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Magnetic tracking and mapping

Niklas Wahlström

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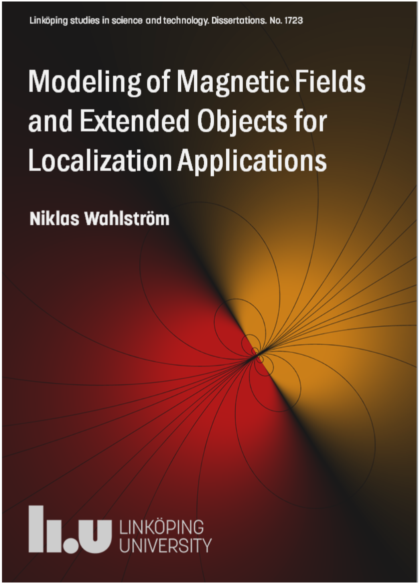
Short about me

- 2005 - 2010: Applied Physics and Electrical Engineering - International, Linköping University.
  - 2007-2008: Exchange student, ETH Zürich, Swizerland
- 2010-2015 : PhD student in Automatic Control, Linköping University
  - Spring 2014, Research visit, Imperial College, London, UK
- *2016- : Postdoctoral researcher at Department of Information Technology, Uppsala University*

My thesis



Three areas:

- **Magnetic tracking and mapping**
- Extended target tracking
- Deep dynamical models for control

Papers about magnetic tracking/mapping

Paper A: N. Wahlström, F. Gustafsson, **Tracking Position of Magnetic Objects Using Magnetometer Networks** *in Modeling of Magnetic Fields and Extended Object for Localization Applications.* PhD Thesis.

Paper B: N. Wahlström, R. Hostettler, F. Gustafsson and W. Birk, **Classification of Driving Direction in Traffic Surveillance using Magnetometers**. *IEEE Transactions on Intelligent Transportation Systems*. 15(4), pp 1405-1418

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Magnetometer measurement models

1. **Common use:** Magnetometer provides **orientation** heading information.

   Assume that the magnetometer (almost) only measures the local (earth) magnetic field.

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Maxwell's equations

$$\begin{aligned}
\boldsymbol{\nabla} \cdot \mathbf{E} &= \frac{\rho}{\varepsilon_0} \\
\boldsymbol{\nabla} \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} &= \mathbf{0} \\
\boldsymbol{\nabla} \cdot \mathbf{B} &= 0 \\
\boldsymbol{\nabla} \times \mathbf{B} - \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} &= \mu_0 \mathbf{J}
\end{aligned}$$

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Maxwell's equations

Electrostatics

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \times \mathbf{E} + \cancel{\frac{\partial \mathbf{B}}{\partial t}} = \mathbf{0}$$

Magnetostatics

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{B} - \cancel{\mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}} = \mu_0 \mathbf{J}$$

$\mathbf{E}$: Electric field
ρ: Charge density
$\mathbf{B}$: Magnetic field
$\mathbf{J}$: Current density

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$\mathbf{E}$: Electric field

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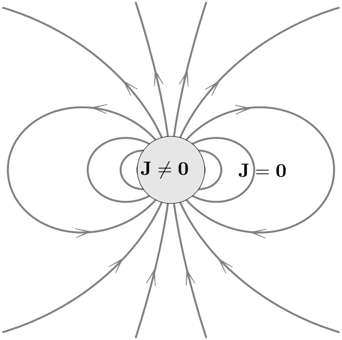
The magnetostatic equations are difficult to solve...

Magnetic dipole

...however, if the current density is localized, an analytical solution exists!



Magnetic dipole field

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \frac{3(\mathbf{r}\cdot\mathbf{m})\mathbf{r} - \|\mathbf{r}\|^2\mathbf{m}}{\|\mathbf{r}\|^5}$$

Magnetic dipole moment

$$\mathbf{m} \triangleq \frac{1}{2}\int \mathbf{r}' \times \mathbf{J}(\mathbf{r}') d^3 r'$$

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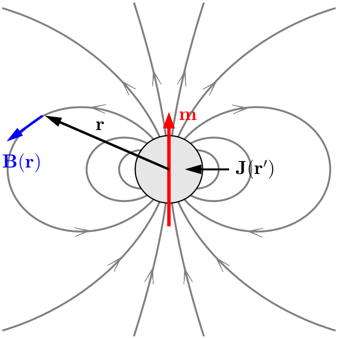

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Sensor model - single dipole

The measurements can be described with a state-space model

$$\mathbf{x}_{k+1} = F_k \mathbf{x}_k + G_k \mathbf{w}_k, \qquad \mathbf{w}_k \sim \mathcal{N}(\mathbf{0}, Q),$$
$$\mathbf{y}_{k,j} = \mathbf{h}_j(\mathbf{x}_k) + \mathbf{e}_k, \qquad \mathbf{e}_k \sim \mathcal{N}(\mathbf{0}, R)$$

Point target sensor model (one dipole)

$$\mathbf{h}_j(\mathbf{x}_k) = C(\mathbf{r}_k - \boldsymbol{\theta}_j)\mathbf{m}_k, \qquad \mathbf{x}_k = [\mathbf{r}_k^\mathsf{T} \quad \mathbf{v}_k^\mathsf{T} \quad \mathbf{m}_k^\mathsf{T} \quad \boldsymbol{\omega}_k^\mathsf{T}]^\mathsf{T}$$
$$C(\mathbf{r}) = \frac{\mu_0}{4\pi\|\mathbf{r}\|^5}(3\mathbf{r}\mathbf{r}^\mathsf{T} - \|\mathbf{r}\|^2 I_3),$$

Measurement from a sensor network of magnetometers positioned at $\{\boldsymbol{\theta}_j\}_{j=1}^J$.

Degrees of freedom

- 3D position
- 2D orientation

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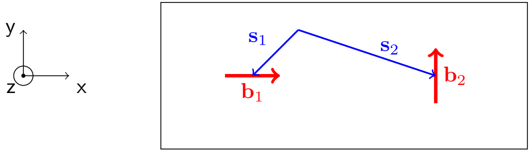
Sensor model - multi-dipole

The measurements can be described with a state-space model

$$\mathbf{x}_{k+1} = F_k \mathbf{x}_k + G_k \mathbf{w}_k, \qquad \mathbf{w}_k \sim \mathcal{N}(\mathbf{0}, Q),$$
$$\mathbf{y}_{k,j} = \mathbf{h}_j(\mathbf{x}_k) + \mathbf{e}_k, \qquad \mathbf{e}_k \sim \mathcal{N}(\mathbf{0}, R)$$

Extended target sensor model (a structure of dipoles)

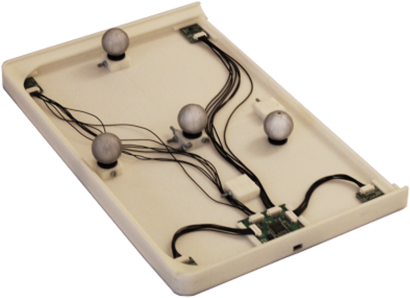
$$\mathbf{h}_j(\mathbf{x}_k) = \sum_{l=1}^{L} C(\mathbf{r}_k + R_k(\mathbf{q}_k)\mathbf{s}_l - \boldsymbol{\theta}_j) m_l R_k(\mathbf{q}_k)\mathbf{b}_l,$$
$$\mathbf{x}_k = [\mathbf{r}_k^{\mathsf{T}} \quad \mathbf{v}_k^{\mathsf{T}} \quad \mathbf{q}_k^{\mathsf{T}} \quad \boldsymbol{\omega}_k^{\mathsf{T}}]^{\mathsf{T}}$$

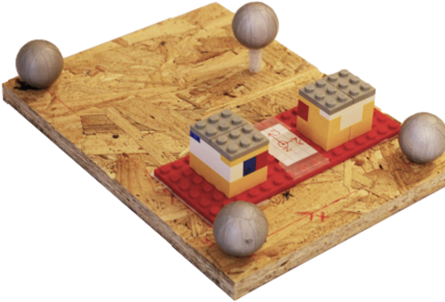


Degrees of freedom

- 3D position
- **3D** orientation

Experiment - setup





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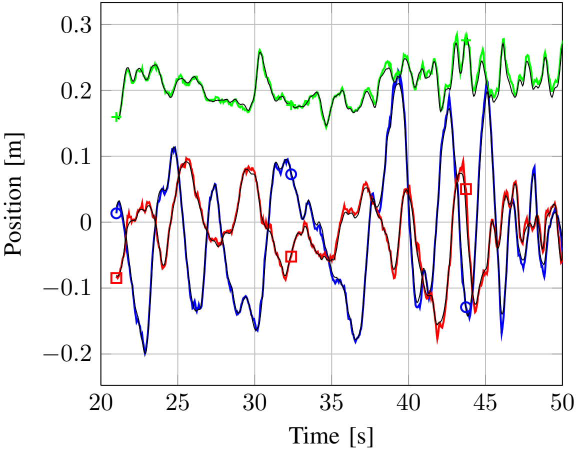
Experiment - results - position



Black: Ground truth position. Color: Estimated position

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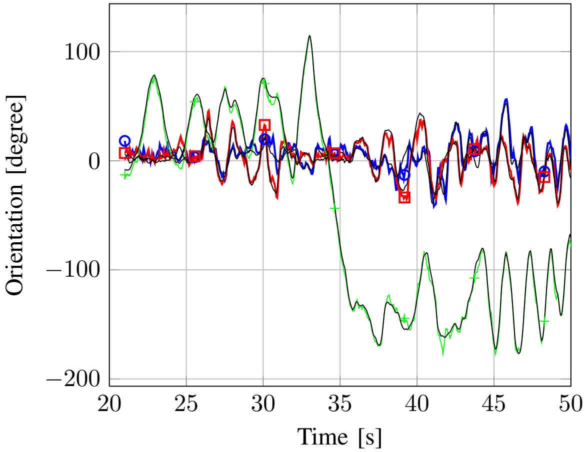
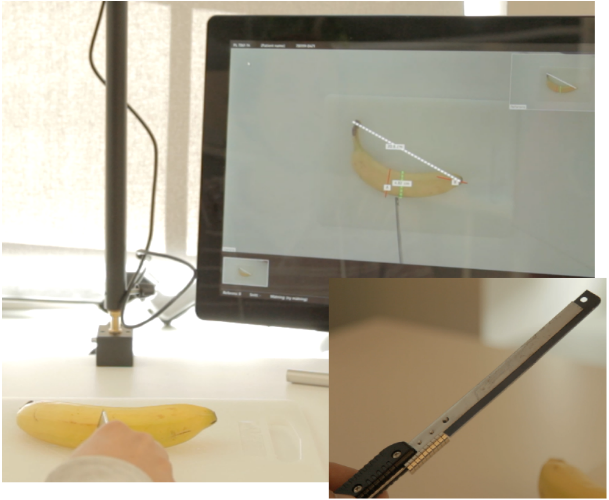
Experiment - results - orientation



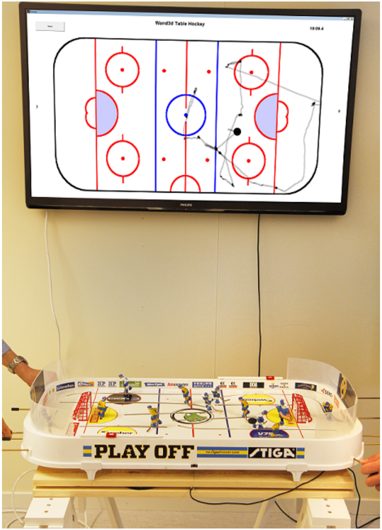
Black: Ground truth orientation. Color: Estimated orientation

Application 1: Digital pathology

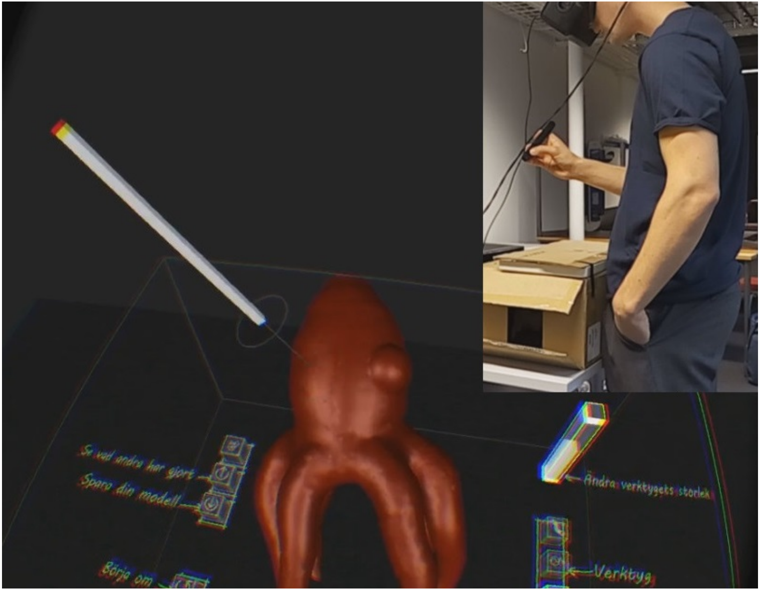


Application 2: Digital water colors

Application 3: Digital table hockey



Application 4: Interactive 3D modeling



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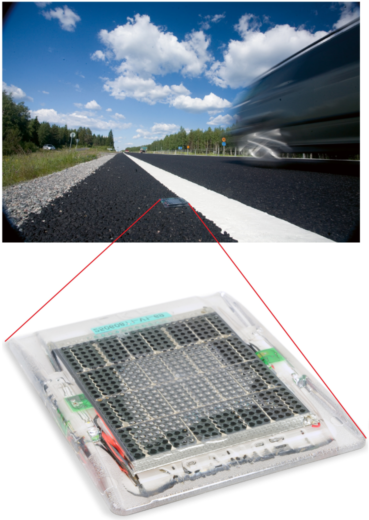
Background



Traffic monitoring in wireless sensor network

- Sensor nodes equipped with a magnetometer.

Limitations:

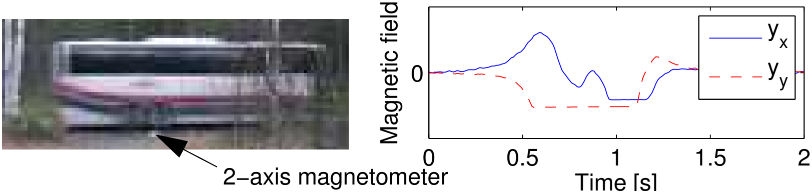
- Energy budget
- Computational resources.

Information you can extract

- Number of vehicles
- Type of vehicle
- *Heading direction*

Problem formulation

- 2-axis magnetometer has been deployed on the roadside



- Magnetometer measures a distortion of the magnetic field.

We want to classify the heading direction of the vehicle!

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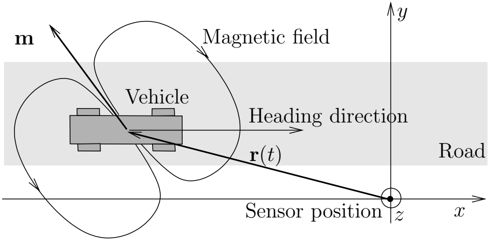
Magnetic dipole

The vehicle can be modeled as a magnetic dipole:

$$\mathbf{h}(t) = \frac{3(\mathbf{r}(t) \cdot \mathbf{m})\mathbf{r}(t) - \|\mathbf{r}(t)\|^2 \mathbf{m}}{\|\mathbf{r}(t)\|^5}$$

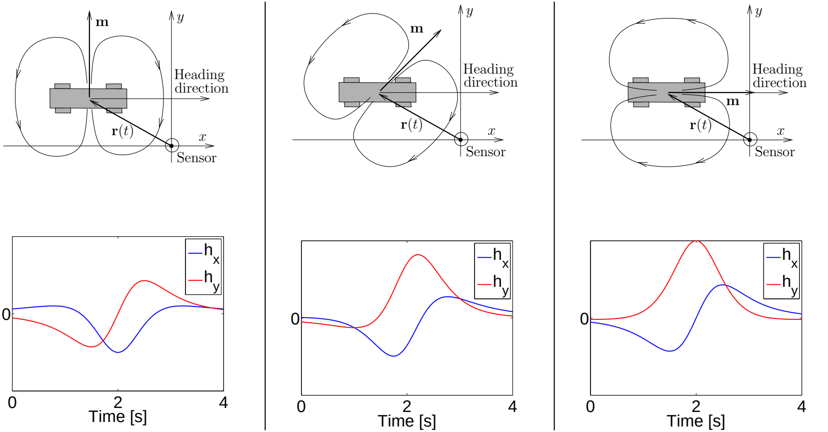


We measure two components of the magnetic field.

$$\mathbf{y}(kT) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \mathbf{h}(kT) + \mathbf{e}(kT), \quad k = 1, \cdots, N$$

Simulated examples

Three different vehicle are heading in positive x-direction



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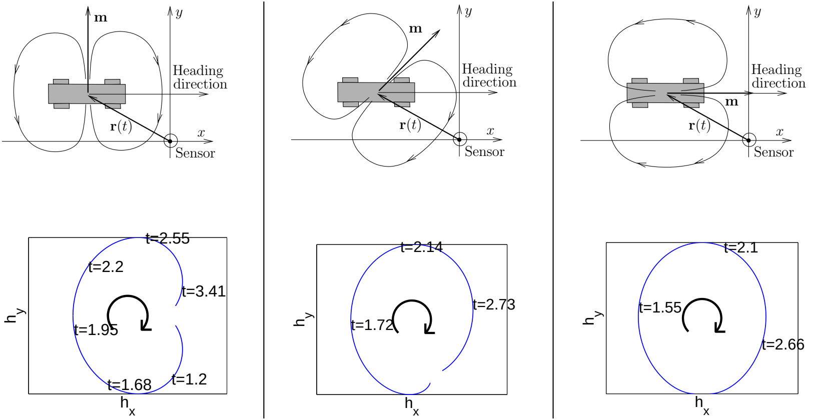
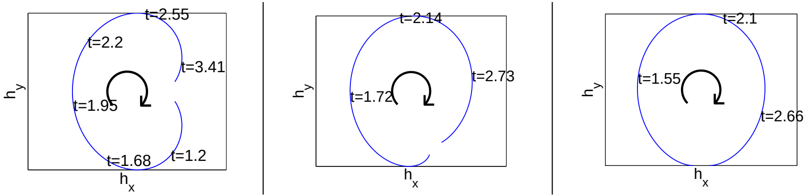


- All measurement trajectories are turning clockwise!

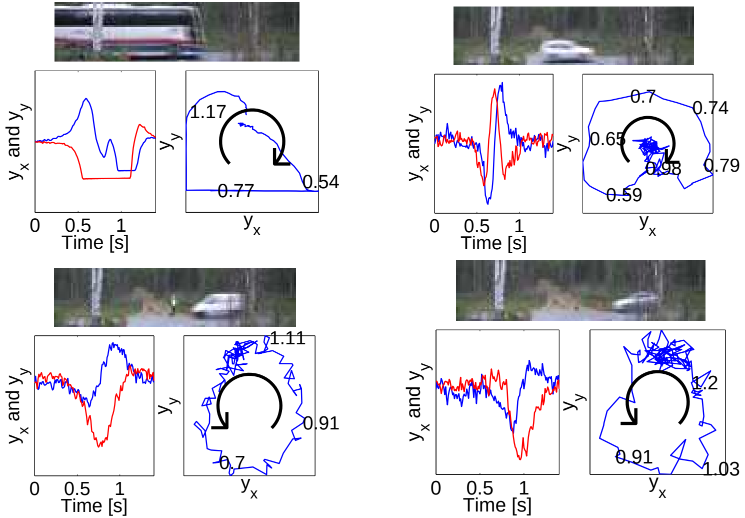
Key idea



Idea:

Classify heading direction by the turn of the measurement trajectory!

Real world data



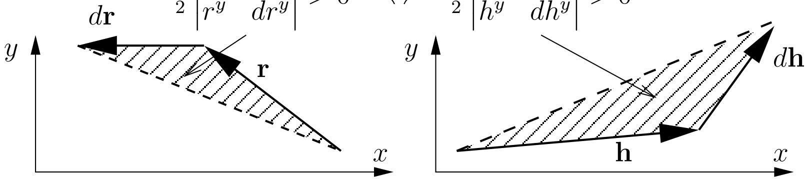
The theorem

Assume the magnetic dipole model

$$\mathbf{h} = \frac{3(\mathbf{r}\cdot\mathbf{m})\mathbf{r} - \|\mathbf{r}\|^2\mathbf{m}}{\|\mathbf{r}\|^5}$$

then

$$\frac{1}{2}\begin{vmatrix} r^x & dr^x \\ r^y & dr^y \end{vmatrix} > 0 \quad \Leftrightarrow \quad \frac{1}{2}\begin{vmatrix} h^x & dh^x \\ h^y & dh^y \end{vmatrix} > 0$$



Observe: Independent of $\mathbf{m}$!

The classifier

Integrate over all infinitesimal area segments

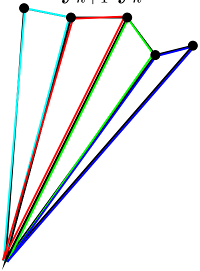
$$f = \int \begin{vmatrix} h^x & dh^x \\ h^y & dh^y \end{vmatrix} = \int \begin{vmatrix} h^x(t) & dh^x(t)/dt \\ h^y(t) & dh^y(t)/dt \end{vmatrix} dt.$$

The time discrete version will then be

$$\begin{aligned} f = & \sum_{k=1}^{N-1} \begin{vmatrix} h_k^x & (h_{k+1}^x - h_k^x)/T \\ h_k^y & (h_{k+1}^y - h_k^y)/T \end{vmatrix} T \\ = & \sum_{k=1}^{N-1} (h_k^x h_{k+1}^y - h_k^y h_{k+1}^x) \\ = & (\mathbf{h}_{1:(N-1)}^x)^T \mathbf{h}_{(1+1):N}^y - (\mathbf{h}_{1:(N-1)}^y)^T \mathbf{h}_{(1+1):N}^x \end{aligned}$$

The classifier

- Sum over all triangles
- The enclosed area can be computed as two inner products!



$$\hat{f} = (\mathbf{y}^x_{1:(N-1)})^T \mathbf{y}^y_{(1+1):N} - (\mathbf{y}^y_{1:(N-1)})^T \mathbf{y}^x_{(1+1):N}$$

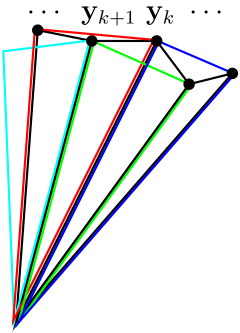
- The sign of $\hat{f}$ determines the heading direction.

Note: $\mathbf{h}$ has been replaced with the measurement $\mathbf{y}$ which contains noise.

The improved classifier

The variance of $\hat{f}$ can be reduced by trading for some bias.

- Idea: Average over larger triangles!

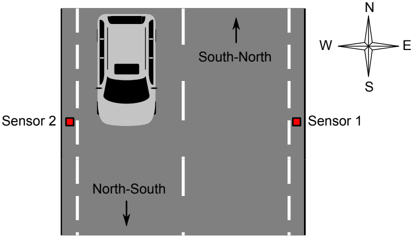


$$\hat{f}_p = (\mathbf{y}^x_{1:(N-p)})^T \mathbf{y}^y_{(1+p):N} - (\mathbf{y}^y_{1:(N-p)})^T \mathbf{y}^x_{(1+p):N}$$

Observe: The feature still only consists of two inner products!

Experimental results

- 2 sensor nodes
- 45 min
- 88 vehicles travelling south-north
- 99 vehicles travelling north-south



Correct classification by the two sensors

	South-North (Sensor 1)	North-South (Sensor 2)
Sensor 1	87/88	91/99
Sensor 2	82/88	99/99

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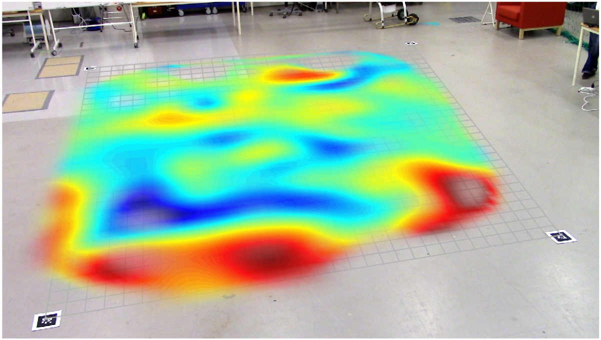
Magnetic mapping

Build a map of the indoor magnetic field using Gaussian processes.



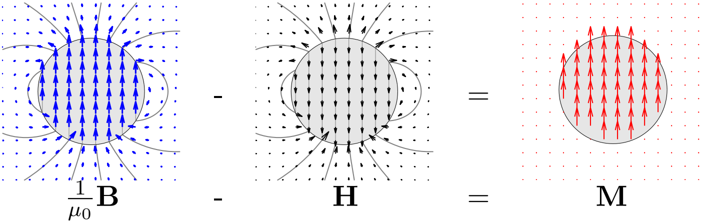
Magnetic fields

We use a slightly different version of the magnetostatic equations

$$\nabla \cdot \mathbf{B} = 0, \qquad \frac{1}{\mu_0}\mathbf{B} - \mathbf{H} = \mathbf{M},$$

$$\nabla \times \mathbf{H} = \mathbf{0}$$

Example



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Gaussian processes

Gaussian processes can be seed as a distribution over functions

$$\mathbf{f}(\mathbf{u}) \sim \mathcal{GP}\big(\boldsymbol{\mu}(\mathbf{u}), K(\mathbf{u}, \mathbf{u}')\big),$$

Mean function ↑ ↑ Covariance function

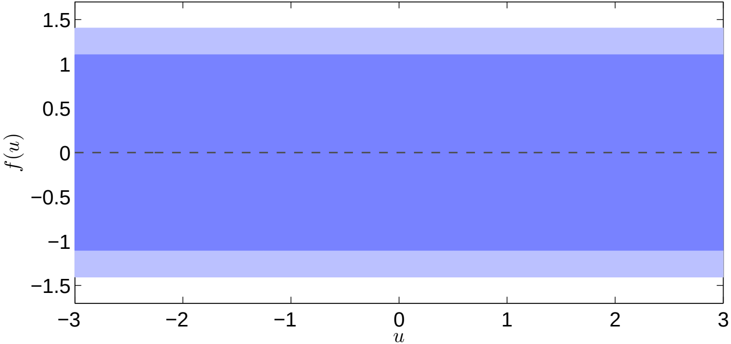
It is a generalization of the multivariate Gaussian distribution

$$\begin{bmatrix} \mathbf{f}(\mathbf{u}_1) \\ \vdots \\ \mathbf{f}(\mathbf{u}_N) \end{bmatrix} \sim \mathcal{N}(\boldsymbol{\mu}, K), \quad \text{where} \quad \boldsymbol{\mu} = \begin{bmatrix} \boldsymbol{\mu}(\mathbf{u}_1) \\ \vdots \\ \boldsymbol{\mu}(\mathbf{u}_N) \end{bmatrix},$$

$$K = \begin{bmatrix} K(\mathbf{u}_1, \mathbf{u}_1) & \cdots & K(\mathbf{u}_1, \mathbf{u}_N) \\ \vdots & & \vdots \\ K(\mathbf{u}_N, \mathbf{u}_1) & \cdots & K(\mathbf{u}_N, \mathbf{u}_N) \end{bmatrix}.$$

Gaussian process regression

Objective: Estimate $f(u)$ from noisy observations $y_k = f(u_k) + e_k$



Gaussian process + magnetic fields

- The animation illustrated regression for one scalar function $f : \mathbb{R} \rightarrow \mathbb{R}$

Gaussian process + magnetic fields

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- We want to learn three different vector fields $\mathbf{f} : \mathbb{R}^3 \to \mathbb{R}^3$ In addition, these fields should obey

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  - $\boldsymbol{\nabla} \cdot \mathbf{B} = 0$ (divergence free)
  - $\boldsymbol{\nabla} \times \mathbf{H} = 0$ (curl free)

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$\leftarrow$ There exist covariance functions for this!

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  - $\frac{1}{\mu_0}\mathbf{B} - \mathbf{H} = \mathbf{M}$

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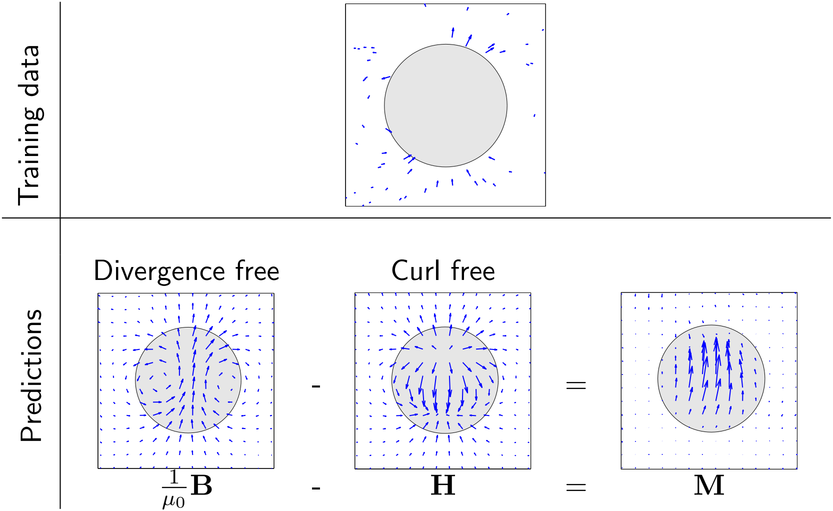
Simulation


Training data
Predictions
Divergence free
Curl free
-
=
$\frac{1}{\mu_0}\mathbf{B}$ - $\mathbf{H}$ = $\mathbf{M}$

Building magnetic field maps (1)

Build a map of the indoor magnetic field using Gaussian processes.

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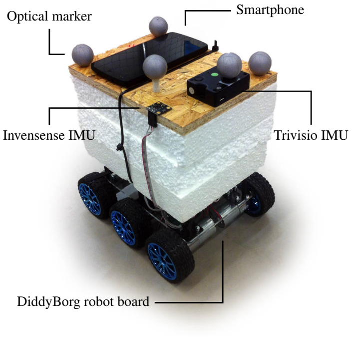

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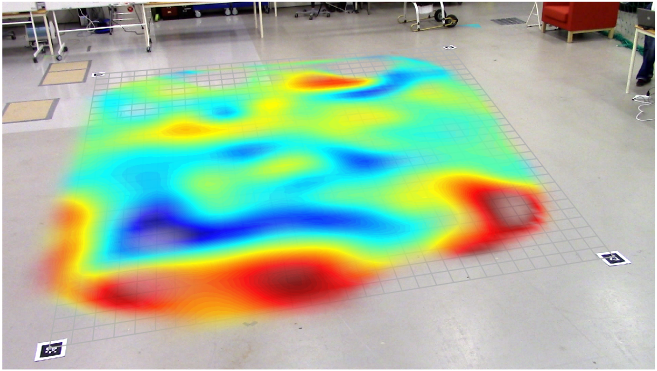

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Modeling the magnetic field

- The magnetic field $\mathbf{H}$ is curl-free, i.e. $\nabla \times \mathbf{H} = \mathbf{0}$ [1]

$$\mathbf{y}_k = \mathbf{f}(\mathbf{x}_k) + \boldsymbol{\varepsilon}_k$$
$$\mathbf{f}(\mathbf{x}) \sim \mathcal{GP}(\mathbf{0}, \sigma^2_{\text{const.}} I_3 + K_{\text{curl}}(\mathbf{x}, \mathbf{x}'))$$

[1] Niklas Wahlström, Manon Kok, Thomas B. Schön and Fredrik Gustafsson, **Modeling magnetic fields using Gaussian processes** *The 38th International Conference on Acoustics, Speech, and Signal Processing (ICASSP)*, Vancouver, Canada, May 2013.

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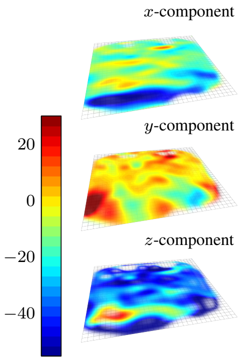
- If a vector-field is curl-free, a scalar potential φ exists $\mathbf{H} = -\nabla \varphi$ [2]

$$\mathbf{y}_k = -\nabla \varphi(\mathbf{x})\Big|_{\mathbf{x}=\mathbf{x}_i} + \boldsymbol{\varepsilon}_k$$
$$\varphi(\mathbf{x}) \sim \mathcal{GP}\left(0,\; k_{\text{lin.}}(\mathbf{x}, \mathbf{x}') + k_{\text{SE}}(\mathbf{x}, \mathbf{x}')\right)$$

[1] Niklas Wahlström, Manon Kok, Thomas B. Schön and Fredrik Gustafsson, **Modeling magnetic fields using Gaussian processes** *The 38th International Conference on Acoustics, Speech, and Signal Processing (ICASSP)*, Vancouver, Canada, May 2013.
[2] Arno Solin, Manon Kok, Niklas Wahlström, Thomas B. Schön and Simo Särkkä, **Modeling and interpolation of the ambient magnetic field by Gaussian processes** *ArXiv e-prints*, September 2015. arXiv:1509.04634.

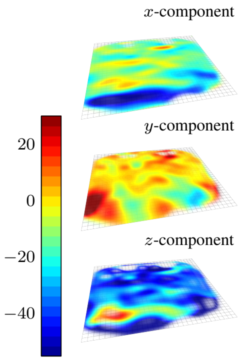
Building magnetic field maps (2)

- Encode physical knowledge in the kernel.



Building magnetic field maps (2)

- Encode physical knowledge in the kernel.
- Use reduced-rank GP regression based on the method from [1].



[1] Hilbert Space Methods for Reduced-Rank Gaussian Process Regression – A. Solin, S. Särkkä.

Building magnetic field maps (2)

- Encode physical knowledge in the kernel.
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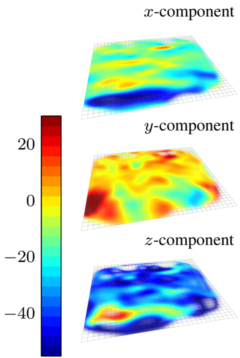

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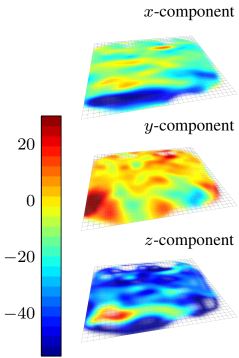
Building magnetic field maps (2)

- Encode physical knowledge in the kernel.
- Use reduced-rank GP regression based on the method from [1].
- Use a Kalman filter formulation to allow for sequential updating.
- Use a spatio-temporal model to allow for changes in the magnetic field.



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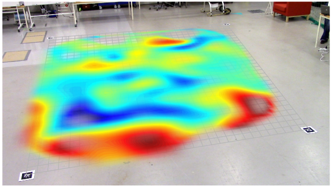
Localization in the map



Building the map:

Arno Solin, Manon Kok, Niklas Wahlström, Thomas B. Schön and Simo Särkkä, **Modeling and interpolation of the ambient magnetic field by Gaussian processes** *IEEE Transactions on Robotics*, 34(4):1112 1127, 2018

Localization in the map:

Arno Solin, Simo Särkkä, Juho Kannala, and Esa Rahtu. **Terrain navigation in the magnetic landscape: Particle filtering for indoor positioning** *In Proceedings of the European Navigation Conference*, Helsinki, Finland, May–June 2016.

SLAM:

Arno Solin, Manon Kok, **Scalable Magnetic Field SLAM in 3D Using Gaussian Process Maps** *ArXiv e-prints*, March 2018. arXiv:1804.01926.

Summary

- **Tracking** position and orientation of magnetic objects with magnetometer sensor network

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- **Vehicle heading direction classification** using a 2-axis magnetometer.

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- **Tracking** position and orientation of magnetic objects with magnetometer sensor network
- **Vehicle heading direction classification** using a 2-axis magnetometer.
- **Mapping magnetic fields** using Gaussian processes.

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Thank you!

Reduced-Rank GPR

Hilbert-space approximation of the covariance operator in terms of an eigenfunction expansion of the Laplace operator in a compact subset of $\mathbb{R}^d$.

- Assume that the measurements are confined to a certain domain.
- Approximate the covariance using the spectral density and a number of eigenvalues and eigenfunctions. For $d = 1$:

$$k(x, x') \approx \sum_{j=1}^{m} S(\lambda_j)\phi_j(x)\phi_j(x')$$

$$\phi_j(x) = \frac{1}{\sqrt{L}} \sin\left(\frac{\pi n_j(x+L)}{2L}\right), \qquad \lambda_j = \frac{\pi j}{2L},$$

- Converges to the true GP when the number of basis functions and the size of the domain goes to infinity.

Hilbert Space Methods for Reduced-Rank Gaussian Process Regression – Arno Solin and Simo Särkkä
(http://arxiv.org/pdf/1401.5508v1.pdf)

Reduced-Rank GPR

Consequences for our problem:

Original formulation:

50 or 100 Hz magnetometer data (in 3D)
$\Rightarrow$ Size of the matrix to invert grows very quickly with each additional second of data
$\Rightarrow$ Downsampling needed and large buildings become infeasible

Reduced-rank formulation:

Possible to use all data
$\Rightarrow$ Size of the problem does not grow for longer data sets

Sequential updating

Initialize $\mu_0 = 0$ and $\Sigma_0 = \Lambda_\theta$ (from the GP prior). For each new observation $i = 1, 2, \ldots, n$ update the estimate according to

$$
\begin{aligned}
S_i &= \nabla\Phi_i \Sigma_{i-1} [\nabla\Phi_i]^\mathsf{T} + \sigma^2_{\text{noise}} \mathcal{I}_3, \\
K_i &= \Sigma_{i-1} [\nabla\Phi_i]^\mathsf{T} S_i^{-1}, \\
\mu_i &= \mu_{i-1} + K_i (y_i - \nabla\Phi_i \mu_{i-1}), \\
\Sigma_i &= \Sigma_{i-1} - K_i S_i K_i^\mathsf{T}.
\end{aligned}
$$

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Spatio-temporal modeling

Model the scalar potential magnetic field instead as

$$\varphi(x,t) \sim \mathcal{GP}(0, \kappa_{\text{lin.}}(x,x') + \kappa_{\text{SE}}(x,x')\kappa_{\text{exp}}(t,t')),$$

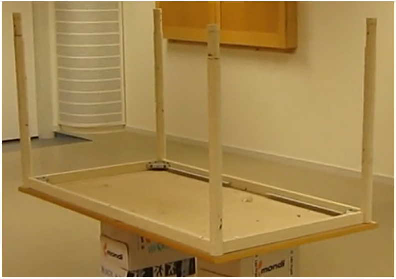
with

$$\kappa_{\text{exp}}(t,t') = \exp\left(- \frac{|t-t'|}{\ell_{\text{time}}} \right).$$

The scalar potential can then sequentially be estimated by adding a *time update* to the *measurement update* from before as

$$\begin{aligned} \tilde{\mu}_i &= A_{i-1}\mu_{i-1}, \\ \tilde{\Sigma}_i &= A_{i-1}\Sigma_{i-1}A_{i-1}^{\mathsf{T}} + Q_{i-1}. \end{aligned}$$

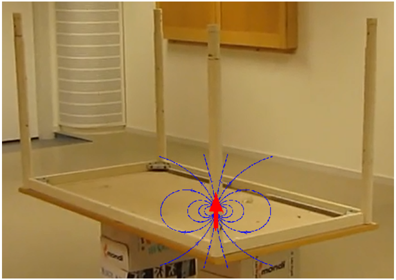
Problem formulation



How should the map be modeled?

We want to find a magnetic map of this object!

Problem formulation



How should the map be modeled?

- Use the dipole model?

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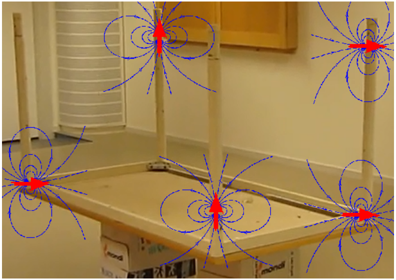

How should the map be modeled?

- Use the dipole model?
- Use multiple dipoles?

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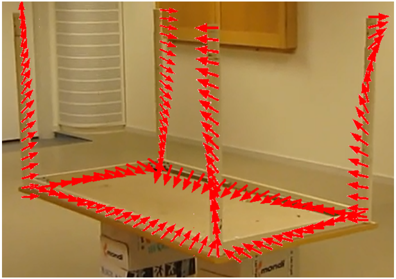
niklas.wahlstrom@it.uu.se

Problem formulation



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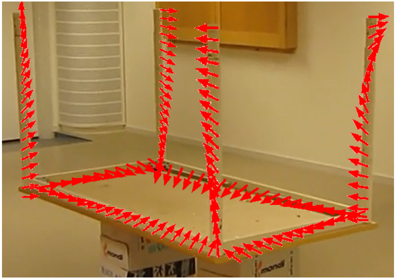

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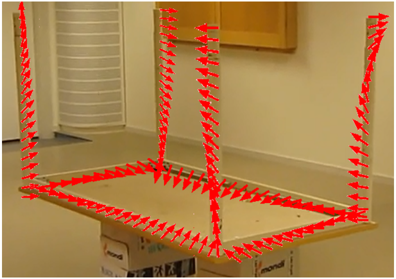

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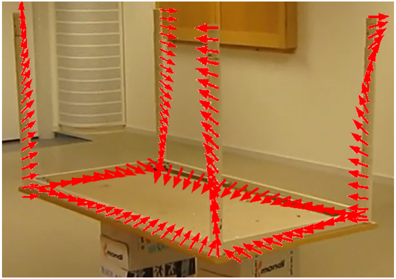

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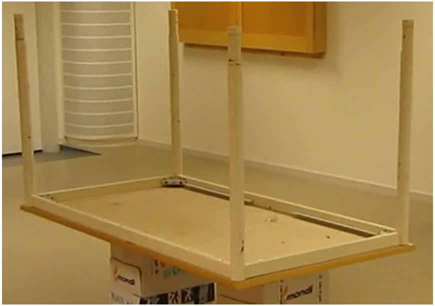
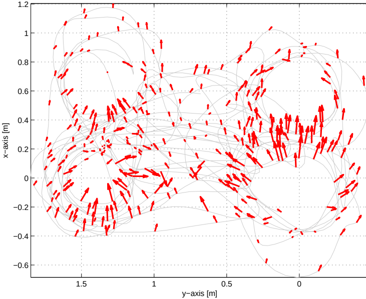
Real world experiment

- Measurements have been collected with a magnetometer
- An optical reference system (Vicon) has been used for determining the position and orientation of the sensor



The magnetic environment



Training data

Real world experiment

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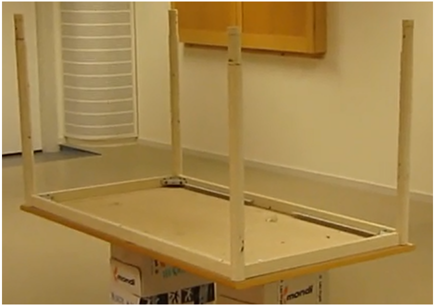

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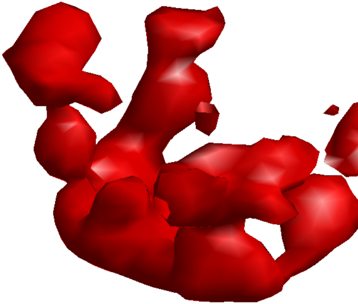

Estimated magnetic content