Modeling of Magnetic Fields and Extended Objects for Localization Applications

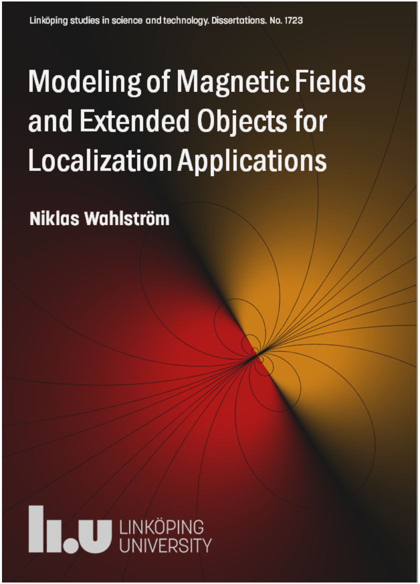
Niklas Wahlström

LINKÖPING
UNIVERSITY

About me

- Grew up in Lindsberg (north of Örebro)
- 2005 - 2010: Applied Physics and Electrical Engineering - International, Linköping University.
  - 2007-2008: Exchange student, ETH Zürich, Swizerland
- 2010- : PhD candidate in Automatic Control, Linköping University
  - Spring 2014, Research visit, Imperial College, London, UK

My thesis


Linköping studies in science and technology. Dissertations. No. 1723
Modeling of Magnetic Fields and Extended Objects for Localization Applications
Niklas Wahlström
LINKÖPING UNIVERSITY

1 Extended Target Tracking Using Gaussian Processes
2 Tracking of Magnetic Objects

LINKÖPING UNIVERSITY

1 Extended Target Tracking Using Gaussian Processes
2 Tracking of Magnetic Objects

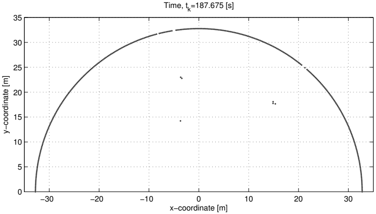
LINKÖPING UNIVERSITY

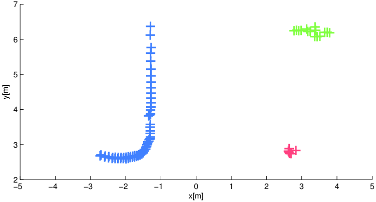
N. Wahlström and E. Özkan.
Extended target tracking using Gaussian processes.
IEEE Transactions on Signal Processing, 63(16):4165–4178, 2015.

LINKÖPING UNIVERSITY

Extended target tracking

Many sensors generate more than one measurement/target





Extended target (definition)

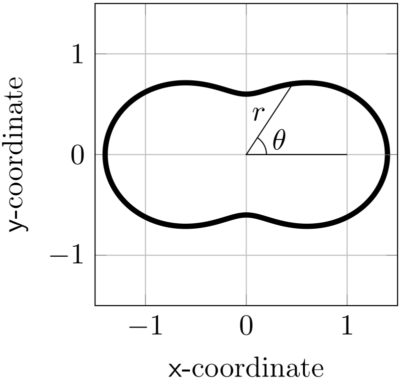
Targets that potentially give rise to multiple measurements at each time step

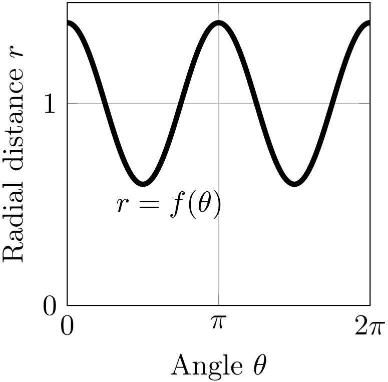
Goal: We want to estimate target position, target orientation and target extent *jointly*.

Related work

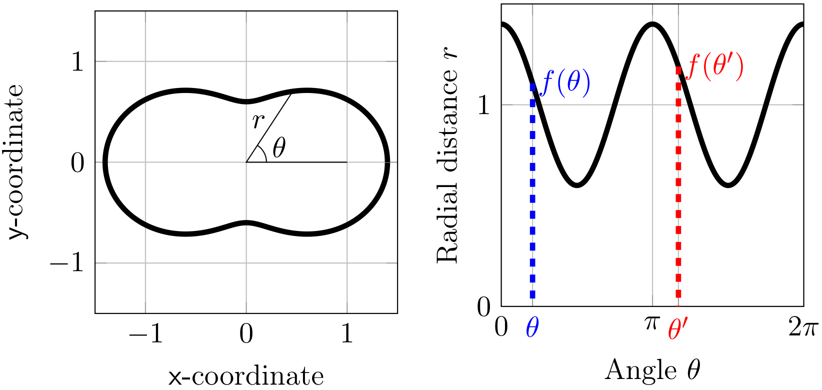
- Elliptical targets using inverse Wishart prior
  - First paper (Koch, Trans. Aerospace 2008).
  - Multiple ellipses per target (Lan and Li, Fusion 2012).
  - Encoding orientation (Granström and Orguner, Trans. Aeorspace 2014)
- Parametrized objects, rectangles, ellipses etc. (Granström, Fusion 2011)
- Random hyper-surface model (Baum and Hanebeck, Fusion 2011)
- Computer vision: B-splines (Blake et al., 1993, 1995, 1998)

Modeling using polar coordinates





Modeling using polar coordinates



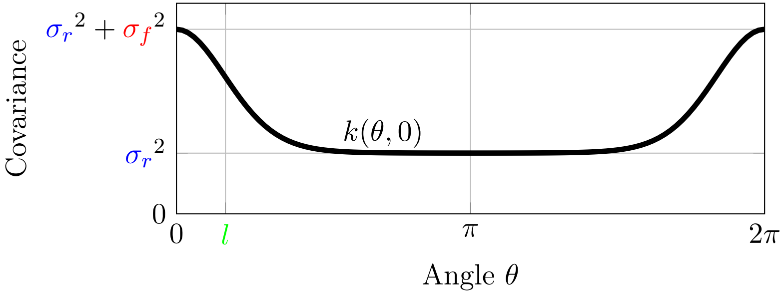
We model $f(\theta)$ using a Gaussian process.

$$f(\theta) \sim \mathcal{GP}(0, k(\theta, \theta')), \qquad \mathbb{E}[f(\theta)f(\theta')] = k(\theta, \theta')$$

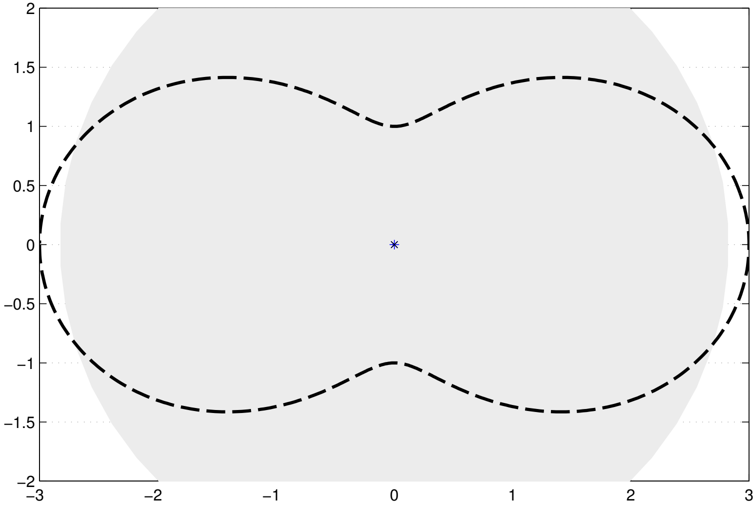
Covariance function

- We use a periodic covariance function to model the periodicity
- We use a additional constant covariance to model a constant (but unknown) mean, corresponding to the mean radius of the target

$$k(\theta, \theta') = {\sigma_f}^2 e^{-\frac{2\sin^2\left(\frac{|\theta-\theta'|}{2}\right)}{l^2}} + {\sigma_r}^2$$



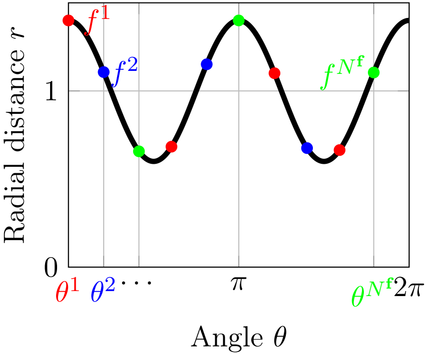
Example



Recursive GP regression

Idea: Consider function values $f^1, f^2, \ldots, f^{N^\mathbf{f}}$ to be the state components

$$\mathbf{x}^\mathbf{f} = \begin{bmatrix} f^1 \\ f^2 \\ \vdots \\ f^{N^\mathbf{f}} \end{bmatrix}$$



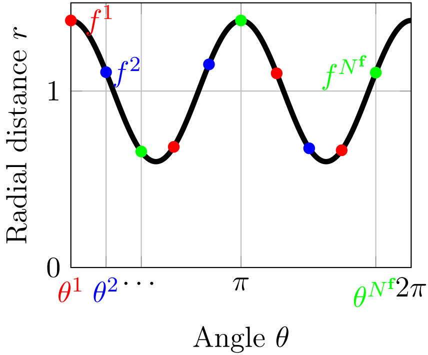
This can be cast into a state space model

$$\mathbf{x}^\mathbf{f}_{k+1} = \mathbf{x}^\mathbf{f}_k$$
$$\mathbf{y}_k = H\mathbf{x}^\mathbf{f}_k + \mathbf{e}_k$$
$$\mathbf{x}^\mathbf{f}_0 \sim \mathcal{N}(0, P^\mathbf{f}_0)$$

Recursive GP regression

Idea: Consider function values $f^1, f^2, \ldots, f^{N^\mathbf{f}}$ to be the state components

$$\mathbf{x}^\mathbf{f} = \begin{bmatrix} f^1 \\ f^2 \\ \vdots \\ f^{N^\mathbf{f}} \end{bmatrix}$$



This can be cast into a state space model

$$\mathbf{x}^\mathbf{f}_{k+1} = \mathbf{x}^\mathbf{f}_k$$
$$\mathbf{y}_k = H\mathbf{x}^\mathbf{f}_k + \mathbf{e}_k$$
$$\mathbf{x}^\mathbf{f}_0 \sim \mathcal{N}(0, P^\mathbf{f}_0)$$

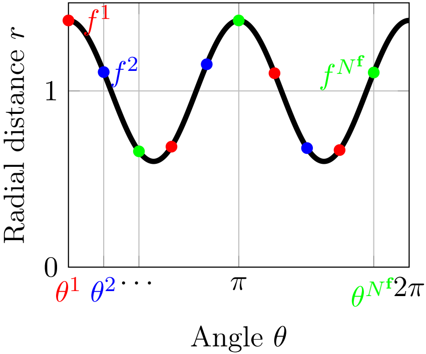
Advantages

- Recursive update with KF

Recursive GP regression

Idea: Consider function values $f^1, f^2, \ldots, f^{N^{\mathbf{f}}}$ to be the state components

$$\mathbf{x}^{\mathbf{f}} = \begin{bmatrix} f^1 \\ f^2 \\ \vdots \\ f^{N^{\mathbf{f}}} \end{bmatrix}$$



This can be cast into a state space model

$$\mathbf{x}^{\mathbf{f}}_{k+1} = F\mathbf{x}^{\mathbf{f}}_k + \mathbf{w}_k$$
$$\mathbf{y}_k = H\mathbf{x}^{\mathbf{f}}_k + \mathbf{e}_k$$
$$\mathbf{x}^{\mathbf{f}}_0 \sim \mathcal{N}(0, P^{\mathbf{f}}_0)$$

Advantages

- Recursive update with KF
- Add process noise

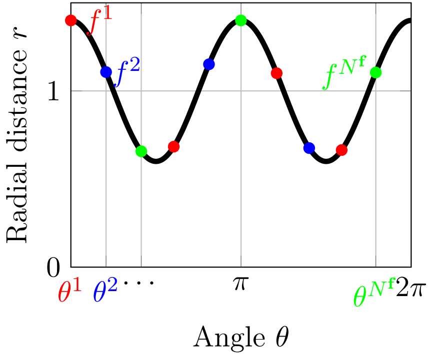
Recursive GP regression

Idea: Consider function values $f^1, f^2, \dots, f^{N^\mathbf{f}}$ to be the state components

$$\mathbf{x}^\mathbf{f} = \begin{bmatrix} f^1 \\ f^2 \\ \vdots \\ f^{N^\mathbf{f}} \end{bmatrix}$$

This can be cast into a state space model

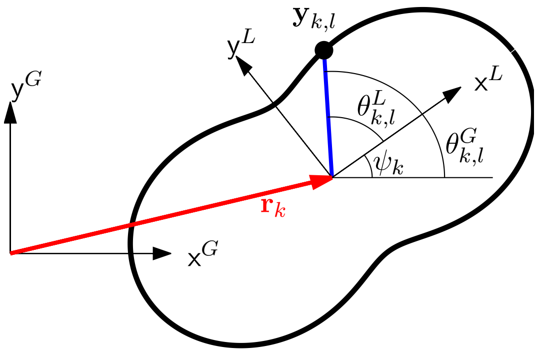
$$\begin{aligned} \mathbf{x}^\mathbf{f}_{k+1} &= F\mathbf{x}^\mathbf{f}_k + \mathbf{w}_k \\ \mathbf{y}_k &= H\mathbf{x}^\mathbf{f}_k + \mathbf{e}_k \\ \mathbf{x}^\mathbf{f}_0 &\sim \mathcal{N}(0, P^\mathbf{f}_0) \end{aligned}$$



Advantages

- Recursive update with KF
- Add process noise
- $\mathbf{x}^\mathbf{f}_k$ can be augmented with target position and orientation

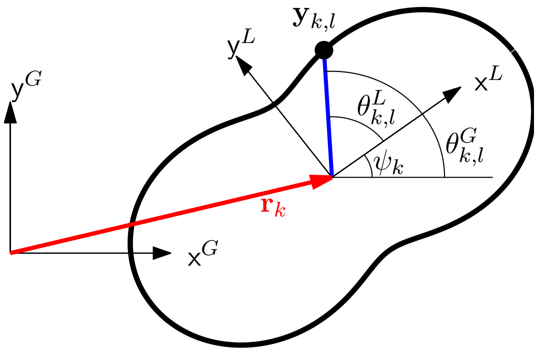
Sensor model



Measurement is the sum of target position and offset due to target extent

$$\mathbf{y}_{k,l} = \mathbf{r}_k + \begin{bmatrix}\cos(\theta^G_{k,l}) \\ \sin(\theta^G_{k,l})\end{bmatrix} f(\theta^L_{k,l}) + \mathbf{e}_{k,l}, \qquad \theta^G_{k,l} = \theta^G_{k,l}(\mathbf{r}_k)$$

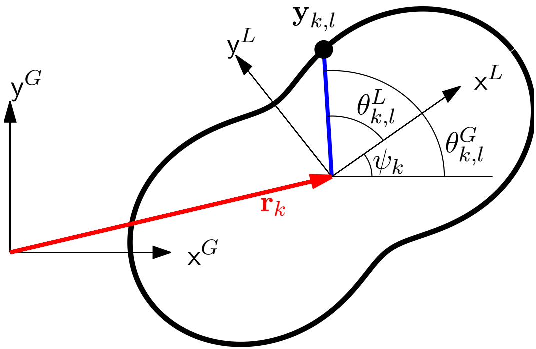
Sensor model



Measurement is the sum of target position and offset due to target extent

$$\mathbf{y}_{k,l} = \mathbf{r}_k + \begin{bmatrix} \cos(\theta^G_{k,l}) \\ \sin(\theta^G_{k,l}) \end{bmatrix} H(\theta^L_{k,l})\mathbf{x}^{\mathbf{f}}_k + \mathbf{e}_{k,l}, \qquad \theta^G_{k,l} = \theta^G_{k,l}(\mathbf{r}_k)$$

Sensor model



This can be summarized into a non-linear sensor model

$$\mathbf{y}_{k,l} = \mathbf{r}_k + H(\mathbf{r}_k, \psi_k)\mathbf{x}^{\mathbf{f}}_k + \mathbf{e}_{k,l}$$

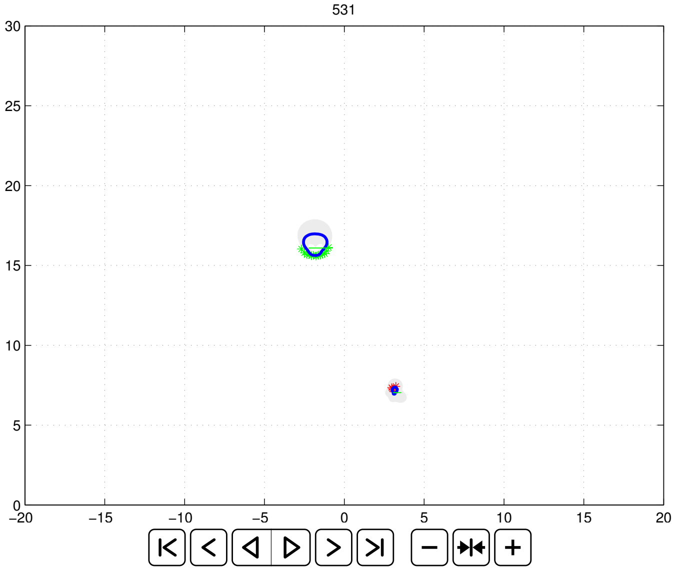
Real data experiment

- Laser range data
- Multi-target scenario (cars, bicycles, humans)
- Almost no clutter

We used a simple logic-based multi-target tracker:

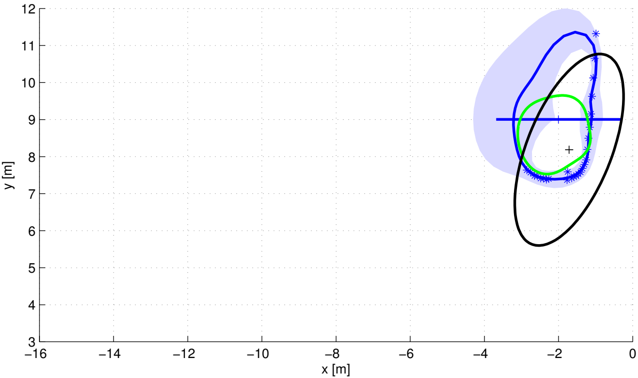
- Gating based likelihood
- Associate a measurement with the most likely target
- Cluster all ungated measurements and form new targets

Real data experiment -result



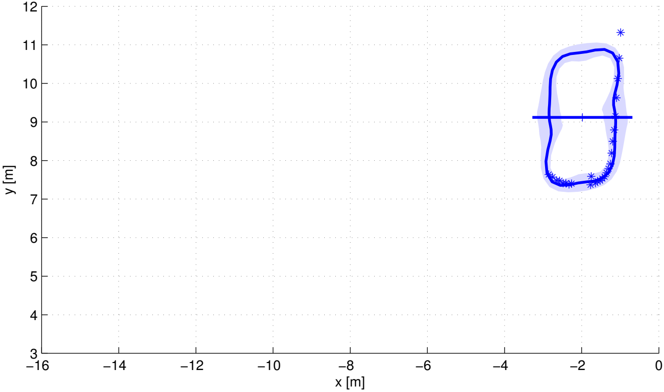
Real data experiment - comparison

Green: RHM (Baum and Hanebeck). Black: Elliptical target (Koch...), Blue: proposed model



Extension - Symmetry assumptions

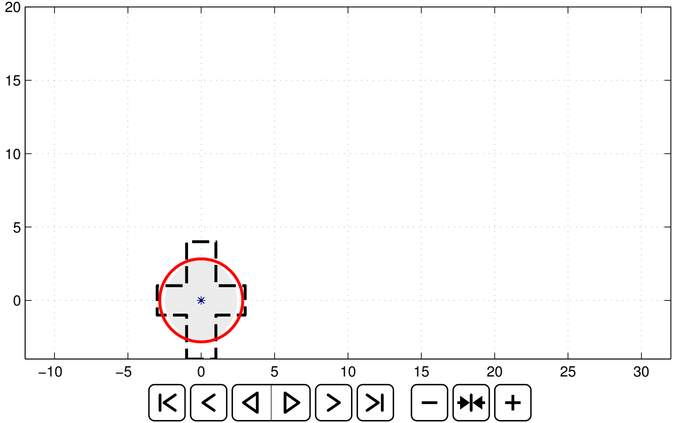
If we assume that $f(\theta)$ has a period of π instead of 2π, we can encode symmetry assumptions.



Extension - Measurements from interior

If the measurements originate from the target interior, we can add a random scalar to compensate for that

$$\mathbf{y}_{k,l} = \mathbf{r}_k + s_{k,l} H(\mathbf{r}_k, \psi_k)\mathbf{x}^{\mathbf{f}}_k + \mathbf{e}_{k,l}, \quad s_{k,l} \in [0, 1]$$



Conclusions

Conclusions

- Model the target extent with a Gaussian process
- Estimate target extent and kinematic state jointly
- Fully recursive update provided

1 Extended Target Tracking Using Gaussian Processes
2 Tracking of Magnetic Objects

LINKÖPING UNIVERSITY

N. Wahlström and F. Gustafsson.
Tracking position and orientation of magnetic objects using magnetometer networks.
IEEE Transactions on Signal Processing, 2015. Submitted.

Magnetic tracking

Advantages

- Cheap sensors

Magnetic tracking

Advantages

- Cheap sensors
- Small sensors

Magnetic tracking

Advantages

- Cheap sensors
- Small sensors
- Low energy consumption

Magnetic tracking

Advantages

- Cheap sensors
- Small sensors
- Low energy consumption
- No weather dependency

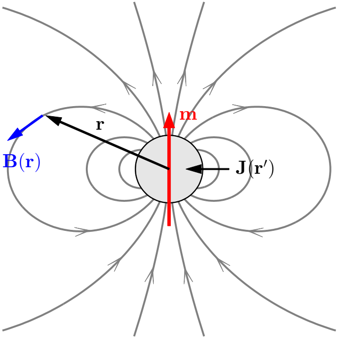
Magnetic tracking

Advantages

- Cheap sensors
- Small sensors
- Low energy consumption
- No weather dependency
- Passive unit, requires no batteries

Mathematical model - dipole field

The magnetic field can be described with a dipole field.



$$\mathbf{B}(\mathbf{r}) = \underbrace{\frac{\mu_0}{4\pi\|\mathbf{r}\|^5}\left(3\mathbf{r}\cdot\mathbf{r}^\mathsf{T} - \|\mathbf{r}\|^2 I_3\right)}_{=C(\mathbf{r})}\mathbf{m}$$

$$\mathbf{m} \triangleq \frac{1}{2}\int \mathbf{r}' \times \mathbf{J}(\mathbf{r}')d^3r'$$

Sensor model - single dipole

The measurements can be described with a state-space model

$$\begin{aligned} \mathbf{x}_{k+1} &= F_k \mathbf{x}_k + G_k \mathbf{w}_k, \qquad \mathbf{w}_k \sim \mathcal{N}(\mathbf{0}, Q), \\ \mathbf{y}_{k,j} &= \mathbf{h}_j(\mathbf{x}_k) + \mathbf{e}_k, \qquad \mathbf{e}_k \sim \mathcal{N}(\mathbf{0}, R) \end{aligned}$$

Point target sensor model (one dipole)

$$\begin{aligned} \mathbf{h}_j(\mathbf{x}_k) &= C(\mathbf{r}_k - \boldsymbol{\theta}_j)\mathbf{m}_k, \qquad \mathbf{x}_k = [\mathbf{r}_k^\mathsf{T} \quad \mathbf{v}_k^\mathsf{T} \quad \mathbf{m}_k^\mathsf{T} \quad \boldsymbol{\omega}_k^\mathsf{T}]^\mathsf{T} \\ C(\mathbf{r}) &= \frac{\mu_0}{4\pi\|\mathbf{r}\|^5}(3\mathbf{r}\mathbf{r}^\mathsf{T} - \|\mathbf{r}\|^2 I_3), \end{aligned}$$

Measurement from a sensor network of magnetometers positioned at $\{\boldsymbol{\theta}_j\}_{j=1}^J$.

Degrees of freedom

- 3D position
- 2D orientation

Sensor model - multi-dipole

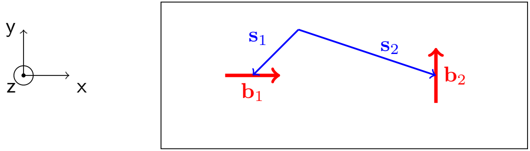
The measurements can be described with a state-space model

$$\begin{aligned} \mathbf{x}_{k+1} &= F_k \mathbf{x}_k + G_k \mathbf{w}_k, \qquad & \mathbf{w}_k &\sim \mathcal{N}(\mathbf{0}, Q), \\ \mathbf{y}_{k,j} &= \mathbf{h}_j(\mathbf{x}_k) + \mathbf{e}_k, \qquad & \mathbf{e}_k &\sim \mathcal{N}(\mathbf{0}, R) \end{aligned}$$

Extended target sensor model (a structure of dipoles)

$$\mathbf{h}_j(\mathbf{x}_k) = \sum_{l=1}^{L} C(\mathbf{r}_k + R_k(\mathbf{q}_k)\mathbf{s}_l - \boldsymbol{\theta}_j) m_l R_k(\mathbf{q}_k)\mathbf{b}_l,$$

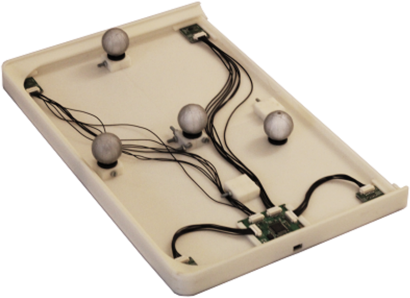
$$\mathbf{x}_k = [\mathbf{r}_k^\mathsf{T} \quad \mathbf{v}_k^\mathsf{T} \quad \mathbf{q}_k^\mathsf{T} \quad \boldsymbol{\omega}_k^\mathsf{T}]^\mathsf{T}$$

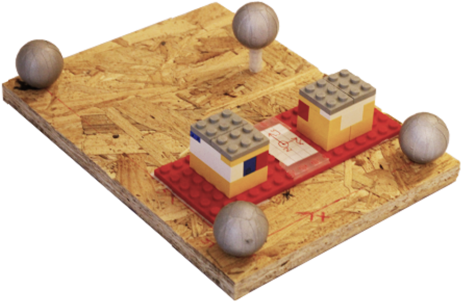


Degrees of freedom

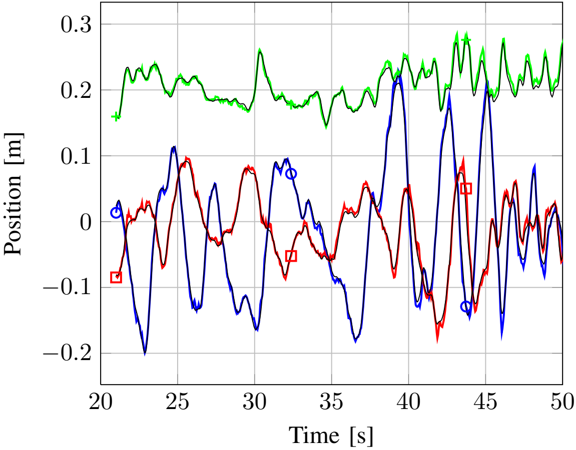
- 3D position
- **3D** orientation

Experiment - setup



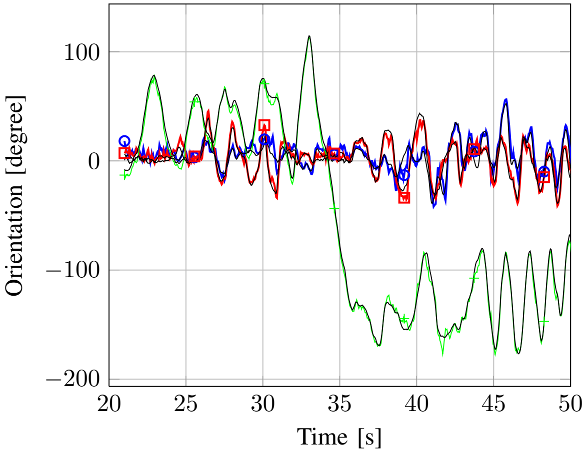


Experiment - results - position



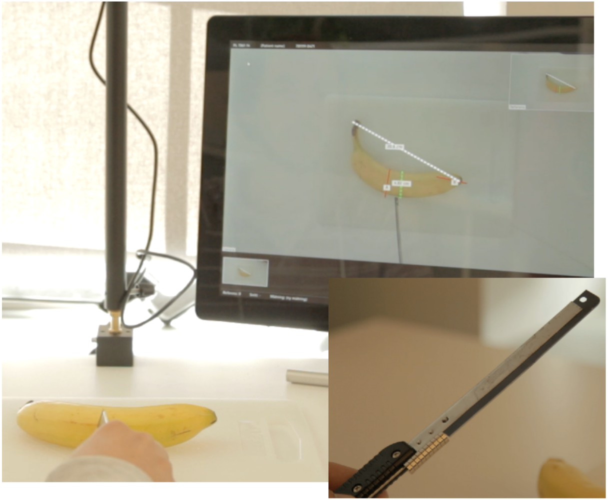
Black: Ground truth position. Color: Estimated position

Experiment - results - orientation



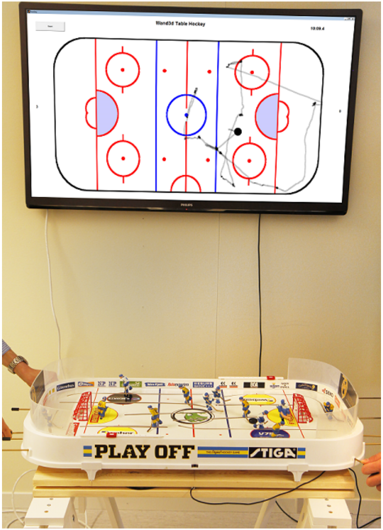
Black: Ground truth orientation. Color: Estimated orientation

Application 1: Digital pathology

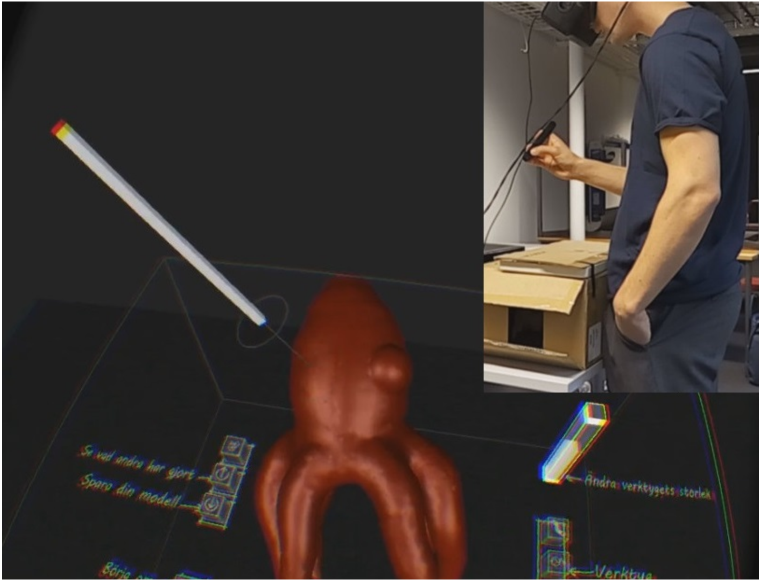


Application 2: Digital water colors

Application 3: Digital table hockey



Application 4: Interactive 3D modeling

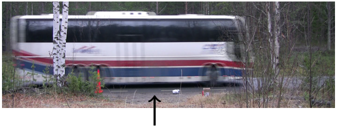


N. Wahlström, R. Hostettler, F. Gustafsson, and W. Birk.
Classification of driving direction in traffic surveillance using magnetometers.
IEEE Transactions on Intelligent Transportation Systems, 15(4):1405–1418, 2014.

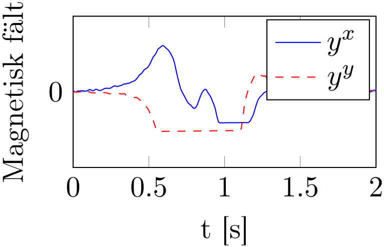
LINKÖPING UNIVERSITY

Problem formulation

- One 2-axis magnetometer has been deployed on the roadside



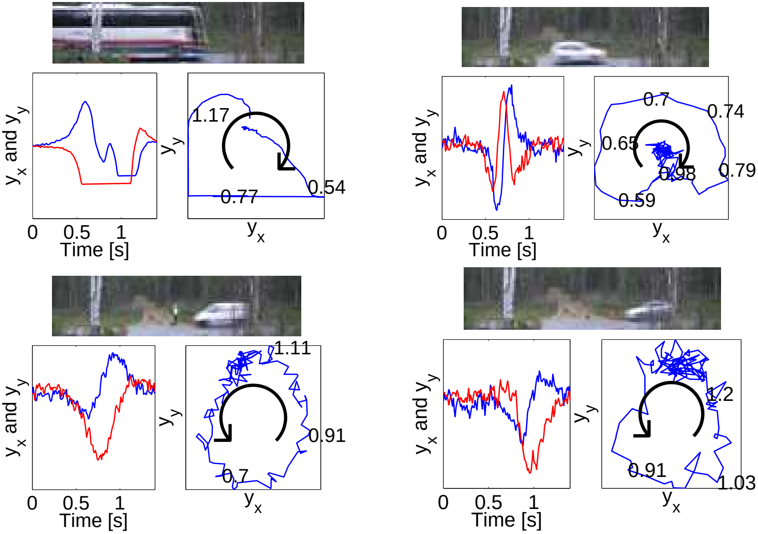
Magnetometer



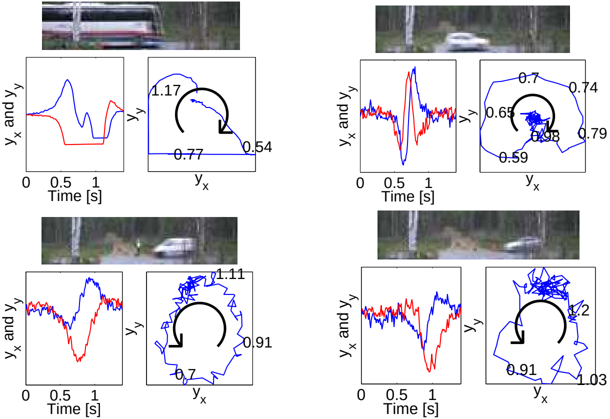
- The magnetometer measures a distortion of the magnetic field.

We want to classify the driving direction of the vehicle!

Real world data



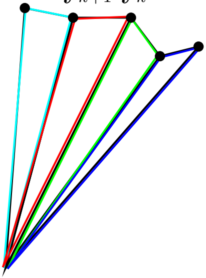
Real world data



Classify driving direction by the turn of the measurement trajectory!

The classifier

The area of one triangle is



$$\frac{1}{2}\begin{vmatrix} y_k^{\mathsf{x}} & y_{k+1}^{\mathsf{x}} \\ y_k^{\mathsf{y}} & y_{k+1}^{\mathsf{y}} \end{vmatrix} = \frac{1}{2}(y_k^{\mathsf{x}} y_{k+1}^{\mathsf{y}} - y_k^{\mathsf{y}} y_{k+1}^{\mathsf{x}})$$

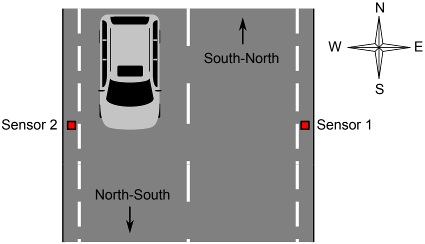
- Sum over all triangles
- The enclosed area can be computed as two inner products!

$$\hat{f} = \frac{1}{2}\big((\mathbf{y}_{1:N}^{\mathsf{x}})^T \mathbf{y}_{2:(N+1)}^{\mathsf{y}} - (\mathbf{y}_{1:N}^{\mathsf{y}})^T \mathbf{y}_{2:(N+1)}^{\mathsf{x}}\big)$$

- The sign of $\hat{f}$ determines the driving direction.

Experimental results

- 2 sensor nodes
- 158 min
- 291 vehicles travelling south-north
- 220 vehicles travelling north-south



Correct classification by the two sensors

	South-North (Sensor 1)	North-South (Sensor 2)
Sensor 1	290/291	189/220
Sensor 2	265/291	220/220

Conclusion

- Extended Target Tracking using GPs

Conclusion

- Extended Target Tracking using GPs
    1. Model the target extent with a Gaussian process

Conclusion

- Extended Target Tracking using GPs
  1. Model the target extent with a Gaussian process

  2. Estimate target extent and kinematic state jointly

Conclusion

- Extended Target Tracking using GPs
  1. Model the target extent with a Gaussian process
  2. Estimate target extent and kinematic state jointly
  3. Fully recursive update provided

Conclusion

- Extended Target Tracking using GPs
  1. Model the target extent with a Gaussian process
  2. Estimate target extent and kinematic state jointly
  3. Fully recursive update provided

Conclusion

- Extended Target Tracking using GPs
  1. Model the target extent with a Gaussian process
  2. Estimate target extent and kinematic state jointly
  3. Fully recursive update provided
- Tracking of Magnetic Objects

Conclusion

- Extended Target Tracking using GPs
    1. Model the target extent with a Gaussian process
    2. Estimate target extent and kinematic state jointly
    3. Fully recursive update provided
- Tracking of Magnetic Objects

Conclusion

- Extended Target Tracking using GPs
  1. Model the target extent with a Gaussian process
  2. Estimate target extent and kinematic state jointly
  3. Fully recursive update provided
- Tracking of Magnetic Objects
  1. Accuracy down to 5 mm and 2°

Conclusion

- Extended Target Tracking using GPs
  1. Model the target extent with a Gaussian process
  2. Estimate target extent and kinematic state jointly
  3. Fully recursive update provided
- Tracking of Magnetic Objects
  1. Accuracy down to 5 mm and 2°
  2. Four different applications were presented

Conclusion

- Extended Target Tracking using GPs
  1. Model the target extent with a Gaussian process
  2. Estimate target extent and kinematic state jointly
  3. Fully recursive update provided
- Tracking of Magnetic Objects
  1. Accuracy down to 5 mm and 2°
  2. Four different applications were presented
  3. Classifying driving direction with only one 2-axis magnetometer

Niklas Wahlström

www.liu.se

LINKÖPING UNIVERSITY