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Modeling magnetic fields using Gaussian processes

Niklas Wahlström

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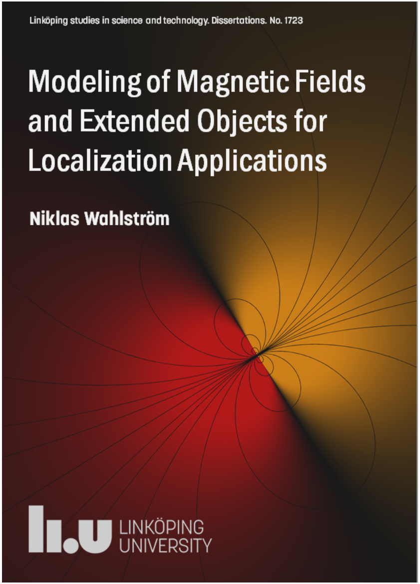
Short about me

- 2005 - 2010: Applied Physics and Electrical Engineering - International, Linköping University.
    - 2007-2008: Exchange student, ETH Zürich, Swizerland
- 2010-2015 : PhD student in Automatic Control, Linköping University
    - Spring 2014, Research visit, Imperial College, London, UK
- *2016- : Researcher at Department of Information Technology, Uppsala University*

My thesis



Three areas:

- **Magnetic tracking and mapping**
- Extended target tracking
- Deep dynamical models for control

Magnetometer measurement models

1. **Common use:** Magnetometer provides **orientation** heading information.

    Assume that the magnetometer (almost) only measures the local (earth) magnetic field.

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Magnetic Tracking - Application

Track position and orientation of a small magnet using a sensor network of magnetometers

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https://www.youtube.com/watch?v=ACFqeFyjq4Y

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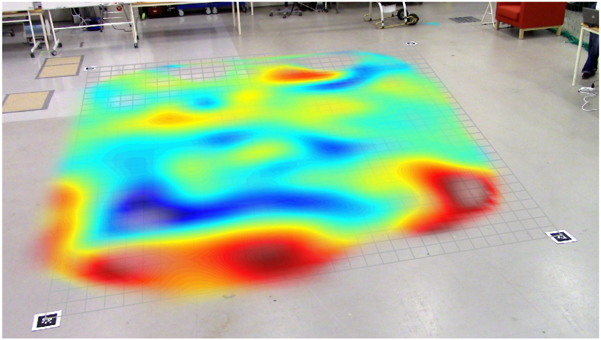
2. **My use:** Magnetometer(s) to provide **position and orientation** information.
   a. **Magnetic tracking:** Measure the position and orientation of a known magnetic source.
   b. **Magnetic mapping:** Build a map of the (indoor) magnetic field.

Magnetic mapping

Build a map of the indoor magnetic field using Gaussian processes.



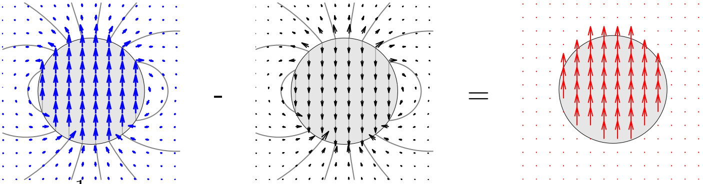
Magnetic fields

We use a slightly different version of the magnetostatic equations

$$\nabla \cdot \mathbf{B} = 0, \qquad \frac{1}{\mu_0}\mathbf{B} - \mathbf{H} = \mathbf{M},$$

$$\nabla \times \mathbf{H} = \mathbf{0}$$

Example



$\frac{1}{\mu_0}\mathbf{B}$ - $\mathbf{H}$ = $\mathbf{M}$

Gaussian processes

Gaussian processes can be seed as a distribution over functions

$$\mathbf{f}(\mathbf{u}) \sim \mathcal{GP}\big(\boldsymbol{\mu}(\mathbf{u}), K(\mathbf{u}, \mathbf{u}')\big),$$

Mean function ↑ ↑ Covariance function

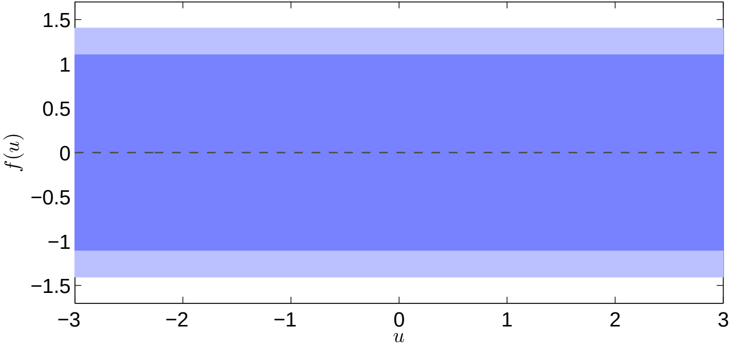
It is a generalization of the multivariate Gaussian distribution

$$\begin{bmatrix} \mathbf{f}(\mathbf{u}_1) \\ \vdots \\ \mathbf{f}(\mathbf{u}_N) \end{bmatrix} \sim \mathcal{N}(\boldsymbol{\mu}, K), \quad \text{where} \quad \boldsymbol{\mu} = \begin{bmatrix} \boldsymbol{\mu}(\mathbf{u}_1) \\ \vdots \\ \boldsymbol{\mu}(\mathbf{u}_N) \end{bmatrix},$$

$$K = \begin{bmatrix} K(\mathbf{u}_1, \mathbf{u}_1) & \cdots & K(\mathbf{u}_1, \mathbf{u}_N) \\ \vdots & & \vdots \\ K(\mathbf{u}_N, \mathbf{u}_1) & \cdots & K(\mathbf{u}_N, \mathbf{u}_N) \end{bmatrix}.$$

Gaussian process regression

Objective: Estimate $f(u)$ from noisy observations $y_k = f(u_k) + e_k$



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Gaussian process + magnetic fields

- The animation illustrated regression for one scalar function $f : \mathbb{R} \to \mathbb{R}$

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Gaussian process + magnetic fields

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- We want to learn three different vector fields $\mathbf{f} : \mathbb{R}^3 \to \mathbb{R}^3$ In addition, these fields should obey

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  - $\boldsymbol{\nabla} \cdot \mathbf{B} = 0$ (divergence free)
  - $\boldsymbol{\nabla} \times \mathbf{H} = 0$ (curl free)

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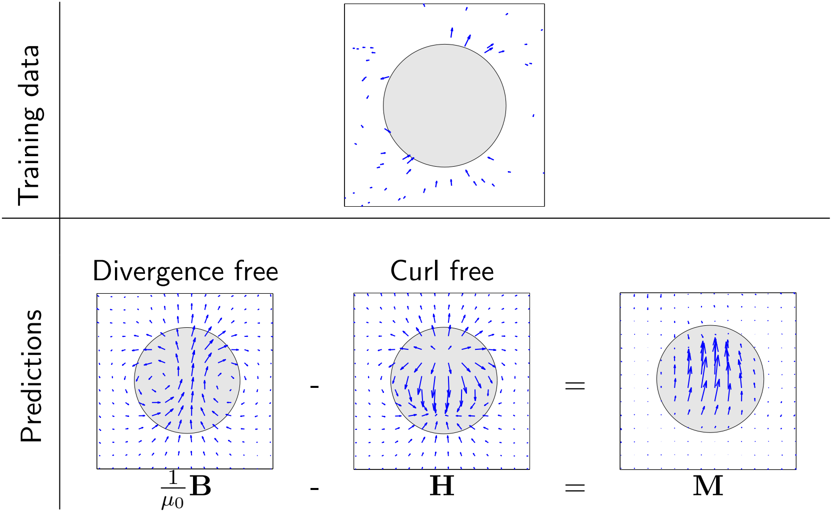
Simulation


Training data
Predictions
Divergence free
Curl free
$\frac{1}{\mu_0}\mathbf{B}$ - $\mathbf{H}$ = $\mathbf{M}$

Building magnetic field maps (1)

Build a map of the indoor magnetic field using Gaussian processes.

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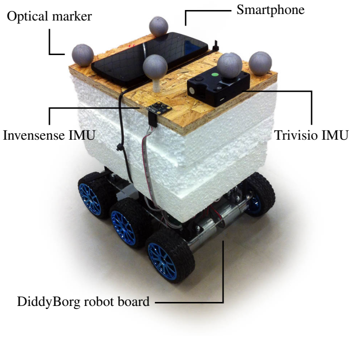

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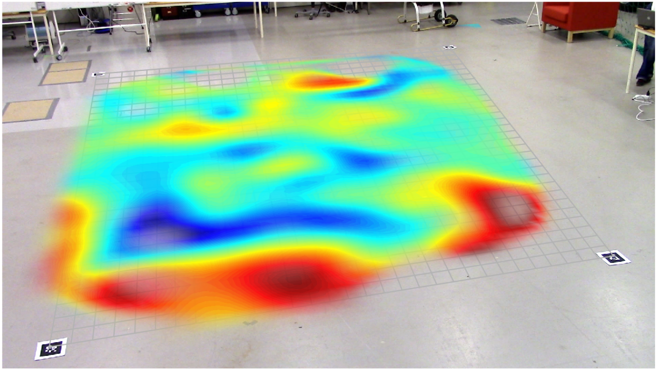

https://www.youtube.com/watch?v=enlMiUqPVJo

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Modeling the magnetic field

- The magnetic field $\mathbf{H}$ is curl-free, i.e. $\boldsymbol{\nabla} \times \mathbf{H} = \mathbf{0}$ [1]

$$\mathbf{y}_k = \mathbf{f}(\mathbf{x}_k) + \boldsymbol{\varepsilon}_k$$
$$\mathbf{f}(\mathbf{x}) \sim \mathcal{GP}(\mathbf{0}, \sigma^2_{\text{const.}} I_3 + K_{\text{curl}}(\mathbf{x}, \mathbf{x}'))$$

[1] Niklas Wahlström, Manon Kok, Thomas B. Schön and Fredrik Gustafsson, **Modeling magnetic fields using Gaussian processes** *The 38th International Conference on Acoustics, Speech, and Signal Processing (ICASSP)*, Vancouver, Canada, May 2013.

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- If a vector-field is curl-free, a scalar potential φ exists $\mathbf{H} = -\nabla\varphi$ [2]

$$\mathbf{y}_k = -\nabla\varphi(\mathbf{x})\Big|_{\mathbf{x}=\mathbf{x}_i} + \boldsymbol{\varepsilon}_k$$
$$\varphi(\mathbf{x}) \sim \mathcal{GP}\left(0,\ k_{\text{lin.}}(\mathbf{x}, \mathbf{x}') + k_{\text{SE}}(\mathbf{x}, \mathbf{x}')\right)$$

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[2] Arno Solin, Manon Kok, Niklas Wahlström, Thomas B. Schön and Simo Särkkä, **Modeling and interpolation of the ambient magnetic field by Gaussian processes** *ArXiv e-prints*, September 2015. arXiv:1509.04634.

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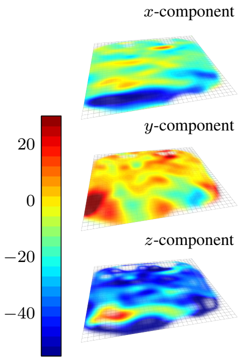
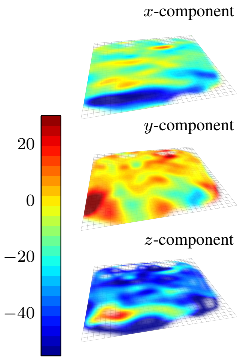
Building magnetic field maps (2)

- Encode physical knowledge in the kernel.



Building magnetic field maps (2)

- Encode physical knowledge in the kernel.
- Use reduced-rank GP regression based on the method from [1].



[1] Hilbert Space Methods for Reduced-Rank Gaussian Process Regression – A. Solin, S. Särkkä.

Building magnetic field maps (2)

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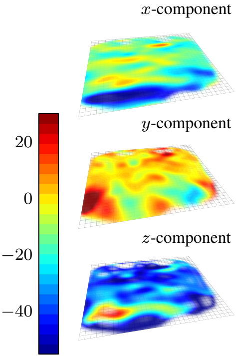
- Use a Kalman filter formulation to allow for sequential updating.



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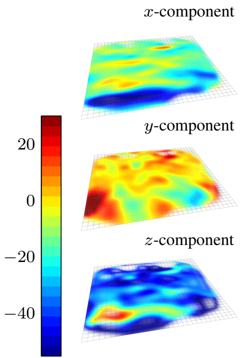
Building magnetic field maps (2)

- Encode physical knowledge in the kernel.
- Use reduced-rank GP regression based on the method from [1].
- Use a Kalman filter formulation to allow for sequential updating.
- Use a spatio-temporal model to allow for changes in the magnetic field.



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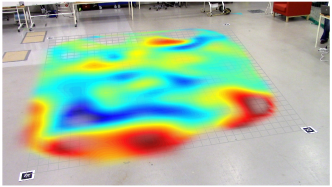
Localization in the map



Building the map:

Arno Solin, Manon Kok, Niklas Wahlström, Thomas B. Schön and Simo Särkkä, **Modeling and interpolation of the ambient magnetic field by Gaussian processes** *ArXiv e-prints*, September 2015. arXiv:1509.04634.

Localization in the map:

Arno Solin, Simo Särkkä, Juho Kannala, and Esa Rahtu. **Terrain navigation in the magnetic landscape: Particle filtering for indoor positioning** *In Proceedings of the European Navigation Conference*, Helsinki, Finland, May–June 2016.

SLAM: . . . future work.

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Summary

- **Mapping magnetic fields** using Gaussian processes

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- We employ covariance functions which **encode the known physical constraints**

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Summary

- **Mapping magnetic fields** using Gaussian processes
- We employ covariance functions which **encode the known physical constraints**
- Magnetic map can be used for **indoor navigation**

Thank you!

Reduced-Rank GPR

Hilbert-space approximation of the covariance operator in terms of an eigenfunction expansion of the Laplace operator in a compact subset of $\mathbb{R}^d$.

- Assume that the measurements are confined to a certain domain.
- Approximate the covariance using the spectral density and a number of eigenvalues and eigenfunctions. For $d = 1$:

$$k(x, x') \approx \sum_{j=1}^{m} S(\lambda_j)\phi_j(x)\phi_j(x')$$

$$\phi_j(x) = \tfrac{1}{\sqrt{L}} \sin\left(\tfrac{\pi n_j(x+L)}{2L}\right), \qquad \lambda_j = \tfrac{\pi j}{2L},$$

- Converges to the true GP when the number of basis functions and the size of the domain goes to infinity.

Hilbert Space Methods for Reduced-Rank Gaussian Process Regression – Arno Solin and Simo Särkkä (http://arxiv.org/pdf/1401.5508v1.pdf)

Reduced-Rank GPR

Consequences for our problem:

Original formulation:

50 or 100 Hz magnetometer data (in 3D)
$\Rightarrow$ Size of the matrix to invert grows very quickly with each additional second of data
$\Rightarrow$ Downsampling needed and large buildings become infeasible

Reduced-rank formulation:

Possible to use all data
$\Rightarrow$ Size of the problem does not grow for longer data sets

Sequential updating

Initialize $\mu_0 = 0$ and $\Sigma_0 = \Lambda_\theta$ (from the GP prior). For each new observation $i = 1, 2, \ldots, n$ update the estimate according to

$$
\begin{aligned}
S_i &= \nabla\Phi_i \Sigma_{i-1} [\nabla\Phi_i]^\mathsf{T} + \sigma^2_{\text{noise}} \mathcal{I}_3, \\
K_i &= \Sigma_{i-1} [\nabla\Phi_i]^\mathsf{T} S_i^{-1}, \\
\mu_i &= \mu_{i-1} + K_i (y_i - \nabla\Phi_i \mu_{i-1}), \\
\Sigma_i &= \Sigma_{i-1} - K_i S_i K_i^\mathsf{T}.
\end{aligned}
$$

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Spatio-temporal modeling

Model the scalar potential magnetic field instead as

$$\varphi(x,t) \sim \mathcal{GP}(0, \kappa_{\text{lin.}}(x,x') + \kappa_{\text{SE}}(x,x')\kappa_{\text{exp}}(t,t')),$$

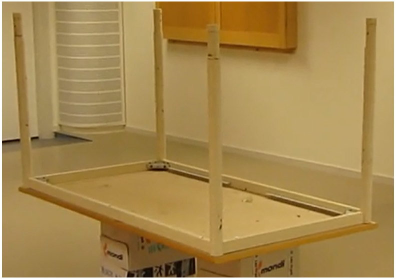
with

$$\kappa_{\text{exp}}(t,t') = \exp\left(-\frac{|t-t'|}{\ell_{\text{time}}}\right).$$

The scalar potential can then sequentially be estimated by adding a *time update* to the *measurement update* from before as

$$\tilde{\mu}_i = A_{i-1}\mu_{i-1},$$
$$\tilde{\Sigma}_i = A_{i-1}\Sigma_{i-1}A_{i-1}^{\mathsf{T}} + Q_{i-1}.$$

Problem formulation



How should the map be modeled?

We want to find a magnetic map of this object!

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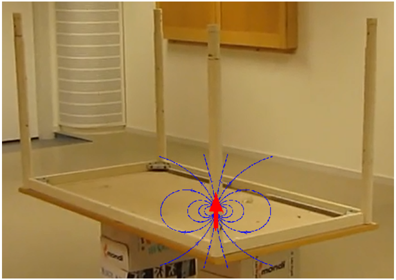

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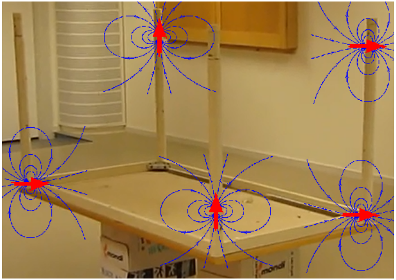

How should the map be modeled?

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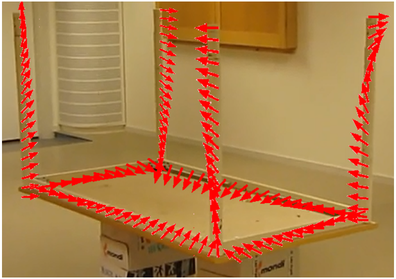

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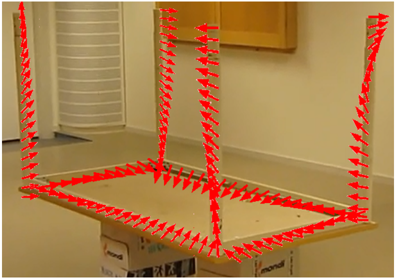

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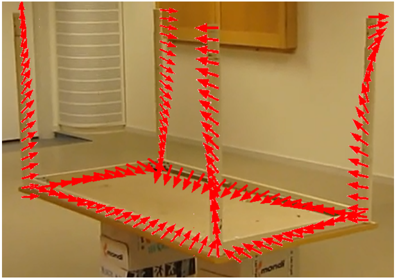

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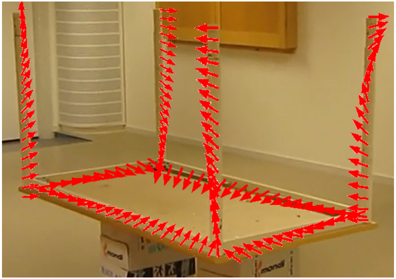

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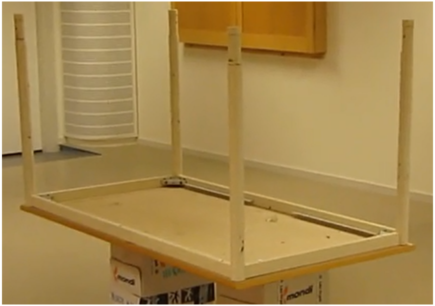
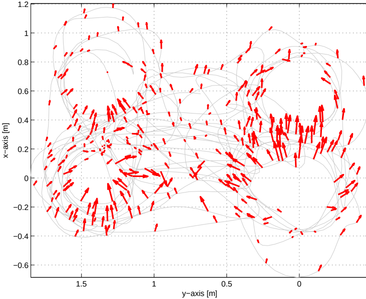
Real world experiment

- Measurements have been collected with a magnetometer
- An optical reference system (Vicon) has been used for determining the position and orientation of the sensor



The magnetic environment



Training data

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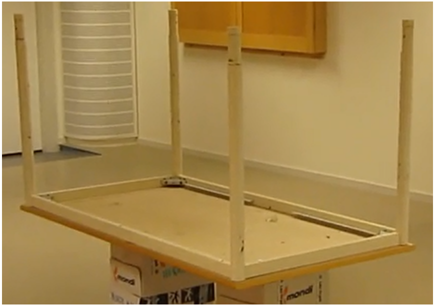

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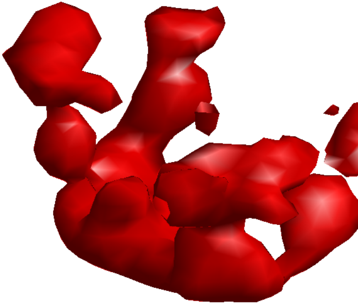

Estimated magnetic content