UPPSALA
UNIVERSITET

Pose tracking of magnetic objects

Niklas Wahlström

Department of Information Technology, Uppsala University, Sweden

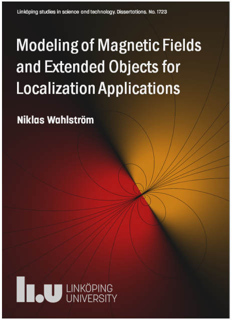
Novmber 13, 2017

UPPSALA UNIVERSITET

Short about me

- 2005 - 2010: Applied Physics and Electrical Engineering - International, Linköping University.
  - 2007-2008: Exchange student, ETH Zürich, Swizerland
- 2010-2015 : PhD student in Automatic Control, Linköping University
  - Spring 2014, Research visit, Imperial College, London, UK
- *2016- : Researcher at Department of Information Technology, Uppsala University*

My thesis and my work at Uppsala



Three areas:

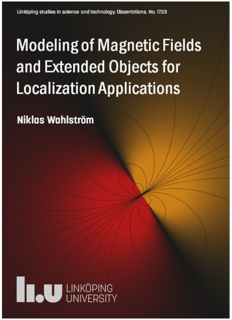
- Magnetic tracking and mapping
- Extended target tracking
- Deep dynamical models for control

UPPSALA
UNIVERSITET

Two areas:

- Constrained Gaussian processes
- Deep learning and system identification

My thesis and my work at Uppsala



Three areas:

- **Magnetic tracking and mapping**
- Extended target tracking
- Deep dynamical models for control

UPPSALA
UNIVERSITET

Two areas:

- Constrained Gaussian processes
- Deep learning and system identification

Magnetometer measurement models

1. **Common use:** Magnetometer provides **orientation** heading information.

   Assume that the magnetometer (almost) only measures the local (earth) magnetic field.

Magnetometer measurement models

1. **Common use:** Magnetometer provides **orientation** heading information.

   Assume that the magnetometer (almost) only measures the local (earth) magnetic field.

2. **My use:** Magnetometer(s) to provide **position and orientation** information.

Magnetometer measurement models

1. **Common use:** Magnetometer provides **orientation** heading information.

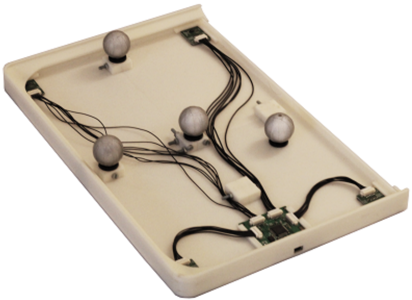
   Assume that the magnetometer (almost) only measures the local (earth) magnetic field.

2. **My use:** Magnetometer(s) to provide **position and orientation** information.

   - **Magnetic tracking:** Measure the position and orientation of a known magnetic source.

Sensor setup

We use a sensor network with four three-axis magnetometers to determine the position and orientation of a magnet.



UPPSALA UNIVERSITET

Magnetic tracking

Advantages

- Cheap sensors

Magnetic tracking

Advantages

- Cheap sensors
- Small sensors

Magnetic tracking

Advantages

- Cheap sensors
- Small sensors
- Low energy consumption

UPPSALA UNIVERSITET

Magnetic tracking

Advantages

- Cheap sensors
- Small sensors
- Low energy consumption
- No weather dependency

UPPSALA UNIVERSITET

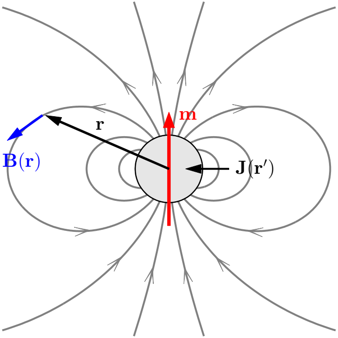
Magnetic tracking

Advantages

- Cheap sensors
- Small sensors
- Low energy consumption
- No weather dependency
- Passive unit, requires no batteries

Mathematical model - dipole field

The magnetic field can be described with a dipole field.



$$\mathbf{B}(\mathbf{r}) = \underbrace{\frac{\mu_0}{4\pi\|\mathbf{r}\|^5}\left(3\mathbf{r}\cdot\mathbf{r}^\mathsf{T} - \|\mathbf{r}\|^2 I_3\right)}_{=C(\mathbf{r})}\mathbf{m}$$

$$\mathbf{m} \triangleq \frac{1}{2}\int \mathbf{r}' \times \mathbf{J}(\mathbf{r}')d^3r'$$

UPPSALA UNIVERSITET

Sensor model - single dipole

The measurements can be described with a state-space model

$$\mathbf{x}_{k+1} = F_k \mathbf{x}_k + G_k \mathbf{w}_k, \qquad \mathbf{w}_k \sim \mathcal{N}(\mathbf{0}, Q),$$
$$\mathbf{y}_{k,j} = \mathbf{h}_j(\mathbf{x}_k) + \mathbf{e}_k, \qquad \mathbf{e}_k \sim \mathcal{N}(\mathbf{0}, R)$$

Point target sensor model (one dipole)

$$\mathbf{h}_j(\mathbf{x}_k) = C(\mathbf{r}_k - \boldsymbol{\theta}_j)\mathbf{m}_k, \qquad \mathbf{x}_k = [\mathbf{r}_k^\mathsf{T} \quad \mathbf{v}_k^\mathsf{T} \quad \mathbf{m}_k^\mathsf{T} \quad \boldsymbol{\omega}_k^\mathsf{T}]^\mathsf{T}$$
$$C(\mathbf{r}) = \frac{\mu_0}{4\pi\|\mathbf{r}\|^5}(3\mathbf{r}\mathbf{r}^\mathsf{T} - \|\mathbf{r}\|^2 I_3),$$

Measurement from a sensor network of magnetometers positioned at $\{\boldsymbol{\theta}_j\}_{j=1}^J$.

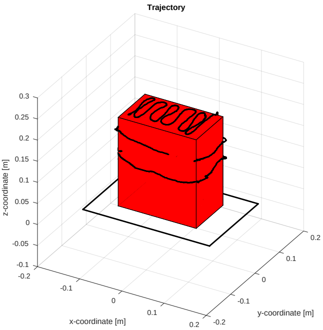
Degrees of freedom

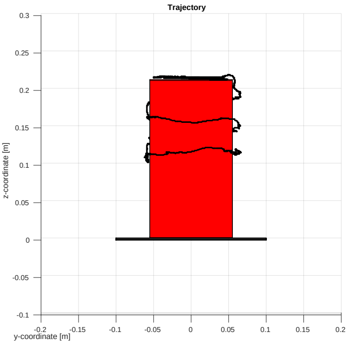
- 3D position
- 2D orientation

niklas.wahlstrom@it.uu.se

UPPSALA
UNIVERSITET

Experiment 1



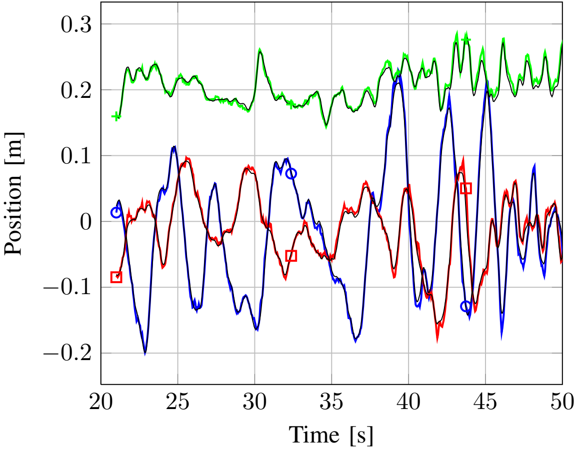


accuracy is

UPPSALA UNIVERSITET

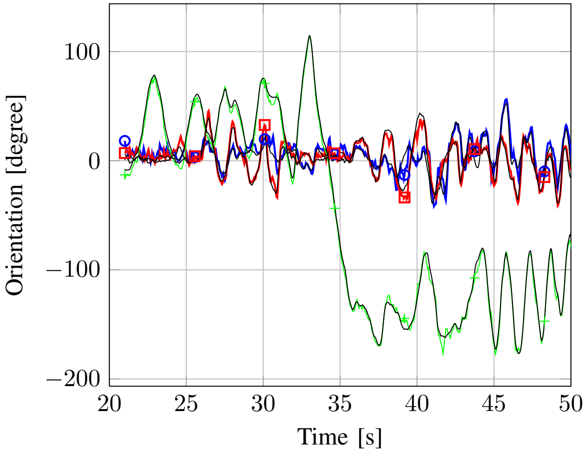
Experiment 2 - setup

Experiment 2 - results - position



Black: Ground truth position. Color: Estimated position

Experiment - results - orientation

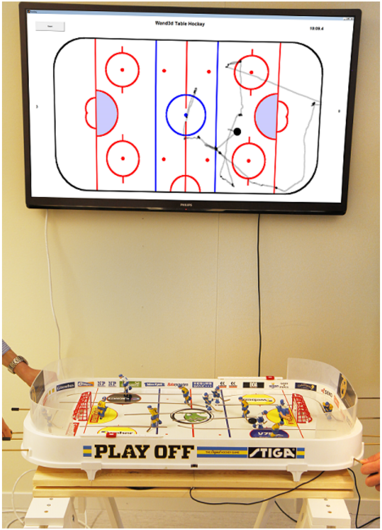


Black: Ground truth orientation. Color: Estimated orientation

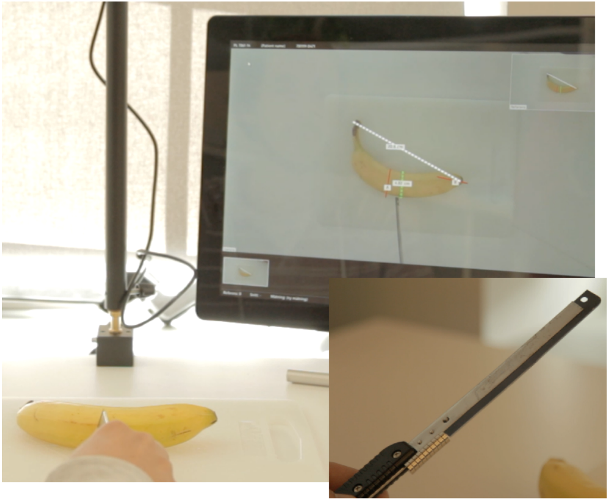
Application 1: 3D painting book

Application 2: Digital water colors

Application 3: Digital table hockey



Application 4: Digital pathology



Stylaero AB

- In February in this year a company was started around this technology
- The areas the company has so far one employee.
- Collaborations with gaming companied and industrial partners have been initiated.

STYLAERO

Thank you!

Magnetometer measurement models

1. **Common use:** Magnetometer provides **orientation** heading information.

   Assume that the magnetometer (almost) only measures the local (earth) magnetic field.

2. **My use:** Magnetometer(s) to provide **position and orientation** information.
   - **Magnetic tracking:** Measure the position and orientation of a known magnetic source.
   - **Magnetic mapping:** Build a map of the (indoor) magnetic field.

Sensor model - multi-dipole

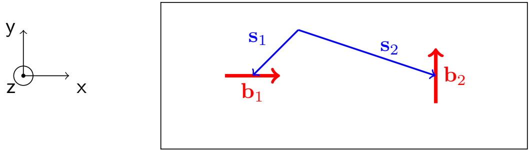
The measurements can be described with a state-space model

$$\begin{aligned} \mathbf{x}_{k+1} &= F_k \mathbf{x}_k + G_k \mathbf{w}_k, & \mathbf{w}_k &\sim \mathcal{N}(\mathbf{0}, Q), \\ \mathbf{y}_{k,j} &= \mathbf{h}_j(\mathbf{x}_k) + \mathbf{e}_k, & \mathbf{e}_k &\sim \mathcal{N}(\mathbf{0}, R) \end{aligned}$$

Extended target sensor model (a structure of dipoles)

$$\mathbf{h}_j(\mathbf{x}_k) = \sum_{l=1}^{L} C(\mathbf{r}_k + R_k(\mathbf{q}_k)\mathbf{s}_l - \boldsymbol{\theta}_j) m_l R_k(\mathbf{q}_k)\mathbf{b}_l,$$

$$\mathbf{x}_k = [\mathbf{r}_k^{\mathsf{T}} \quad \mathbf{v}_k^{\mathsf{T}} \quad \mathbf{q}_k^{\mathsf{T}} \quad \boldsymbol{\omega}_k^{\mathsf{T}}]^{\mathsf{T}}$$

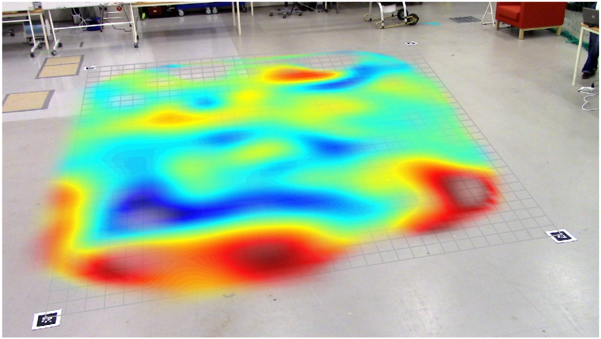


Degrees of freedom

- 3D position
- **3D** orientation

Magnetic mapping

Build a map of the indoor magnetic field. This map can be used for localization.



We want a statistical model of the magnetic field - Gaussian processes!

Gaussian processes

Gaussian processes can be seed as a distribution over functions

$$\mathbf{f}(\mathbf{u}) \sim \mathcal{GP}\big(\boldsymbol{\mu}(\mathbf{u}), K(\mathbf{u}, \mathbf{u}')\big),$$

Mean function ↑ ↑ Covariance function

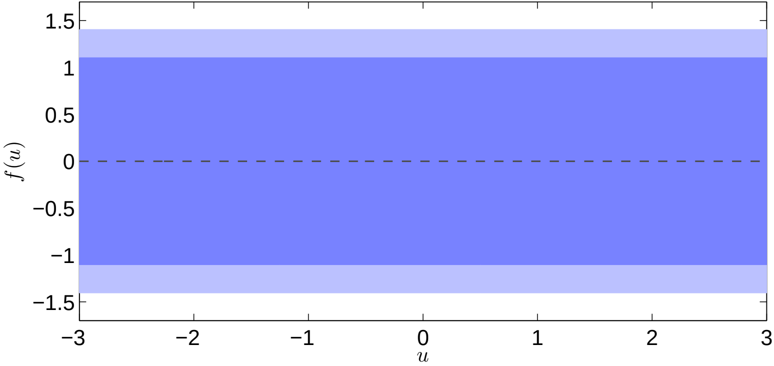
It is a generalization of the multivariate Gaussian distribution

$$\begin{bmatrix} \mathbf{f}(\mathbf{u}_1) \\ \vdots \\ \mathbf{f}(\mathbf{u}_N) \end{bmatrix} \sim \mathcal{N}(\boldsymbol{\mu}, K), \quad \text{where} \quad \boldsymbol{\mu} = \begin{bmatrix} \boldsymbol{\mu}(\mathbf{u}_1) \\ \vdots \\ \boldsymbol{\mu}(\mathbf{u}_N) \end{bmatrix},$$

$$K = \begin{bmatrix} K(\mathbf{u}_1, \mathbf{u}_1) & \cdots & K(\mathbf{u}_1, \mathbf{u}_N) \\ \vdots & & \vdots \\ K(\mathbf{u}_N, \mathbf{u}_1) & \cdots & K(\mathbf{u}_N, \mathbf{u}_N) \end{bmatrix}.$$

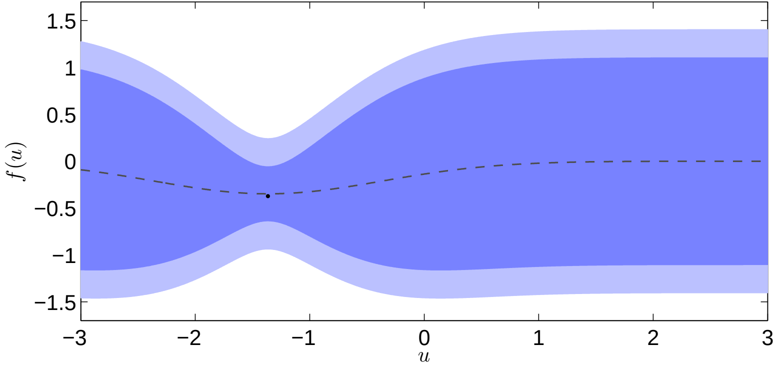
Gaussian process regression

Objective: Estimate $f(u)$ from noisy observations $y_k = f(u_k) + e_k$



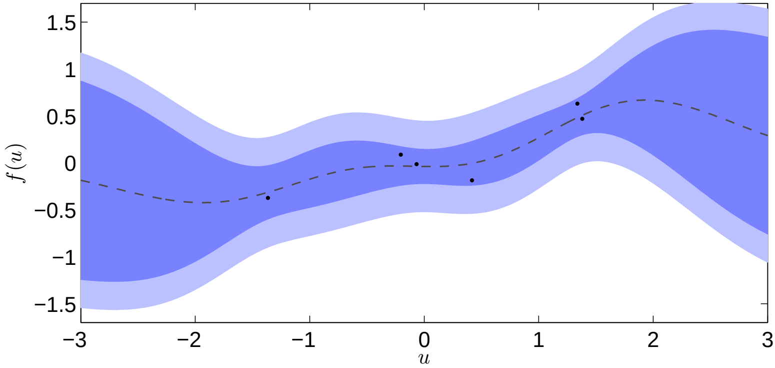
Gaussian process regression

Objective: Estimate $f(u)$ from noisy observations $y_k = f(u_k) + e_k$



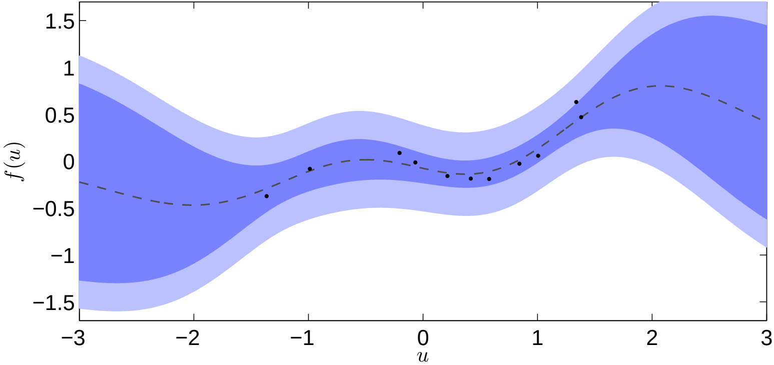
Gaussian process regression

Objective: Estimate $f(u)$ from noisy observations $y_k = f(u_k) + e_k$



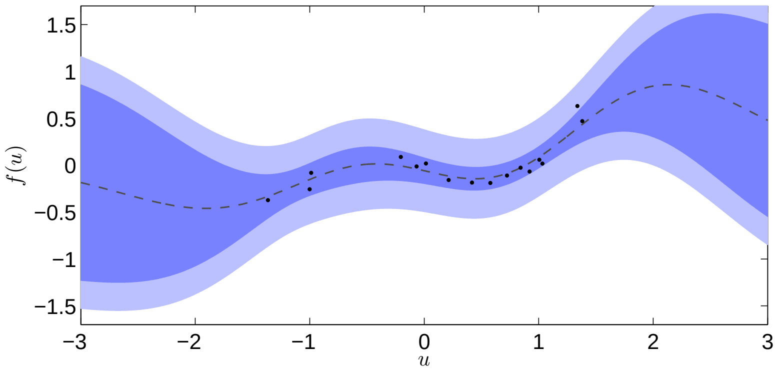
Gaussian process regression

Objective: Estimate $f(u)$ from noisy observations $y_k = f(u_k) + e_k$



Gaussian process regression

Objective: Estimate $f(u)$ from noisy observations $y_k = f(u_k) + e_k$



UPPSALA UNIVERSITET

Modeling the magnetic field

- The magnetic field $\mathbf{H}$ is curl-free, i.e. $\boldsymbol{\nabla} \times \mathbf{H} = \mathbf{0}$ [1]

$$\mathbf{y}_k = \mathbf{f}(\mathbf{x}_k) + \boldsymbol{\varepsilon}_k$$
$$\mathbf{f}(\mathbf{x}) \sim \mathcal{GP}(\mathbf{0}, \sigma^2_{\text{const.}} I_3 + K_{\text{curl}}(\mathbf{x}, \mathbf{x}'))$$

[1] Niklas Wahlström, Manon Kok, Thomas B. Schön and Fredrik Gustafsson, **Modeling magnetic fields using Gaussian processes** *The 38th International Conference on Acoustics, Speech, and Signal Processing (ICASSP)*, Vancouver, Canada, May 2013.

UPPSALA UNIVERSITET

Modeling the magnetic field

- The magnetic field $\mathbf{H}$ is curl-free, i.e. $\nabla \times \mathbf{H} = \mathbf{0}$ [1]

$$\mathbf{y}_k = \mathbf{f}(\mathbf{x}_k) + \boldsymbol{\varepsilon}_k$$
$$\mathbf{f}(\mathbf{x}) \sim \mathcal{GP}(\mathbf{0}, \sigma^2_{\text{const.}} I_3 + K_{\text{curl}}(\mathbf{x}, \mathbf{x}'))$$

- If a vector-field is curl-free, a scalar potential φ exists $\mathbf{H} = -\nabla \varphi$ [2]

$$\mathbf{y}_k = -\nabla \varphi(\mathbf{x})\big|_{\mathbf{x}=\mathbf{x}_i} + \boldsymbol{\varepsilon}_k$$
$$\varphi(\mathbf{x}) \sim \mathcal{GP}\left(0,\ k_{\text{lin.}}(\mathbf{x}, \mathbf{x}') + k_{\text{SE}}(\mathbf{x}, \mathbf{x}')\right)$$

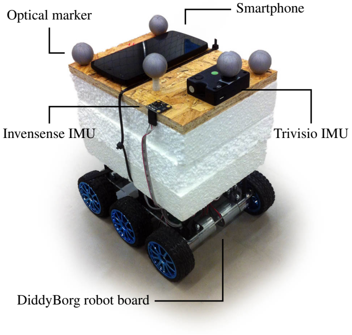
[1] Niklas Wahlström, Manon Kok, Thomas B. Schön and Fredrik Gustafsson, **Modeling magnetic fields using Gaussian processes** *The 38th International Conference on Acoustics, Speech, and Signal Processing (ICASSP)*, Vancouver, Canada, May 2013.
[2] Arno Solin, Manon Kok, Niklas Wahlström, Thomas B. Schön and Simo Särkkä, **Modeling and interpolation of the ambient magnetic field by Gaussian processes** *IEEE Transactions on Robotics*, 2017. Accepted.

Experiment

Build a map of the indoor magnetic field using Gaussian processes.

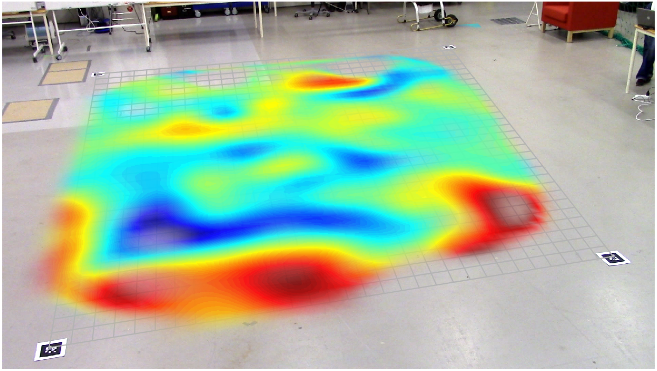
Experiment

Build a map of the indoor magnetic field using Gaussian processes.



Experiment

Build a map of the indoor magnetic field using Gaussian processes.



https://www.youtube.com/watch?v=enlMiUqPVJo

Reduced-Rank GPR

Hilbert-space approximation of the covariance operator in terms of an eigenfunction expansion of the Laplace operator in a compact subset of $\mathbb{R}^d$.

- Assume that the measurements are confined to a certain domain.
- Approximate the covariance using the spectral density and a number of eigenvalues and eigenfunctions. For $d = 1$:

$$k(x, x') \approx \sum_{j=1}^{m} S(\lambda_j)\phi_j(x)\phi_j(x')$$

$$\phi_j(x) = \tfrac{1}{\sqrt{L}} \sin\left(\tfrac{\pi n_j (x+L)}{2L}\right), \qquad \lambda_j = \tfrac{\pi j}{2L},$$

- Converges to the true GP when the number of basis functions and the size of the domain goes to infinity.

Hilbert Space Methods for Reduced-Rank Gaussian Process Regression – Arno Solin and Simo Särkkä (http://arxiv.org/pdf/1401.5508v1.pdf)

Reduced-Rank GPR

Consequences for our problem:

Original formulation:

50 or 100 Hz magnetometer data (in 3D)
$\Rightarrow$ Size of the matrix to invert grows very quickly with each additional second of data
$\Rightarrow$ Downsampling needed and large buildings become infeasible

Reduced-rank formulation:

Possible to use all data
$\Rightarrow$ Size of the problem does not grow for longer data sets

 niklas.wahlstrom@it.uu.se

UPPSALA UNIVERSITET

Sequential updating

Initialize $\mu_0 = 0$ and $\Sigma_0 = \Lambda_\theta$ (from the GP prior). For each new observation $i = 1, 2, \dots, n$ update the estimate according to

$$
\begin{aligned}
S_i &= \nabla\Phi_i \Sigma_{i-1} [\nabla\Phi_i]^\mathsf{T} + \sigma^2_{\text{noise}} \mathcal{I}_3, \\
K_i &= \Sigma_{i-1} [\nabla\Phi_i]^\mathsf{T} S_i^{-1}, \\
\mu_i &= \mu_{i-1} + K_i (y_i - \nabla\Phi_i \mu_{i-1}), \\
\Sigma_i &= \Sigma_{i-1} - K_i S_i K_i^\mathsf{T}.
\end{aligned}
$$

UPPSALA UNIVERSITET

Spatio-temporal modeling

Model the scalar potential magnetic field instead as

$$\varphi(x,t) \sim \mathcal{GP}(0, \kappa_{\text{lin.}}(x,x') + \kappa_{\text{SE}}(x,x')\kappa_{\text{exp}}(t,t')),$$

with

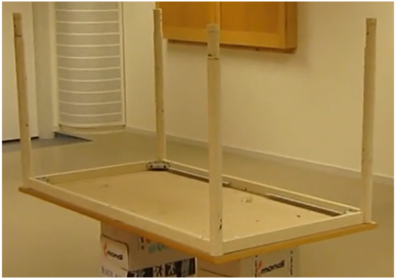
$$\kappa_{\text{exp}}(t,t') = \exp\left(-\frac{|t-t'|}{\ell_{\text{time}}} \right).$$

The scalar potential can then sequentially be estimated by adding a *time update* to the *measurement update* from before as

$$\begin{aligned}\tilde{\mu}_i &= A_{i-1}\mu_{i-1},\\ \tilde{\Sigma}_i &= A_{i-1}\Sigma_{i-1}A_{i-1}^{\mathsf{T}} + Q_{i-1}.\end{aligned}$$

 niklas.wahlstrom@it.uu.se

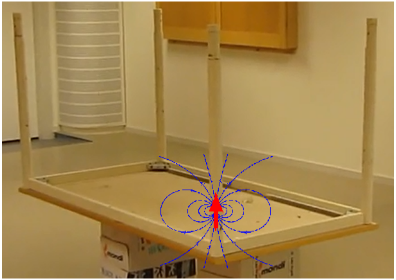
Problem formulation



How should the map be modeled?

We want to find a magnetic map of this object!

Problem formulation

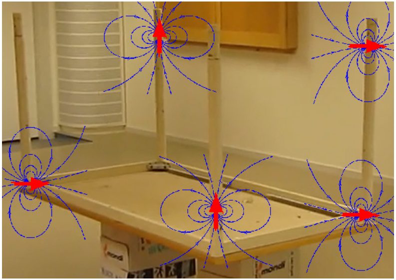


How should the map be modeled?

- Use the dipole model?

We want to find a magnetic map of this object!

Problem formulation

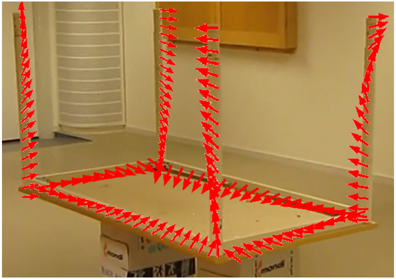


How should the map be modeled?

- Use the dipole model?
- Use multiple dipoles?

We want to find a magnetic map of this object!

Problem formulation

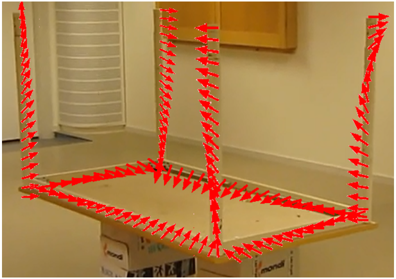


How should the map be modeled?

- Use the dipole model?
- Use multiple dipoles?
- Use a continuum of dipoles!

We want to find a magnetic map of this object!

Problem formulation

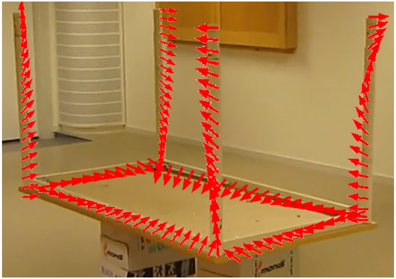


We want to find a magnetic map of this object!

How should the map be modeled?

- Use the dipole model?
- Use multiple dipoles?
- Use a continuum of dipoles!

Problem formulation

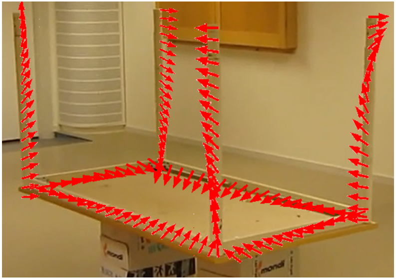


We want to find a magnetic map of this object!

How should the map be modeled?

- Use the dipole model?
- Use multiple dipoles?
- Use a continuum of dipoles!
- Spatial correlation

Problem formulation



How should the map be modeled?

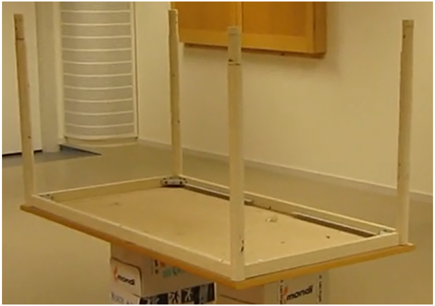
- Use the dipole model?
- Use multiple dipoles?
- Use a continuum of dipoles!
- Spatial correlation

We want to find a magnetic map of this object!

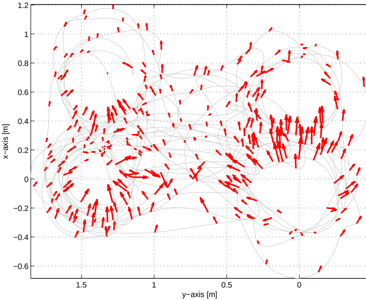
niklas.wahlstrom@it.uu.se

Real world experiment

- Measurements have been collected with a magnetometer
- An optical reference system (Vicon) has been used for determining the position and orientation of the sensor



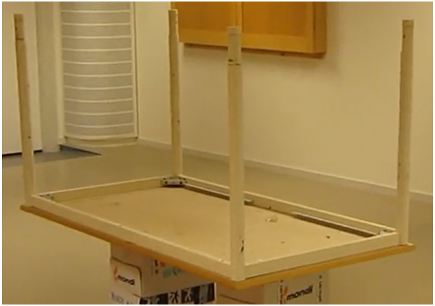
The magnetic environment



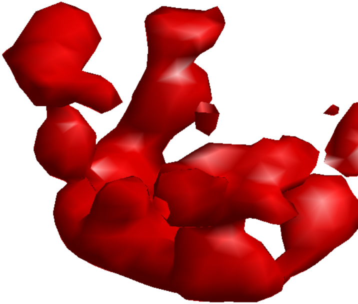
Training data

Real world experiment

- Measurements have been collected with a magnetometer
- An optical reference system (Vicon) has been used for determining the position and orientation of the sensor



The magnetic environment



Estimated magnetic content

UPPSALA UNIVERSITET

Modeling the magnetic field

- The magnetic field $\mathbf{H}$ is curl-free, i.e. $\nabla \times \mathbf{H} = \mathbf{0}$ [1]

$$\mathbf{y}_k = \mathbf{f}(\mathbf{x}_k) + \boldsymbol{\varepsilon}_k$$
$$\mathbf{f}(\mathbf{x}) \sim \mathcal{GP}(\mathbf{0}, \sigma^2_{\text{const.}} I_3 + K_{\text{curl}}(\mathbf{x}, \mathbf{x}'))$$

[1] Niklas Wahlström, Manon Kok, Thomas B. Schön and Fredrik Gustafsson, **Modeling magnetic fields using Gaussian processes** *The 38th International Conference on Acoustics, Speech, and Signal Processing (ICASSP)*, Vancouver, Canada, May 2013.

UPPSALA UNIVERSITET

Modeling the magnetic field

- The magnetic field $\mathbf{H}$ is curl-free, i.e. $\boldsymbol{\nabla} \times \mathbf{H} = \mathbf{0}$ [1]

$$\mathbf{y}_k = \mathbf{f}(\mathbf{x}_k) + \boldsymbol{\varepsilon}_k$$
$$\mathbf{f}(\mathbf{x}) \sim \mathcal{GP}(\mathbf{0}, \sigma^2_{\text{const.}} I_3 + K_{\text{curl}}(\mathbf{x}, \mathbf{x}'))$$

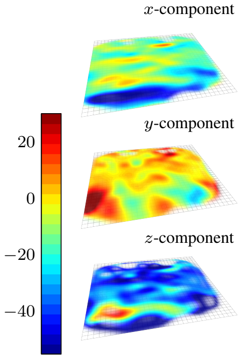
- If a vector-field is curl-free, a scalar potential φ exists $\mathbf{H} = -\boldsymbol{\nabla}\varphi$ [2]

$$\mathbf{y}_k = -\boldsymbol{\nabla}\varphi(\mathbf{x})\Big|_{\mathbf{x}=\mathbf{x}_i} + \boldsymbol{\varepsilon}_k$$
$$\varphi(\mathbf{x}) \sim \mathcal{GP}\left(0,\ k_{\text{lin.}}(\mathbf{x}, \mathbf{x}') + k_{\text{SE}}(\mathbf{x}, \mathbf{x}')\right)$$

[1] Niklas Wahlström, Manon Kok, Thomas B. Schön and Fredrik Gustafsson, **Modeling magnetic fields using Gaussian processes** *The 38th International Conference on Acoustics, Speech, and Signal Processing (ICASSP)*, Vancouver, Canada, May 2013.
[2] Arno Solin, Manon Kok, Niklas Wahlström, Thomas B. Schön and Simo Särkkä, **Modeling and interpolation of the ambient magnetic field by Gaussian processes** *ArXiv e-prints*, September 2015. arXiv:1509.04634.

Building magnetic field maps (2)

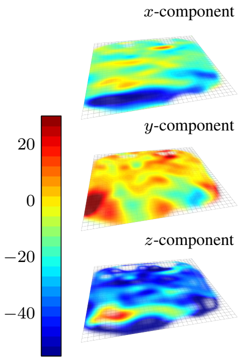
- Encode physical knowledge in the kernel.



niklas.wahlstrom@it.uu.se

Building magnetic field maps (2)

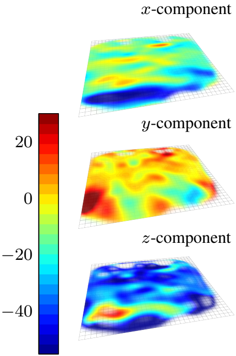
- Encode physical knowledge in the kernel.
- Use reduced-rank GP regression based on the method from [1].



[1] Hilbert Space Methods for Reduced-Rank Gaussian Process Regression – A. Solin, S. Särkkä.

Building magnetic field maps (2)

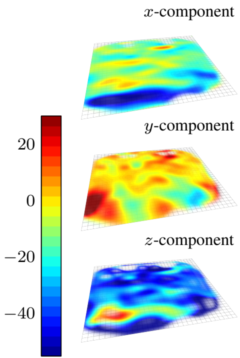
- Encode physical knowledge in the kernel.
- Use reduced-rank GP regression based on the method from [1].
- Use a Kalman filter formulation to allow for sequential updating.



[1] Hilbert Space Methods for Reduced-Rank Gaussian Process Regression – A. Solin, S. Särkkä.

Building magnetic field maps (2)

- Encode physical knowledge in the kernel.
- Use reduced-rank GP regression based on the method from [1].
- Use a Kalman filter formulation to allow for sequential updating.
- Use a spatio-temporal model to allow for changes in the magnetic field.



[1] Hilbert Space Methods for Reduced-Rank Gaussian Process Regression – A. Solin, S. Särkkä.

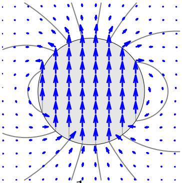
Magnetic fields

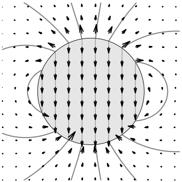
We use a slightly different version of the magnetostatic equations

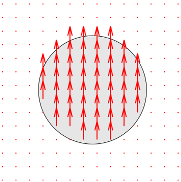
$$\nabla \cdot \mathbf{B} = 0, \qquad \frac{1}{\mu_0}\mathbf{B} - \mathbf{H} = \mathbf{M},$$

$$\nabla \times \mathbf{H} = \mathbf{0}$$

Example







$$\frac{1}{\mu_0}\mathbf{B} \quad - \quad \mathbf{H} \quad = \quad \mathbf{M}$$

Gaussian process + magnetic fields

- The animation illustrated regression for one scalar function $f : \mathbb{R} \to \mathbb{R}$

Gaussian process + magnetic fields

- The animation illustrated regression for one scalar function $f : \mathbb{R} \rightarrow \mathbb{R}$
- We want to learn three different vector fields $\mathbf{f} : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ In addition, these fields should obey

Gaussian process + magnetic fields

- The animation illustrated regression for one scalar function $f : \mathbb{R} \to \mathbb{R}$
- We want to learn three different vector fields $\mathbf{f} : \mathbb{R}^3 \to \mathbb{R}^3$ In addition, these fields should obey
  - $\mathbf{\nabla} \cdot \mathbf{B} = 0$ (divergence free)
  - $\mathbf{\nabla} \times \mathbf{H} = 0$ (curl free)

UPPSALA UNIVERSITET

Gaussian process + magnetic fields

- The animation illustrated regression for one scalar function $f : \mathbb{R} \to \mathbb{R}$
- We want to learn three different vector fields $\mathbf{f} : \mathbb{R}^3 \to \mathbb{R}^3$ In addition, these fields should obey
  - $\boldsymbol{\nabla} \cdot \mathbf{B} = 0$ (divergence free)
  - $\boldsymbol{\nabla} \times \mathbf{H} = 0$ (curl free)

$\leftarrow$ There exist covariance functions for this!

UPPSALA UNIVERSITET

Gaussian process + magnetic fields

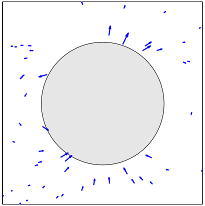
- The animation illustrated regression for one scalar function $f : \mathbb{R} \rightarrow \mathbb{R}$
- We want to learn three different vector fields $\mathbf{f} : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ In addition, these fields should obey
  - $\boldsymbol{\nabla} \cdot \mathbf{B} = 0$ (divergence free)
  - $\boldsymbol{\nabla} \times \mathbf{H} = 0$ (curl free)
  - $\frac{1}{\mu_0}\mathbf{B} - \mathbf{H} = \mathbf{M}$

$\leftarrow$ There exist covariance functions for this!

UPPSALA UNIVERSITET

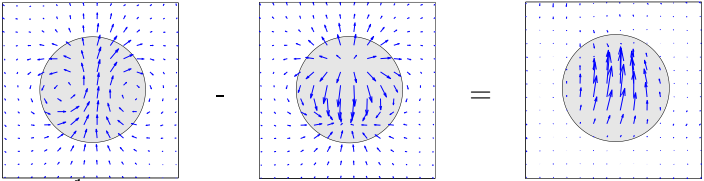
Simulation

Training data



Predictions

Divergence free — Curl free



$$\frac{1}{\mu_0}\mathbf{B} - \mathbf{H} = \mathbf{M}$$