Physics-informed machine learning

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Joint work with
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Celsius-Linnaeus Lecturers with Symposium
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Models

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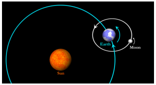
Example A: *Predicting next solar eclipse*

Input

Model

Output

Present position of celestial bodies



Next total solar eclips in Swe-den (16 October 2126)

Models

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Example B: *Image classification*

Input — *An image*

Model

Output — *What the image depicts* — A church

Finding models

How do we find models?

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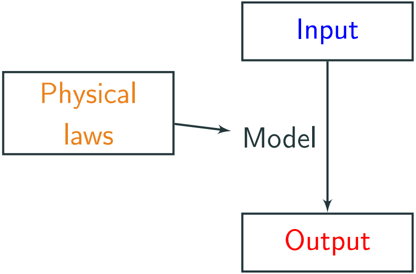
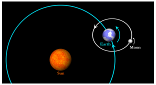


Present position of celestial bodies



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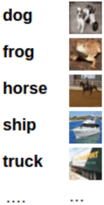
Example B: *Image classification*

Training data



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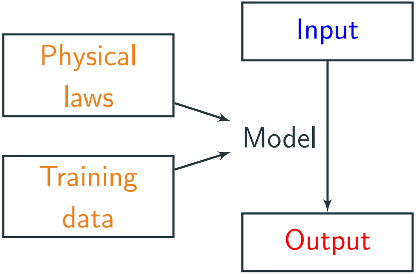
An image

What the image depicts

A church

Physics-informed machine learning

In **physics-based machine learning** we make use first principles to leverage the performance of data-driven machine learning models.



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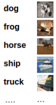

First principles ⟷ Data-driven

Finding models that can make use for both first principles and data-driven approaches is an active research field

Physics-informed machine learning

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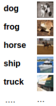
Example 1: Magnetic tracking ↓

Example 3: Strain field reconstruction ↓



First principles ⟷ Data-driven

↑ *Example 2: Magnetic mapping*

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A magnetometer is a sensor that **measure magnetic fields**

Advantages with magnetometers

- Cheap sensors

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- Small sensors
- Low energy consumption

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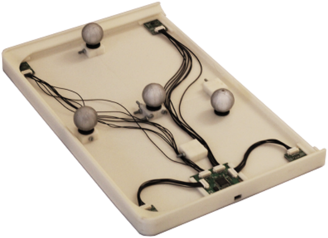
We have:

- A sensor network of magnetometers
- A moving magnetic object in the vicinity of that network



Example 1: Magnetic tracking
Problem setup

We have:

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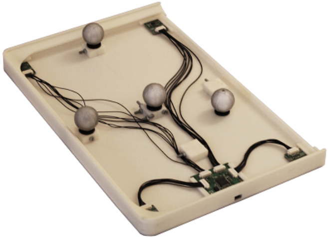


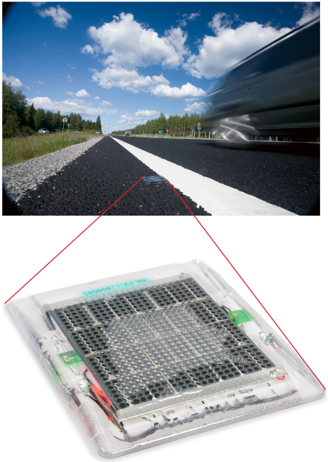
What we want to know: The position and orientation of the magnetic object

What we observe: The magnetic field measured by the magnetometers

Physical laws: The magnetic dipole model

Example 1: Magnetic tracking
Interactive 3D-modelling

Example 1: Magnetic tracking
Traffic surveillance



Traffic surveillance with wireless sensors

- Each unit is equipped with a magnetometer

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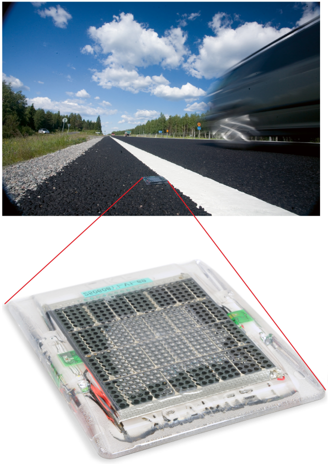
Traffic surveillance



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Limitations:

- Powered by solar cells
- Small computational budget

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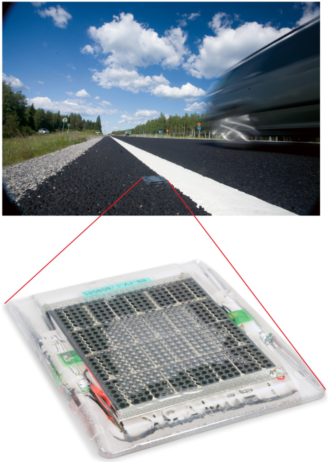

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Information that can be retrieved

- Count number of cars
- Type of car
- Driving direction

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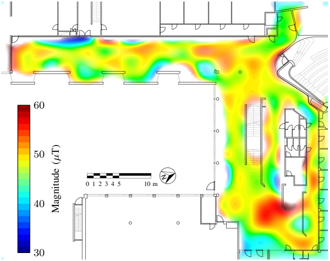
   a. **Example 1, Magnetic tracking:** Measure the position and orientation of a known magnetic source.

   b. **Example 2, Magnetic mapping:** Build a map of the (indoor) magnetic field.

Example 2: Magnetic mapping - Build magnetic maps

- The magnetic field changes due to metal structures in the buildings.
- These changes can be used as a map!



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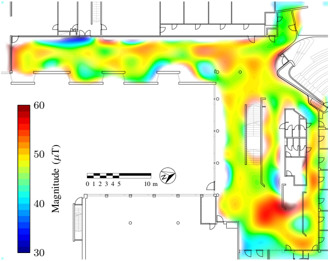

What we want to know: The magnetic field

Training data: The magnetic field at a few places

Physical laws: The magnetic field must satisfy Maxwell's equations

Example 2: Magnetic mapping - Build magnetic maps

Arno Solin, Manon Kok, Niklas Wahlström, Thomas B. Schön and Simo Särkkä, **Modeling and interpolation of the ambient magnetic field by Gaussian processes** *IEEE Transactions on Robotics*, 34(4):1112 – 1127, 2018

Example 3: Strain field reconstruction - intuition

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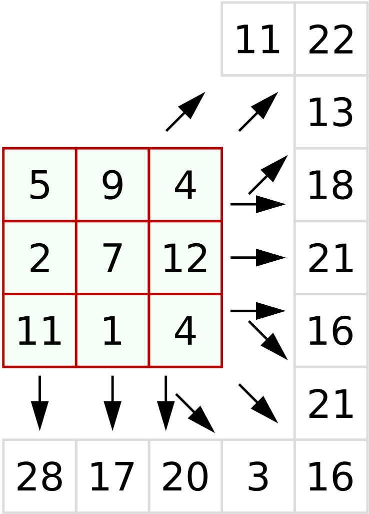

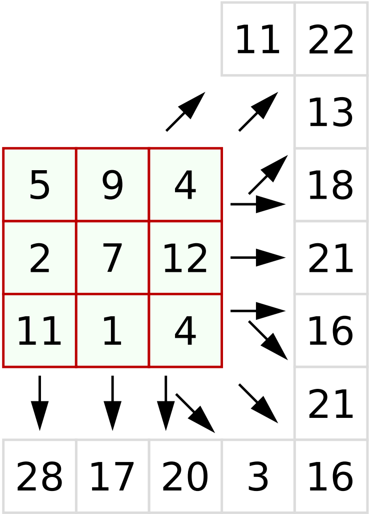
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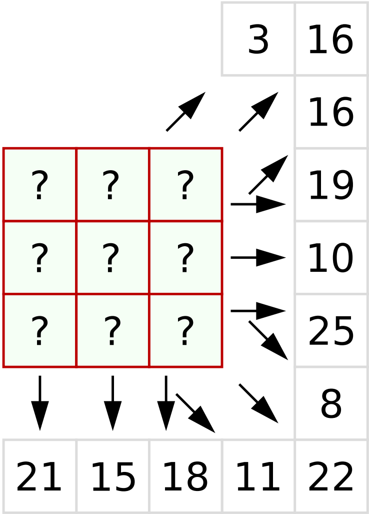
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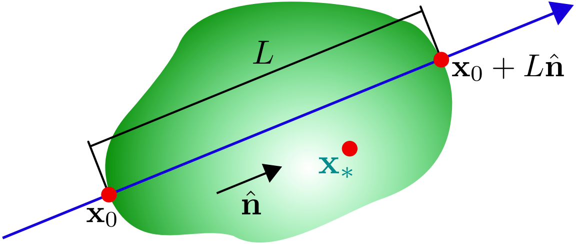
Example 3: Strain field reconstruction – problem setup



What we want to know: The strain field

Training data: Neutron beams through the object in various directions

Physical laws: A physical strain field must satisfy the *equilibrium constraints*

$$0 = \frac{\partial f_{\mathbf{xx}}(\vec{x})}{\partial x} + (1-\nu)\frac{\partial f_{\mathbf{xy}}(\vec{x})}{\partial y} + \nu\frac{\partial f_{\mathbf{yy}}(\vec{x})}{\partial x}$$

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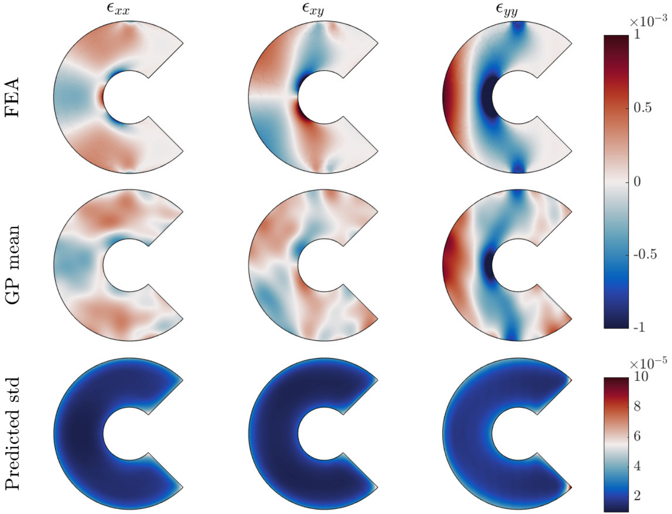
Example 3: Strain field reconstruction – experimental results



Carl Jidling, Johannes Hendriks, Niklas Wahlström, Alexander Gregg, Thomas B. Schön, Chris Wensrich, Adrian Wills. **Probabilistic modelling and reconstruction of strain**, *Nuclear instruments and methods in physics research section B*, 436:141-155, 2018.

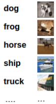
Summary

In physics-based machine learning we make use first principles to leverage the performance of data-driven machine learning models.

I have presented three examples from our research which exemplify this

Example 1:
Magnetic tracking ↓

Example 3:
Strain field reconstruction ↓



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