belongs to $W_1(G')$. Otherwise, Player 0 could have won from v in G by following e (the play would never have left $W_0(G')$, and any loop formed would have been winning for Player 0).

When e was removed, vertex u changed from being losing for Player 1 to being winning for him. This implies that in G', Player 1 must force the play to v in order to win, since all other plays would have been possible in G as well, and were losing for Player 1. In G, however, no matter how Player 1 forced the play from u to v, Player 0 could win by using some edge other than e and some strategy for the remaining play. All these plays are possible in G' as well, so Player 1 cannot win from u in G'. This contradiction means that there can be no edge e from $W_1(G)$ to $W_0(G)$. By symmetric reasoning, no edges from $W_0(G)$ to $W_1(G)$ are possible either.

Since there are no edges between the winning sets, and no player has any choices within his own winning set, any positional strategy is optimal (winning from all vertices in the players winning set). $\square$

6 Extension to Infinite Duration Parity Games

We now show that a positional strategy that wins an FPG starting in vertex v also wins the infinite Parity Game on the same game graph starting in v.

Let σ be a winning strategy (positional or not) for Player i (for $i \in \{0, 1\}$) in the FPG starting from v. No matter what the opponent does, the first revisit to a previously visited vertex v_l guarantees that the maximal color on the loop from v_l to v_l is winning for Player i. Now forget what happened on the path from v_l to v_l, assume v_l is visited for the first time, and let the FPG develop further. The next time some vertex v'_l (not necessarily equal to v_l) is first revisited for the second time, we also know that the maximal color on the loop from v'_l to v'_l is winning for Player i. Again, forget what happened on the path from v'_l to v'_l, assume v'_l is visited for the first time, and let the FPG develop further. In this way, using the winning strategy for Player i, we construct an infinite sequence of loops and the corresponding infinite sequence of maximal colors $S = \{c_i\}_{i=1}^{\infty}$ winning for Player i. It follows that the maximal color hit in the infinite game infinitely often is the maximum appearing in S infinitely often.

This also means that the winning sets of the players in the infinite game coincides with the ones in the finite game.

The following theorem makes the above argument formal.

Theorem 6.1 *A positional strategy σ of Player i wins in an infinite PG if and only if it wins in the FPG.*

As direct consequences, the winning sets in the infinite and finite games coincide, and the memoryless determinacy from Theorem 5.1 holds also for infinite games.

Proof. We will prove that σ wins the infinite game if and only if the highest color on every loop reachable from v in G_σ is winning for Player i. This in turn is trivially equivalent to σ winning the finite game.

If G_σ contains a loop with losing highest color that is reachable from v, then σ is obviously losing: Player $1 - i$ can go to this loop and stay in it forever.

Assume G_σ does not contain any loops with losing highest color reachable from v. We will prove that there is no infinite path starting from v in G_σ on which the highest color appearing infinitely often is losing. Assume, towards a contradiction, that there is such a path p with highest infinitely occurring color c. There must be some vertex u with color c that appears infinitely often on p. But between any two appearances of u on the path, all other vertices on some loop containing u must appear. Among these vertices, at least one has a winning color higher than c. This means that some winning color higher than c appears infinitely often on p, contradicting the assumption. This finishes the proof. $\square$

7 Extension to Mean Payoff Games

This section shows how the proof of memoryless determinacy for Parity Games extends to Mean Payoff Games.

7.1 Finite and Infinite Mean Payoff Games

Mean Payoff Games (Gurvich et al. 1988, Ehrenfeucht & Mycielski 1979, Zwick & Paterson 1996) are similar to Parity Games. Let $V = V_0 \cup V_1$, $E \subseteq V \times V$ (where, for each $u \in V$, there is some v with $(u, v) \in E$), and $c : E \to \mathbb{R}$. Define the game graph $\Gamma = \Gamma(V_0, V_1, E, c)$. Starting in some predefined vertex v, the two players move by selecting edges from their respective vertex sets in the same way as in Parity Games. This yields an infinite play (sequence of vertices) $p = v_0 v_1 v_2 \ldots$. Player 0 wants to maximize

$$\nu_0(p) = \liminf_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} c(v_i, v_{i+1})$$

and Player 1 wants to minimize

$$\nu_1(p) = \limsup_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} c(v_i, v_{i+1}).$$

Analogously to the finite version of Parity Games, we define Finite Mean Payoff Games (FMPGs). Like in FPGs, a play starts in the initial vertex v and ends as soon as a loop is formed. The value of the play $p_0 p_1 \ldots p_m \ldots p_n$, where $p_m = p_n$, is

$$\nu(p) = \frac{1}{n-m} \sum_{i=m}^{n-1} c(p_i, p_{i+1}).$$

Player 0 wants to maximize this value and Player 1 wants to minimize it.

Theorem 7.1 *In any FMPG, starting from any vertex u, there is a value $\nu(u)$ such that Player 0 can ensure that he wins at least $\nu(u)$, and Player 1 can ensure that he loses at most $\nu(u)$.*

Proof. In analogy to the proof of Theorem 3.1, consider the finite tree of all possible plays from u. Assign to each leaf the value of the play it represents. Let all internal vertices corresponding to choices of Player 0 be `MAX`-vertices, and let the choices of Player 1 be `MIN`-vertices. The root of the resulting `MIN-MAX`-tree can be straightforwardly evaluated in a bottom-up fashion, and its value is the value $\nu(u)$ of the game starting from u. $\square$

It is easy to show (see, e.g., Lemma 1 of (Ehrenfeucht & Mycielski 1979)), using only elementary means, that a strategy in an FMPG G yields a strategy in the corresponding MPG Γ.

Lemma 7.2 (Ehrenfeucht-Mycielski 1979) *A strategy σ of Player i in G can be modified to a strategy $\tilde{\sigma}$ in Γ. If σ secures the value ν in G, then $\tilde{\sigma}$ secures ν in Γ. If σ is positional, then $\tilde{\sigma} = \sigma$ so $\tilde{\sigma}$ is also positional.*

7.2 Main Theorem for the Decision Version of Mean Payoff Games

Consider the *decision* version of Finite Mean Payoff Games, denoted FMPG(D), where we are only interested in whether the value of a play is greater than some threshold t, not in the actual value. The winning condition for Player 0 is either $\nu(p) > t$ or $\nu(p) \geq t$. This allows us to define winning sets as for Finite Parity Games.

The reader may straightforwardly verify that the proof of Theorem 5.1 works without any modifications for a broader class of games satisfying the following

> **Assumption:** *a play on a game graph stops as soon as a loop is formed, and the winner is determined by the sequence of vertices on the loop (modulo cyclic permutations).*

It is easily seen (by inspecting the proofs of Theorem 5.1 and the lemmas it depends upon) that this is a sufficient assumption on the properties of the winning condition. It clearly holds for FMPG(D)s, where a sequence of vertices uniquely determines a sequence of edges (since there are no multiple edges), and their average cost determines the winner. Therefore, we obtain the following

Theorem 7.3 *In every* FMPG(D)*, each player has a positional strategy that allows him to win from all vertices in his winning set.*

7.3 Ergodic Partition Theorem for Mean Payoff Games

From Theorem 7.3 it does not directly follow that whenever Player 1 secures a loss $\leq t$, then Player 0 can secure a gain $\geq t$. In this section we prove that each vertex has a value, which both players can secure by means of positional strategies whenever a play starts in this vertex. Moreover, the same pair of strategies can be used independently of the starting vertex. More formally:

Theorem 7.4 (Memoryless Determinacy and Ergodic Partition) *The vertices of every FMPG partition into equivalence classes C_i with corresponding values ν_i. There are positional strategies σ_0, σ_1 of Player 0 and 1 such that if a play starts in C_i, then σ_0 secures a gain $\geq \nu_i$ for Player 0, and σ_1 secures a loss $\leq \nu_i$ for Player 1 (i.e., all vertices in C_i have the same value ν_i).*

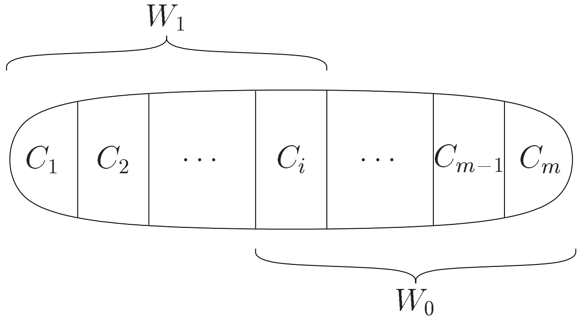
Moreover, Player 0 has no edges leading from C_i to C_j with $\nu_i < \nu_j$, and Player 1 has no edges leading from C_i to C_j with $\nu_i > \nu_j$.

(Such a partition $C_1, \ldots C_m$ is called *ergodic*.)

Proof. By Theorem 7.1, there exist values $\nu_1 < \nu_2 < \cdots < \nu_m$ and a partition $C_1, C_2, \ldots, C_m$ such that for every starting vertex $u \in C_i$ of an FMPG G both players ensure for themselves (possibly by non-positional strategies) the value ν_i. Now, for every value ν_i solve two FMPG(D) problems (using Theorem 7.3):

1. find the winning set W_0 and corresponding strategy σ_0 of Player 0 securing a gain $\geq \nu_i$ when a play starts in W_0.

2. find the winning set W_1 and corresponding strategy σ_1 of Player 1 securing a loss $\leq \nu_i$ when a play starts in W_1.



Consider $W_0 \cap W_1$. In this (nonempty) set both Player 0 can secure a gain $\geq \nu_i$ by means of σ_0, and Player 1 can secure a loss $\leq \nu_i$ by means of σ_1. In other words, $W_0 \cap W_1 = C_i$.

By Lemma 4.2, any play starting from W_0 always stays in this set, when Player 0 uses σ_0 (Player 1 has no edges out of it), and symmetrically for W_1.

We repeat the argument above for all values $\nu_1, \ldots, \nu_m$ getting winning positional strategies σ_0^i, σ_1^i. Using the property from the preceding paragraph we merge all these strategies into positional strategies σ_0 and σ_1 for Player 0 and Player 1 as stated in the theorem. Simply define σ_0 as coinciding with σ_0^i on the set of vertices $V_0 \cap C_i$, and similarly for σ_1. $\square$

By Lemma 7.2, the same theorem holds also for (infinite) Mean Payoff Games. Moreover, it is easy to see that it holds (with the same proof) in the version of Parity Games where Player 0 and 1 not only want to win, but want to win with highest possible color.

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